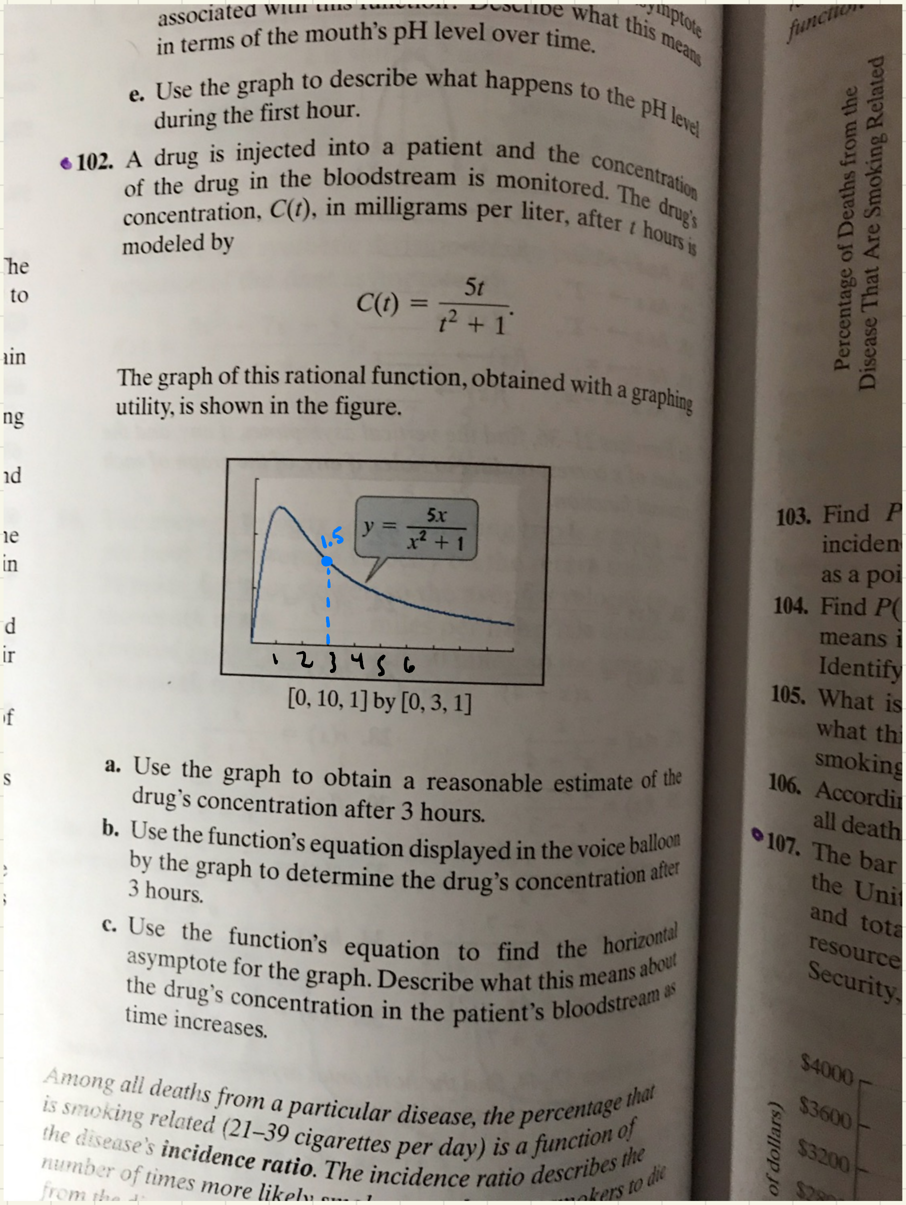
Book work for Application and Partial Fraction Decompostion:

Page 400: EXERCISE SET 101, 102, 107, 111,125 (Apps)

Page 841: EXERCISE SET (9-18) all 25 and 26 (Partial Fraction Decomposition)

102



associated with this function. Describe what this asymptote means in terms of the mouth's pH level over time.

e. Use the graph to describe what happens to the pH level during the first hour.

102. A drug is injected into a patient and the concentration of the drug in the bloodstream is monitored. The drug's concentration, $C(t)$, in milligrams per liter, after t hours is modeled by

$$C(t) = \frac{5t}{t^2 + 1}.$$

The graph of this rational function, obtained with a graphing utility, is shown in the figure.

