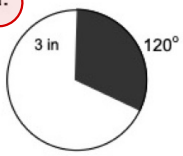
Arc Length and Sectors

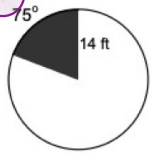
Use your knowledge of arc length and area of sectors to solve the following problems. Work problems on your own paper. Show all work.

1. Use the given information to find the arc length and area of each labeled sector in the following circles. Round to the nearest tenth.

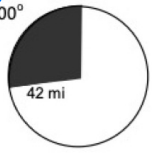
a.



b.



c.



d. central angle 72°, radius 15 cm

e. central angle 60°, radius 25 yd

a. $\frac{\theta d}{360^\circ} = \frac{S}{2\pi(r)} \rightarrow \frac{120^\circ}{360^\circ} = \frac{S}{2\pi(3)} \rightarrow \frac{1}{3} = \frac{S}{6\pi} \rightarrow S = \frac{6\pi}{3} \rightarrow$ $S = 2\pi$ inches, $S \approx 6.28$ inches

$\frac{\theta d}{360^\circ} = \frac{A}{\pi r^2} \rightarrow \frac{120^\circ}{360^\circ} = \frac{A}{\pi(3)^2} \rightarrow \frac{1}{3} = \frac{A}{9\pi} \rightarrow A = \frac{9\pi}{3} \rightarrow$ $A = 3\pi \text{ in}^2$, $A \approx 9.42 \text{ in}^2$

b. $\frac{\theta d}{360^\circ} = \frac{S}{2\pi(r)} \rightarrow \frac{75^\circ}{360^\circ} = \frac{S}{2\pi(14)} \rightarrow \frac{5}{24} = \frac{S}{28\pi} \rightarrow \frac{5 \cdot 28\pi}{24} = S \rightarrow S = \frac{35\pi}{6}$ ft or $S \approx 18.33$ ft

$\frac{\theta d}{360^\circ} = \frac{A}{\pi r^2} \rightarrow \frac{75^\circ}{360^\circ} = \frac{A}{\pi(14)^2} \rightarrow \frac{5}{24} = \frac{A}{144\pi} \rightarrow \frac{5 \cdot 144\pi}{24} = A \rightarrow$ $A = 30\pi \text{ ft}^2$, $A \approx 94.25 \text{ ft}^2$

c. $\frac{\theta d}{360^\circ} = \frac{S}{2\pi r} \rightarrow \frac{100^\circ}{360^\circ} = \frac{S}{2\pi(42)} \rightarrow \frac{5}{18} = \frac{S}{84\pi} \rightarrow \frac{5 \cdot 84\pi}{18} = S \rightarrow S = \frac{70\pi}{3}$ mi, $S \approx 73.30$ mi

$\frac{\theta d}{360^\circ} = \frac{A}{\pi(r)^2} \rightarrow \frac{100^\circ}{360^\circ} = \frac{A}{\pi(42)^2} \rightarrow \frac{5}{18} = \frac{A}{1764\pi} \rightarrow A = \frac{5 \cdot 1764\pi}{16} \rightarrow$ $A = \frac{2205\pi}{4} \text{ mi}^2$, $A \approx 1731.80 \text{ mi}^2$

d. $\frac{\theta d}{360^\circ} = \frac{S}{2\pi r} \rightarrow \frac{72^\circ}{360^\circ} = \frac{S}{2\pi(15)} \rightarrow \frac{1}{5} = \frac{S}{30\pi} \rightarrow S = \frac{30\pi}{5} \rightarrow$ $S = 6\pi$ cm, $S \approx 18.85$ cm

