Sum/Difference, Double Angle and Half-Angle Practice

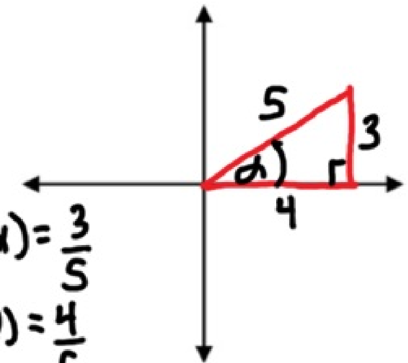
Given $\sin\alpha = \frac{3}{5}$, $0 < \alpha < \frac{\pi}{2}$, $\cos\beta = -\frac{5}{13}$, $\pi < \beta < \frac{3\pi}{2}$, find

- a. $\sin(\alpha + \beta)$
- b. $\cos(\alpha + \beta)$
- c. $\tan(\alpha + \beta)$

a) $\sin(\alpha)\cos(\beta) - \sin(\beta)\cos(\alpha)$

$$\left(\frac{3}{5}\right)\cdot\left(-\frac{5}{13}\right) - \left(-\frac{12}{13}\right)\cdot\left(\frac{4}{5}\right)$$

$$\frac{-15}{65} + \frac{48}{65} = \boxed{\frac{33}{65}}$$



$\sin(\alpha) = \frac{3}{5}$

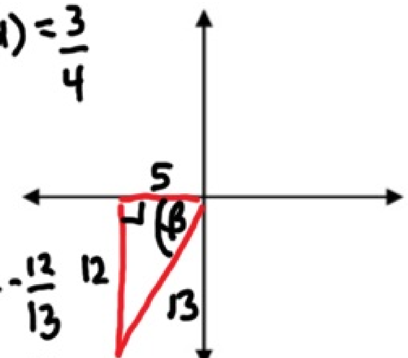
$\cos(\alpha) = \frac{4}{5}$

$\tan(\alpha) = \frac{3}{4}$

b) $\cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$$\left(\frac{4}{5}\right)\left(-\frac{5}{13}\right) - \left(\frac{3}{5}\right)\left(-\frac{12}{13}\right)$$

$$-\frac{20}{65} + \frac{36}{65} = \boxed{\frac{16}{65}}$$



$\sin(\beta) = -\frac{12}{13}$

$\cos(\beta) = -\frac{5}{13}$

$\tan(\beta) = \frac{12}{5}$

c) $\dfrac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)} \rightarrow \dfrac{\left(\frac{3}{4}\right) + \left(\frac{12}{5}\right)}{1 - \left(\frac{3}{4}\right)\left(\frac{12}{5}\right)}$

$$\rightarrow \frac{\frac{63}{20}}{-\frac{4}{5}} \rightarrow \boxed{-\frac{63}{15}}$$

Given that $\tan\theta = -\frac{3}{4}$ and θ is in quadrant II, find the following:

- a. $\sin(2\theta)$
- b. $\cos(2\theta)$
- c. $\tan(2\theta)$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = \boxed{-\frac{24}{25}}$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\left(-\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

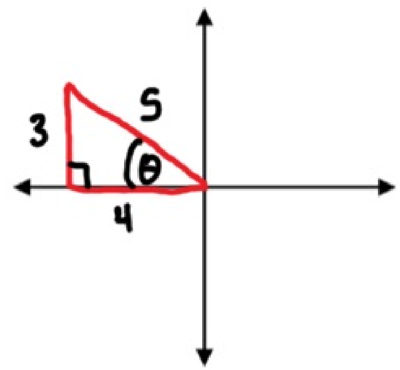
$$\frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$$

$$= \frac{2\left(-\frac{3}{4}\right)}{1-\left(-\frac{3}{4}\right)^2}$$

$$\frac{-\frac{3}{2}}{\frac{7}{16}}$$

$$\boxed{-\frac{24}{7}}$$



$$\sin(\theta) = \frac{3}{5}$$

$$\cos(\theta) = -\frac{4}{5}$$

$$\tan(\theta) = -\frac{3}{4}$$

Prove the following identity

$$\boxed{\frac{\sin(x-y)}{\cos x \cos y}} = \tan x - \tan y.$$

$$\frac{\sin(x)\cos(y) - \sin(y)\cos(x)}{\cos(x)\cos(y)}$$

$$\frac{\sin(x)\cancel{\cos(y)}}{\cos(x)\cancel{\cos(y)}} - \frac{\sin(y)\cancel{\cos(x)}}{\cancel{\cos(x)}\cos(y)}$$