$y = \frac{5x}{x^2 + 1}$

1.5

1 2 3 4 5 6

[0, 10, 1] by [0, 3, 1]

a. Use the graph to obtain a reasonable estimate of the drug's concentration after 3 hours.

b. Use the function's equation displayed in the voice balloon by the graph to determine the drug's concentration after 3 hours.

c. Use the function's equation to find the horizontal asymptote for the graph. Describe what this means about the drug's concentration in the patient's bloodstream as time increases.

Among all deaths from a particular disease, the percentage that is smoking related (21–39 cigarettes per day) is a function of the disease's ***incidence ratio.*** *The incidence ratio describes the number of times more likely smokers to die from the ...*

a) ≈ 1.5

b) $y = \frac{5(3)}{(3)^2+1}$

$\frac{15}{10} \to 1.5$

c) $y = \frac{5x}{x^2+1}$

Denominator power > numerator so horizontal asymptote is $y=0$

so over time the drug's concentration will gravitate toward zero.

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$$\frac{1}{x(x-1)} \rightarrow \frac{A}{x} + \frac{B}{(x-1)} \rightarrow \frac{A(x-1)+Bx}{x(x-1)}$$

equating coefficients

$1 = A(x-1) + Bx$

$1 = Ax - A + Bx$

$1 = (A+B)x - A$

$$\begin{cases} A+B=0 \\ -A=1 \end{cases}$$

so $A=-1$

$B=1$

convenient values

$1 = A(x-1) + Bx$

let $x=1$

$1 = \cancel{A(1-1)} + B(1)$

$1 = B$

let $x=0$

$1 = A(0-1) + \cancel{B(0)}$

$1 = -A$

$A=1$

$$\boxed{\frac{1}{x} + \frac{-1}{(x-1)}}$$

12 $\dfrac{5x-1}{(x-2)(x+1)} \rightarrow \dfrac{A}{(x-2)} + \dfrac{B}{(x+1)} \rightarrow \dfrac{A(x+1)+B(x-2)}{(x-2)(x+1)}$

equating coefficients

$5x-1 = A(x+1)+B(x-2)$

$5x-1 = Ax+A+Bx-2B$

$5x-1 = (A+B)x+A-2B$

$\begin{cases} A+B=5 \rightarrow B=5-A \\ A-2B=-1 \end{cases}$

$A-2(5-A)=-1$

$A-10+2A=-1$

$3A=9$

$A=3$

so $B=2$

convenient values

$5x-1 = A(x+1)+B(x-2)$

let $x=-1$

$5(-1)-1 = \cancel{A(-1+1)}+B(-1-2)$

$-6=-3B$

$B=2$

let $x=2$

$5(2)-1 = A(2+1)+\cancel{B(2-2)}$

$9=3A$

$A=3$

$$\boxed{\frac{3}{(x-2)} + \frac{2}{(x+1)}}$$

(14) $$\frac{9x+21}{x^2+2x-15} \rightarrow \frac{A}{(x+5)} + \frac{B}{(x-3)} \rightarrow \frac{A(x-3)+B(x+5)}{(x+5)(x-3)}$$

$\hookrightarrow (x+5)(x-3)$

equating coefficients

$$9x+21 = A(x-3)+B(x+5)$$
$$9x+21 = Ax-3A+Bx+5B$$
$$9x+21 = (A+B)x-3A+5B$$

$$\begin{cases} A+B=9 \rightsquigarrow B=9-A \\ -3A+5B=21 \end{cases}$$

$$-3A+5(9-A)=21$$
$$-3A+45-5A=21$$
$$-8A=-24$$
$$A=3$$
so $B=6$

convenient values

$$9x+21 = A(x-3)+B(x+5)$$
let $x=3$
$$9(3)+21 = \cancel{A(3-3)}+B(3+5)$$
$$48=8B$$
$$B=6$$
let $x=-5$
$$9(-5)+21 = A(-5-3)+\cancel{B(-5+5)}$$
$$-24=-8A$$
$$A=3$$

$$\boxed{\frac{3}{x+5}+\frac{6}{x-3}}$$

16 $$\frac{x}{x^2+2x-3} \rightarrow \frac{A}{(x-3)} + \frac{B}{(x+1)} \rightarrow \frac{A(x+1)+B(x-3)}{(x-3)(x+1)}$$