$\frac{\theta d}{360^\circ} = \frac{A}{\pi r^2} \rightarrow \frac{75^\circ}{360^\circ} = \frac{A}{\pi(15)^2} \rightarrow \frac{1}{5} = \frac{A}{225\pi} \rightarrow A = \frac{225\pi}{5} \rightarrow$ $A = 51\pi \text{ cm}^2$, $A \approx 160.22 \text{ cm}^2$

e. $\frac{\theta d}{360^\circ} = \frac{S}{2\pi r} \rightarrow \frac{60^\circ}{360^\circ} = \frac{S}{2\pi(25)} \rightarrow \frac{1}{6} = \frac{S}{50\pi} \rightarrow S = \frac{50\pi}{6} \rightarrow$ $S = \frac{25\pi}{3}$ yd, $S \approx 26.18$ yd

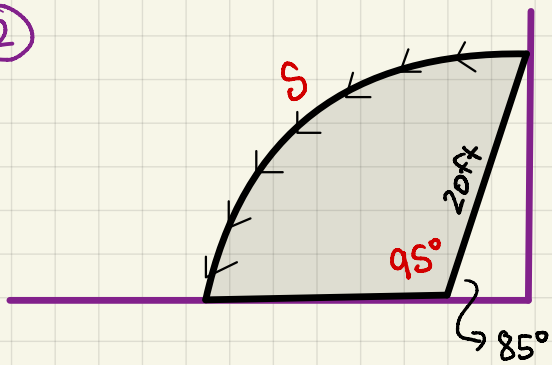
$\frac{\theta d}{360^\circ} = \frac{A}{\pi r^2} \rightarrow \frac{60^\circ}{360^\circ} = \frac{A}{\pi(25)^2} \rightarrow \frac{1}{6} = \frac{A}{625\pi} \rightarrow$ $A = \frac{625\pi}{6} \text{ yd}^2$, $A \approx 104.17 \text{ yd}^2$

2. Ladders can be extremely dangerous if not used correctly. A 20 ft. extension ladder is placed on a wall making an angle of elevation of 85° with the ground. If a person at the top of the ladder leaned back, rotating the ladder away from the wall, how far to the nearest foot would the person fall before he hit the ground?

3. The sprocket on the crankshaft of an engine powers the camshaft by a chain assembly. If the engine crankshaft is turning 3250 revolutions per minute and the sprocket has a radius of 1.5 inches, to the nearest inch how many inches of chain travel past the sprocket in one minute? To the nearest foot how many feet of chain travel past the sprocket in one minute?

4. Peter and his partner are conducting a physics experiment on pendulum motion. Their 30 cm pendulum traverses an arc of 15 cm. To the nearest degree, how many degrees of rotation did the pendulum swing?

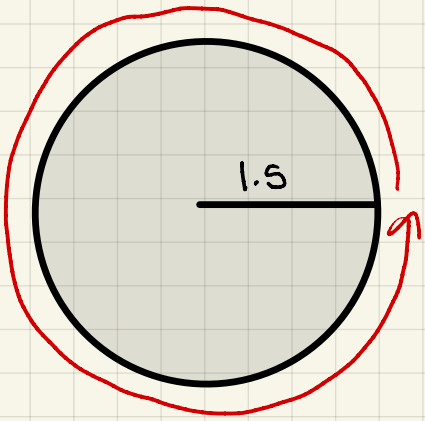
(2)



$\frac{\theta d}{360^\circ} = \frac{S}{2\pi r}$ $\quad \frac{95^\circ}{360^\circ} = \frac{S}{2\pi(20)}$ $\quad \frac{19}{72} = \frac{S}{40\pi}$

$S = \frac{19 \cdot 40\pi}{72} = \frac{95}{9}\pi \text{ ft} \approx 33.16 \text{ ft}$

(3)



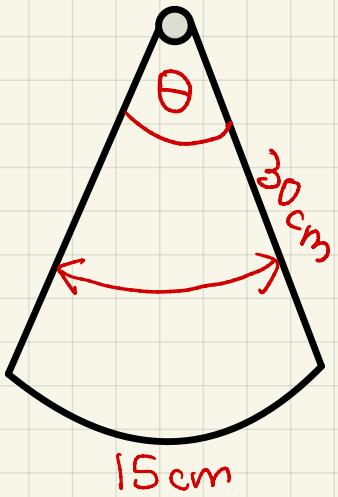
3250 rpm

(3250)(2π · 1.5) = 9750π ← revs per min

$\approx$ 30631 in/min

$\frac{30631 \text{ in}}{1 \text{ min}} \mid \frac{1 \text{ foot}}{12 \text{ in}} \approx 2553. \text{ ft/min}$