$$\frac{\sin(x)}{\cos(x)} - \frac{\sin(y)}{\cos(y)}$$

$$\boxed{\tan(x) - \tan(y)}$$

Prove the following identity

$$\boxed{\sin(x+y)\cdot\sin(x-y)} = \sin^2 x - \sin^2 y$$

$$[S(x)C(y) + S(y)C(x)][S(x)C(y) - S(y)C(x)]$$

$$S^2(x)C^2(y) - \cancel{S(x)C(y)S(y)C(x)} + \cancel{S(x)C(y)S(y)C(x)} - S^2(y)C^2(x)$$

$$S^2(x)C^2(y) - S^2(y)C^2(x)$$

$$S^2(x)[1 - S^2(y)] - S^2(y)[1 - S^2(x)]$$

$$S^2(x) - \cancel{S^2(x)S^2(y)} - S^2(y) + \cancel{S^2(x)S^2(y)}$$

$$\boxed{\sin^2(x) - \sin^2(y)}$$

Prove the following identity

$$\cos(\alpha-\beta)-\cos(\alpha+\beta)=2\sin\alpha\cdot\sin\beta$$

$$c(\alpha)c(\beta)+s(\alpha)s(\beta)-[c(\alpha)c(\beta)-s(\alpha)s(\beta)]$$

$$\cancel{c(\alpha)c(\beta)}+s(\alpha)s(\beta)-\cancel{c(\alpha)c(\beta)}+s(\alpha)s(\beta)$$

$$\boxed{2\sin(\alpha)\cos(\beta)}$$

Prove the following identity

$$\sin\alpha+\sin\left(\alpha+\frac{2}{3}\pi\right)+\sin\left(\alpha+\frac{4}{3}\pi\right)=0$$

$$s(\alpha)+s(\alpha)c\left(\tfrac{2\pi}{3}\right)-s\left(\tfrac{2\pi}{3}\right)c(\alpha)+s(\alpha)c\left(\tfrac{4\pi}{3}\right)-s\left(\tfrac{4\pi}{3}\right)c(\alpha)$$

$$s(\alpha)+s(\alpha)\left(-\tfrac{1}{2}\right)-\left(\tfrac{\sqrt{3}}{2}\right)c(\alpha)+s(\alpha)\left(-\tfrac{1}{2}\right)-\left(-\tfrac{\sqrt{3}}{2}\right)c(\alpha)$$

$$s(\alpha)\underbrace{-\tfrac{1}{2}s(\alpha)-\cancel{\tfrac{\sqrt{3}}{2}c(\alpha)}-\tfrac{1}{2}s(\alpha)}+\cancel{\tfrac{\sqrt{3}}{2}c(\alpha)}$$

$$\sin(\alpha)-\sin(\alpha)$$

$$\boxed{0}$$

Prove the following identity

$$\frac{1-\cos^2 x}{\sin 2x} = \frac{1}{2}\tan x$$

$$\frac{\sin^{\cancel{2}}(x)}{2\cancel{\sin(x)}\cos(x)}$$

$$\frac{\sin(x)}{2\cos(x)}$$

$$\boxed{\frac{1}{2}\tan(x)}$$

Prove the following identity

$$\tan x = \frac{\sin 2x}{1+\cos 2x}$$

$$\frac{2\sin(x)\cos(x)}{1+\cos^2(x)-\sin^2(x)}$$

$$\frac{2\sin(x)\cos(x)}{\cos^2(x)+\cos^2(x)}$$

$$\frac{\cancel{2}\sin(x)\cancel{\cos(x)}}{\cancel{2}\cos^{\cancel{2}}(x)}$$

$$\frac{\sin(x)}{\cos(x)} \rightarrow \boxed{\tan(x)}$$

Prove the following identity

$$\boxed{\sec 2x} = \frac{1}{1-2\sin^2 x}$$

$$\frac{1}{\cos(2x)}$$

$$\frac{1}{\cos^2(x)-\sin^2(x)}$$

$$\frac{1}{1-\sin^2(x)-\sin^2(x)}$$

$$\boxed{\frac{1}{1-2\sin^2(x)}}$$

Prove the following identity

$$\boxed{\frac{\cot^2 x-1}{\csc^2 x}} = \cos 2x$$

$$\frac{\cot^2(x)}{\csc^2(x)} - \frac{1}{\csc^2(x)}$$

$$\frac{\frac{\cos^2(x)}{\sin^2(x)}}{\frac{1}{\sin^2(x)}} - \frac{1}{\frac{1}{\sin^2(x)}}$$

$$\cos^2(x)-\sin^2(x)$$

$$\boxed{\cos(2x)}$$

If $A+B+C=\pi$, and $A+B=\pi-C$, then $\tan(A+B)=\tan(\pi-C)$. Using these relations to prove that: $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$.