$(x-3)(x+1)$

equating coefficients

$x = A(x+1) + B(x-3)$

$x = Ax + A + Bx - 3B$

$x = (A+B)x + A - 3B$

$$\begin{cases} A+B=1 \rightarrow B=1-A \\ A-3B=0 \end{cases}$$

$A - 3(1-A) = 0$

$A - 3 + 3A = 0$

$4A = 3$

$A = 3/4$ so $B = 1/4$

convenient values

$x = A(x+1) + B(x-3)$

let $x=-1$

$-1 = \cancel{A(-1+1)} + B(-1-3)$

$-1 = -4B$

$B = 1/4$

let

$3 = A(3+1) + \cancel{B(3-3)}$

$3 = 4A$

$A = 3/4$

$$\boxed{\frac{3/4}{(x-3)} + \frac{1/4}{(x+1)} \rightarrow \frac{3}{4(x-3)} + \frac{1}{4(x+1)}}$$

(18) $\dfrac{4x^2-5x-15}{x(x+1)(x-5)} \rightarrow \dfrac{A}{x}+\dfrac{B}{(x+1)}+\dfrac{C}{(x-5)}$

$\hookrightarrow A(x+1)(x-5)+B(x)(x-5)+C(x)(x+1)$

equating coefficients

$4x^2-5x-15=A(x+1)(x-5)+B(x)(x-5)+C(x)(x+1)$

$4x^2-5x-15=A(x^2-4x-5)+B(x^2-5x)+C(x^2+x)$

$4x^2-5x-15=Ax^2-4xA-5A+Bx^2-5xB+Cx^2+Cx$

$4x^2-5x-15=(A+B+C)x^2+(-4A-5B+C)x-5A$

$$\begin{cases} A+B+C=4 \\ -4A-5B+C=-5 \\ -5A=-15 \end{cases}$$

$\rightarrow A=3$

$$\boxed{\frac{3}{x}+\frac{-1}{x+1}+\frac{2}{x-5}}$$

$3+B+C=4 \rightarrow B=1-C$

$-4(3)-5B+C=-5$

$-12-5(1-C)+C=-5$

$-12-5+5C+C=-5$

$\rightarrow 6C=12$

$C=2$

So $B=-1$

convenient values

$$4x^2-5x-15=A(x+1)(x-5)+B(x)(x-5)+C(x)(x+1)$$

let $x=0$

$$4(0)^2-5(0)-15=A(0+1)(0-5)+\cancel{B(0)(0-5)}+\cancel{C(0)(0+1)}$$

$$-15=-5A \rightarrow A=3$$

let $x=5$

$$4(5)^2-5(5)-15=\cancel{A(5+1)(5-5)}+\cancel{B(5)(5-5)}+C(5)(5+1)$$

$$60=30C \rightarrow C=2$$

let $x=-1$

$$4(-1)^2-5(-1)-15=\cancel{A(-1+1)(-1+5)}+B(-1)(-1-5)+\cancel{C(-1)(-1+1)}$$

$$-6=6B \rightarrow B=-1$$

26 $\dfrac{3x^2+49}{x(x+7)^2} \rightarrow \dfrac{A}{x} + \dfrac{B}{(x+7)} + \dfrac{C}{(x+7)^2}$

$$\rightarrow (x+7)^2A + x(x+7)B + xC$$

equating coefficients

$$3x^2+49 = (x+7)^2A + x(x+7)B + xC$$

$$3x^2+49 = (x^2+14x+49)\,A + (x^2+7x)B + xC$$

$$3x^2+49 = Ax^2 + 14xA + 49A + Bx^2 + 7xB + xC$$

$$3x^2+49 = (A+B)x^2 + (14A+7B+C)x + 49A$$

$A+B=3 \longrightarrow 1+B=3 \rightarrow B=2$

$14A+7B+C=0$ $\qquad 14(1)+7(2)+C=0$

$49A=49 \longrightarrow A=1$ $\qquad C=-28$

$$\frac{1}{x} + \frac{2}{(x+7)} + \frac{-28}{(x+7)^2}$$