(4)



$\frac{\theta d}{360^\circ} = \frac{S}{2\pi r} \rightarrow \frac{\theta d}{360^\circ} = \frac{15}{2\pi(30)}$

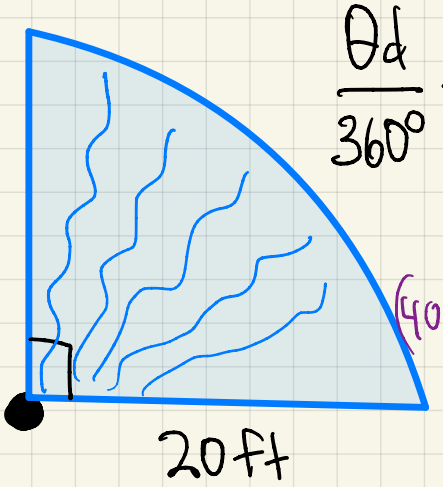
$360^\circ \cdot \frac{\theta d}{360^\circ} = \frac{1}{4\pi} 360^\circ \rightarrow \theta d = \frac{90}{\pi} \approx 28.65^\circ$

5. Ashley has a sprinkler that has several varieties of coverage. The quarter-circle sprinkler head sprays water a distance of up to 20 feet from the head. What area will be covered by the spray of the quarter-circle sprinkler head to the nearest square foot?

6. On a basketball court, the free throw lane is marked off geometrically. This area of the court is called the key and is topped by a semicircle that has a diameter of 12 feet. Find the arc length of the semicircle to the nearest foot. Find the area of the semicircle to the nearest square foot.

7. Mom baked a Dutch apple pie in a 9-inch pie pan. She cut the pie into 6 equal pies so that everyone gets the same sized piece.
   a. Determine the central angle created by two pieces of pie.
   b. Determine the area covered by each piece of pie to the nearest tenth of a square inch.
   c. Determine the circumference of the original pie to the nearest tenth of an inch.
   d. If a piece of pie had an arc length of $\frac{9}{8}\pi$, determine the area of the piece of pie to the nearest tenth of a square inch. What would be the central angle of this piece of pie?

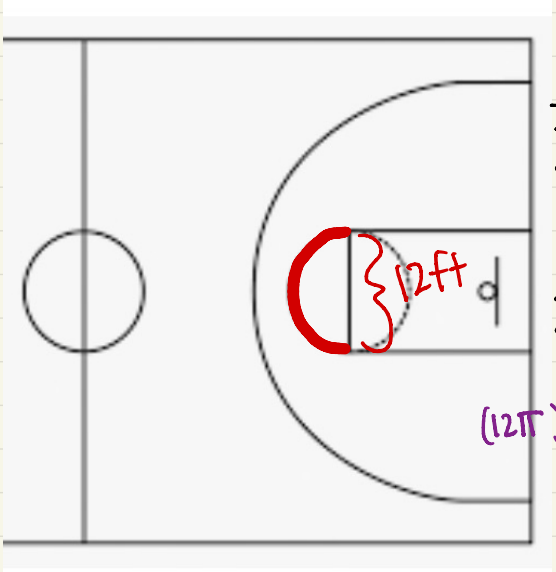
5



$$\frac{\theta d}{360^\circ} = \frac{A}{\pi r^2} \rightarrow \frac{90^\circ}{360^\circ} = \frac{A}{\pi(20)^2}$$

$$(400\pi)\frac{1}{4} = \frac{A}{400\pi}(400\pi) \rightarrow A = 100\pi \approx 314 ft^2$$