$$\tan(A+B)=\tan(\pi-C)$$

$$\frac{\tan(A)+\tan(B)}{1-\tan(A)\tan B}=\frac{\tan(\pi)-\tan(C)}{1-\tan(\pi)\cancel{\tan(C)}}$$

$$\frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}=\frac{-\tan(C)}{1}$$

$$\tan(A)+\tan(B)=-\tan(C)[1-\tan(A)\tan(B)]$$

$$\tan(A)+\tan B=-\tan(C)+\tan(A)\tan(B)\tan(C)$$

$$+\tan(C) \qquad +\tan(C)$$

$$\boxed{\tan(A)+\tan(B)+\tan(C)=\tan(A)\tan(B)\tan(C)}$$

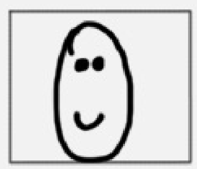
Prove the following identity

$$\frac{\sin(x-y)}{\cos x \cos y}+\frac{\sin(y-z)}{\cos y \cos z}+\frac{\sin(z-x)}{\cos z \cos x}=0$$

$$\frac{\sin(x)\cos(y)-\sin(y)\cos(x)}{\cos(x)\cos(y)}+\frac{\sin(y)\cos(z)-\sin(z)\cos(y)}{\cos(y)\cos(z)}+\frac{\sin(z)\cos(x)-\sin(x)\cos(z)}{\cos(z)\cos(x)}$$

$$\frac{\sin(x)\cancel{\cos(y)}}{\cos(x)\cancel{\cos(y)}}-\frac{\sin(y)\cancel{\cos(x)}}{\cancel{\cos(x)}\cos(y)}+\frac{\sin(y)\cancel{\cos(z)}}{\cos(y)\cancel{\cos(z)}}-\frac{\sin(z)\cancel{\cos(y)}}{\cancel{\cos(y)}\cos(z)}+\frac{\sin(z)\cancel{\cos(x)}}{\cos(z)\cancel{\cos(x)}}-\frac{\sin(x)\cancel{\cos(z)}}{\cancel{\cos(z)}\cos(x)}$$

$$\cancel{\tan(x)}-\cancel{\tan(y)}+\cancel{\tan(y)}-\cancel{\tan(z)}+\cancel{\tan(z)}-\cancel{\tan(x)}$$



A ball attached to a spring is raised 2 feet and released with an initial vertical velocity of 3 feet per second. The distance of the ball from its rest position after t seconds is given by $d = 2\cos t + 3\sin t$. Show that

$$2\cos t + 3\sin t = \boxed{\sqrt{13}\cos(t-\theta)}$$

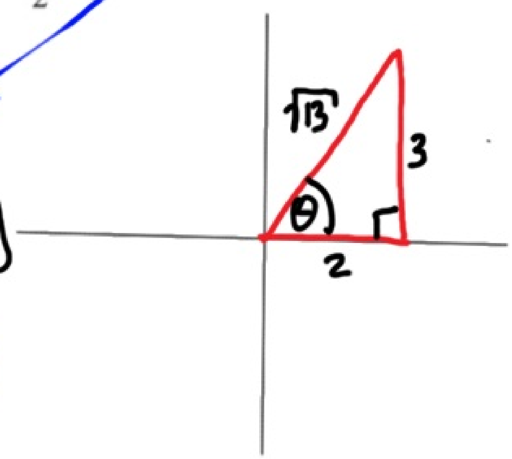
where θ lies in quadrant I and $\tan\theta = \frac{3}{2}$

$$\sqrt{13}\cos(t-\theta)$$

$$\sqrt{13}\left[\cos(t)\cos(\theta) + \sin(t)\sin(\theta)\right]$$

$$\sqrt{13}\left[\cos(t)\left(\frac{2}{\sqrt{13}}\right) + \sin(t)\left(\frac{3}{\sqrt{13}}\right)\right]$$

$$\boxed{2\cos(t) + 3\sin(t)}$$



$$2^2 + 3^2 = c^2$$

$$13 = c^2$$

$$c = \sqrt{13}$$

Use Sum/Difference Formulas to give the exact value of the trigonometric ratios below. Confirm you solution using a calculator

$\sin 195°$

$\sin(150°+45°)$

$\sin(150°)\cos(45°)+\sin(45°)\cos(150°)$

$\frac{1}{2}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\cdot -\frac{\sqrt{3}}{2}$