6



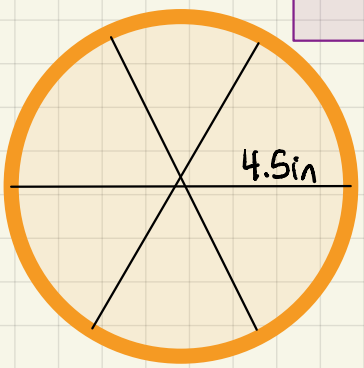
$$\frac{\theta d}{360^\circ} = \frac{S}{2\pi r}$$

$$\frac{180^\circ}{360^\circ} = \frac{S}{2\pi(6)}$$

$$(12\pi)\frac{1}{2} = \frac{S}{12\pi}(12\pi)$$

$$S = \frac{12\pi}{2} = 6\pi \approx 19 ft$$

7



a) 120°

b) $\frac{\theta d}{360^\circ} = \frac{A}{\pi(4.5)^2}$

$\frac{60^\circ}{360^\circ} = \frac{A}{20.25\pi}$

$\frac{1}{6} = \frac{A}{20.25\pi} \rightarrow A = 10.6 in^2$

c) $C = 2\pi r$

$C = 2\pi(4.5)$

$C = 9\pi in \approx 28.3 in$

d) $S=\frac{9\pi}{8}$ $r=4.5$ $A=?$

$$\frac{S}{2\pi r}=\frac{A}{\pi r^2} \rightarrow \frac{\frac{9\pi}{8}}{2\pi(4.5)}=\frac{A}{\pi(4.5)^2} \rightarrow \frac{9\pi}{8\cdot 9\pi}=\frac{A}{20.25\pi}$$

$$(20.25\pi)\frac{1}{8}=\frac{A}{20.25\pi}(20.25\pi) \rightarrow A=\frac{20.25\pi}{8}\approx 8\,in^2$$

$S=\frac{9\pi}{8}$ $r=4.5$ $\theta d=?$

$$\frac{\theta d}{360^\circ}=\frac{S}{2\pi r} \rightarrow \frac{\theta d}{360^\circ}=\frac{\frac{9\pi}{8}}{2\pi(4.5)} \rightarrow \frac{\theta d}{360^\circ}=\frac{9\pi}{8\cdot 9\pi}$$

$$\theta d=\frac{360^\circ}{8}=45^\circ$$

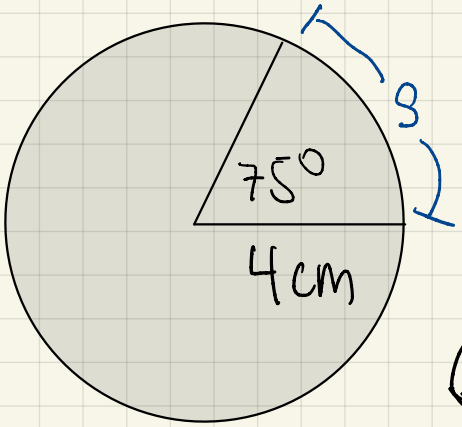
Angular and Linear Motion (and some Arc Length and Sector Area

1) A central angle in a circle of radius 4 cm is 75°. Find the length of the intercepted arc in cm. (Ans: $\frac{5\pi}{3}$)

2) Find the area of a sector in square cm of a circle of a radius 6 cm if the central angle is 60°. (Ans: 6π)

3) The area of a sector of a circle with radius 6 cm is 15 square cm. Find the measure of the central angle of the sector in degrees. (Ans: $\frac{150}{\pi}$)

1)

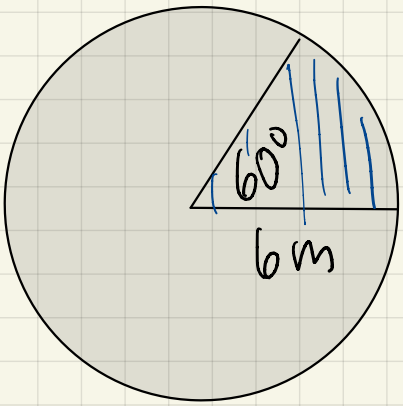


$$\frac{\theta d}{360^\circ} = \frac{s}{2\pi r} \rightarrow \frac{75^\circ}{360^\circ} = \frac{s}{2\pi(4)}$$

$$8\pi \frac{5}{24} = \frac{s}{8\pi} 8\pi$$

$$s = \frac{40\pi}{24} = \frac{8\pi}{5} \text{ cm}$$

2)

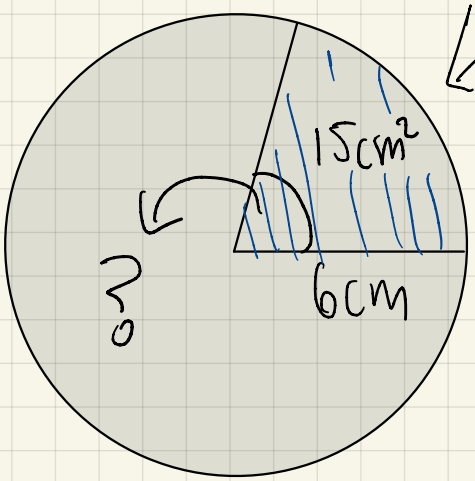


$$\frac{\theta d}{360^\circ} = \frac{A}{\pi r^2} \rightarrow \frac{60^\circ}{360^\circ} = \frac{A}{\pi(6)^2} \rightarrow (36\pi)\frac{1}{6} = \frac{A}{36\pi}(36\pi)$$

$$A = \frac{36\pi}{6} = 6\pi \text{ cm}$$

3)

Not to Scale...



$$\frac{\theta d}{360^\circ} = \frac{A}{\pi r^2} \rightarrow \frac{\theta d}{360^\circ} = \frac{15}{\pi(6)^2}$$

$$360^\circ \frac{\theta d}{360^\circ} = \frac{15}{36\pi} 360^\circ \rightarrow \frac{150^\circ}{\pi} \text{ or } 47.75^\circ$$

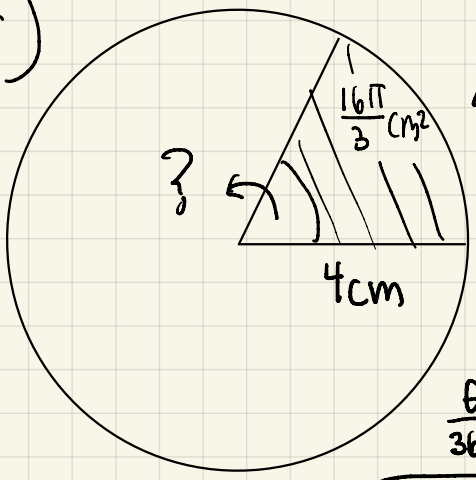
4) The area of a sector of a circle with diameter 8 cm is $\frac{16\pi}{3}$ square cm. Find the measure of the central angle of the sector in degrees. (Ans: 120)

5) Find the area of a circular sector in square cm with angle 15° if the length of the intercepted arc is $\frac{\pi}{2}$ cm. (Ans: $\frac{3\pi}{2}$)

6) A 40 -inch pendulum swings through an angle of 18°. Find the length of the arc in inches through which the tip of the pendulum swings. (Ans: 4π)

7) A point on the rim of a wheel is rotating with angular speed of 110 revolution per second. If the wheel's diameter is 15 feet, find the linear speed in ft/sec. (Ans: 1650π)

4)



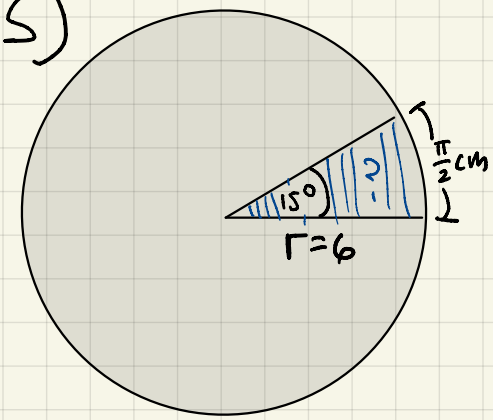
Not to scale

$\frac{\theta d}{360^\circ} = \frac{\frac{16\pi}{3}}{\pi(4)^2}$

$\frac{\theta d}{360^\circ} = \frac{\frac{16\pi}{3}}{48\pi}$

$\theta d = \frac{360^\circ}{3} \rightarrow 120^\circ$

5)