$\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}$

$\boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

NORMAL FLOAT AUTO REAL DEGREE MP

sin(195)
-0.2588190451

(√2−√6)/4
-0.2588190451

$\cos -15°$

$\cos(30°-45°)$

$\cos(30°)\cos(45°)+\sin(30°)\sin(45°)$

$\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}+\frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}$

$\boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

NORMAL FLOAT AUTO REAL DEGREE MP

cos(-15)
0.9659258263

(√6+√2)/4
0.9659258263

$\tan\frac{5\pi}{12}$

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$$\tan\left(\frac{5\pi}{12}\right)$$

$$\tan(30^\circ+45^\circ)$$

$$\frac{\tan(30^\circ)+\tan(45^\circ)}{1-\tan(30^\circ)\tan(45^\circ)}$$

$$\frac{\frac{\sqrt{3}}{3}+1}{1-\left(\frac{\sqrt{3}}{3}\right)(1)}$$

$$\frac{\left(1+\frac{\sqrt{3}}{3}\right)}{\left(1-\frac{\sqrt{3}}{3}\right)}\frac{(3)}{(3)}$$

$$\frac{(3+\sqrt{3})}{(3-\sqrt{3})}\frac{(3+\sqrt{3})}{(3+\sqrt{3})}$$

FOIL

$$\boxed{\frac{(3+\sqrt{3})^2}{6}}$$

radians to degrees.

$$\frac{\frac{5\pi}{12}}{2\pi}=\frac{\theta d}{360^\circ}$$

$$\frac{5}{24}=\frac{\theta d}{360^\circ}$$

$$\theta d=75^\circ$$

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NORMAL FLOAT AUTO REAL RADIAN MP
tan(5π/12)
                    3.732050808
(3+√3)²/6
                    3.732050808
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Use Half Angle Formulas to give the exact value of the trigonometric ratios below. Confirm you solution using a calculator

$\sin\frac{5\pi}{12}$

$\frac{\theta}{2} = \frac{5\pi}{12}$

$\theta = \frac{5\pi}{6}$

$\sin\left(\frac{5\pi/6}{2}\right)$

$\pm\sqrt{\frac{1-\cos(5\pi/6)}{2}}$

$\pm\sqrt{\frac{1-(-\sqrt{3}/2)}{2}}$

$\pm\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}}$

$\pm\sqrt{\frac{2+\sqrt{3}}{4}}$

$\pm\frac{\sqrt{2+\sqrt{3}}}{2}$

original angle $(5\pi/12)$ is in 1^{st} quad so pick "+" sign.

$\boxed{+\frac{\sqrt{2+\sqrt{3}}}{2}}$

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NORMAL FLOAT AUTO REAL RADIAN MP
sin(5π/12)
                    0.9659258263
√(2+√3)/2
                    0.9659258263
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$\cos\frac{7\pi}{8}$

$\frac{\theta}{2} = \frac{7\pi}{8}$

$\theta = \frac{7\pi}{4}$

$\cos\left(\frac{7\pi/4}{2}\right)$

$\pm\sqrt{\frac{1+\cos(7\pi/4)}{2}}$

$\pm\sqrt{\frac{1+\sqrt{2}/2}{2}}$

$\pm\sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}}$

$\pm\sqrt{\frac{2+\sqrt{2}}{4}}$

$\pm\frac{\sqrt{2+\sqrt{2}}}{2}$

original angle $(7\pi/8)$ in 2nd quad where cosine is negative pick "–" sign.

$\boxed{-\frac{\sqrt{2+\sqrt{2}}}{2}}$

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NORMAL FLOAT AUTO REAL RADIAN MP
cos(7π/8)
                    -0.9238795325
-√(2+√2)/2
                    -0.9238795325
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$\tan 22.5°$

$$\tan(22.5^\circ)$$

$$\frac{\theta}{2} = 22.5^\circ$$

$$\theta = 45^\circ$$

$$\tan\left(\frac{45^\circ}{2}\right)$$

$$\frac{1-\cos(45^\circ)}{\sin(45^\circ)}$$

$$\frac{\left(1-\frac{\sqrt{2}}{2}\right)(2)}{\left(\frac{\sqrt{2}}{2}\right)(2)}$$

$$\frac{(2-\sqrt{2})(\sqrt{2})}{\sqrt{2}\,(\sqrt{2})}$$

$$\frac{2\sqrt{2}-2}{2}$$

$$\frac{\cancel{2}(\sqrt{2}-1)}{\cancel{2}}$$

$$\boxed{\sqrt{2}-1}$$

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NORMAL FLOAT AUTO REAL DEGREE MP
tan(22.5)
                  0.4142135624
√2-1
                  0.4142135624
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