Find radius first

$\frac{\theta d}{360^\circ} = \frac{s}{2\pi r} \rightarrow \frac{15^\circ}{360^\circ} = \frac{\pi/2}{2\pi r}$

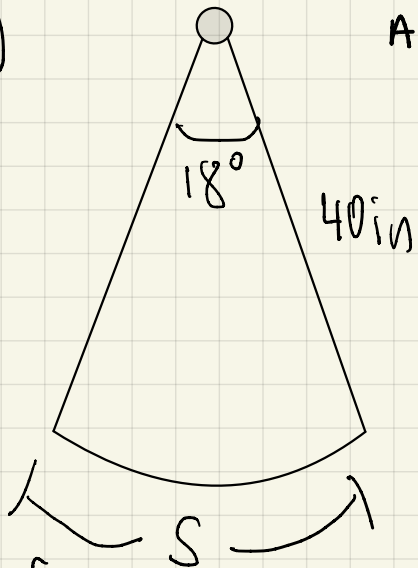
$\frac{1}{24} = \frac{\pi}{4\pi r} \rightarrow 4r = 24 \quad r = 6$

Now sector

$\frac{\theta d}{360^\circ} = \frac{A}{\pi r^2} \rightarrow \frac{15^\circ}{360^\circ} = \frac{A}{\pi(6)^2}$

$A = \frac{36\pi}{24} = \boxed{\frac{3\pi}{2}\text{ cm}^2}$

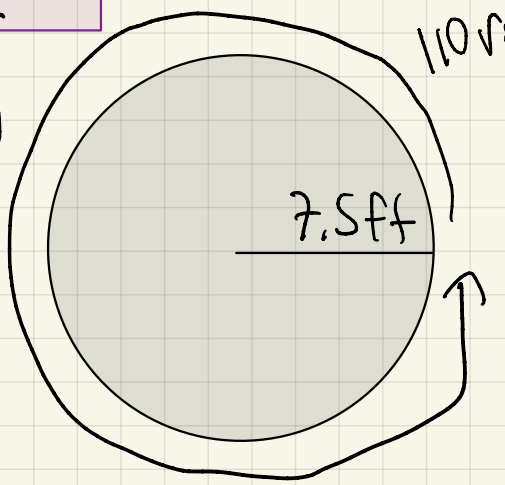
6)



$\frac{\theta d}{360^\circ} = \frac{s}{2\pi r} \rightarrow \frac{18^\circ}{360^\circ} = \frac{s}{2\pi(40)}$

$\frac{1}{20} = \frac{s}{80\pi} \rightarrow s = \frac{80\pi}{20} = \boxed{4\pi \text{ in}}$

7)



one rotation: $2\pi(7.5) = 15\pi$ ft

time rotations per second → $15\pi(110)$

$\rightarrow \boxed{1650\pi \frac{\text{ft}}{\text{sec}}}$

8) A motorcycle is traveling on a curve along a highway. The curve is an arc of a circle with radius of $\frac{1}{4}$ miles. If the motorcycle's speed is 42 mph, what is the angle in radians through which the motorcycle will turn in $\frac{1}{2}$ minute. (Ans: $\frac{7}{5}$)

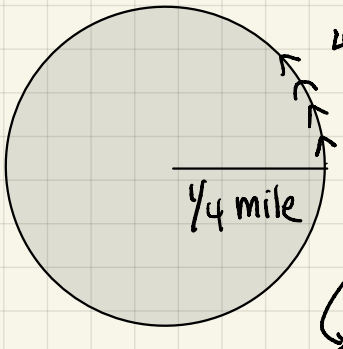
9) The outer diameter of the wheels on a bicycle is 22 inches. If the wheels are turning at a rate of 240 rpm. Find the linear speed of the bike in inches per minute.(Ans: 5280π)

10) A wheel is rotating at 50 rpm. Find the angular speed in radians per second. (Ans: $\frac{5\pi}{3}$)

11) An object is traveling around a circle with diameter 6 cm. If in 12 seconds a central angle of $\frac{1}{3}$ radian is swept out, a) what is the angular speed of the object? b) what is it's linear speed

[Ans: a) $\frac{1}{36}$ rad/sec b) $\frac{1}{12}$ cm/sec]

8)



$\frac{42 \text{ mile}}{1 \text{ hour}} \cdot \frac{1 \text{ hour}}{60 \text{ min}}$

$\frac{42 \text{ miles}}{60 \text{ mins}} \rightarrow$ or $\frac{7}{10}$ miles/min

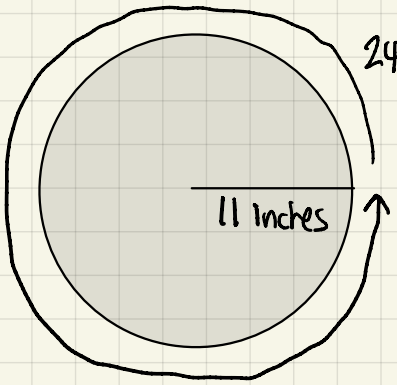
$\frac{\theta r}{2\pi} = \frac{S}{2\pi r}$

$\frac{\theta r}{2\pi} = \frac{7/20}{2\pi(1/4)}$

$d = r \cdot t = \frac{7}{10} \cdot \frac{1}{2} = \frac{7}{20} = S$

$\frac{\theta r}{2\pi} = \frac{7/20}{\pi/2} \rightarrow \frac{\theta r}{2\pi} = \frac{14}{20\pi} \rightarrow \theta r = \frac{28\pi}{20\pi} = \frac{7}{5}$ radians

9)

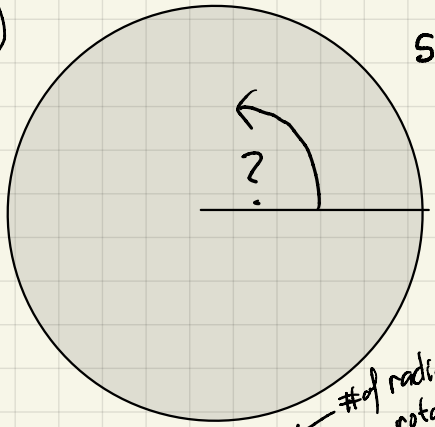


One rotation: $2\pi(11) \rightarrow 22\pi$ inches

times 240 rotations in a minute

$22\pi \times 240 = 5280\pi$ rpm

10)

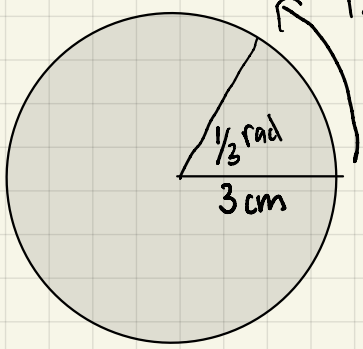


Angular speed $\rightarrow$ $2\pi \cdot 50 \rightarrow \frac{100\pi \text{ radians}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}}$

(# of radians in one turn; rotations per min)

$\frac{100\pi \text{ rad}}{60 \text{ sec}} \rightarrow \frac{5\pi}{3}$ rad/sec

11)



a) $\frac{\frac{1}{3} \text{ rad}}{12 \text{ sec}} \rightarrow \frac{1}{36}$ rad/sec

b) $\frac{\theta r}{2\pi} = \frac{S}{2\pi r}$ $\rightarrow \frac{1/3}{2\pi} = \frac{S}{2\pi(3)} \rightarrow \frac{1}{6\pi} = \frac{S}{6\pi}$

$S = 1$ cm $\rightarrow \frac{1}{12}$ cm/sec