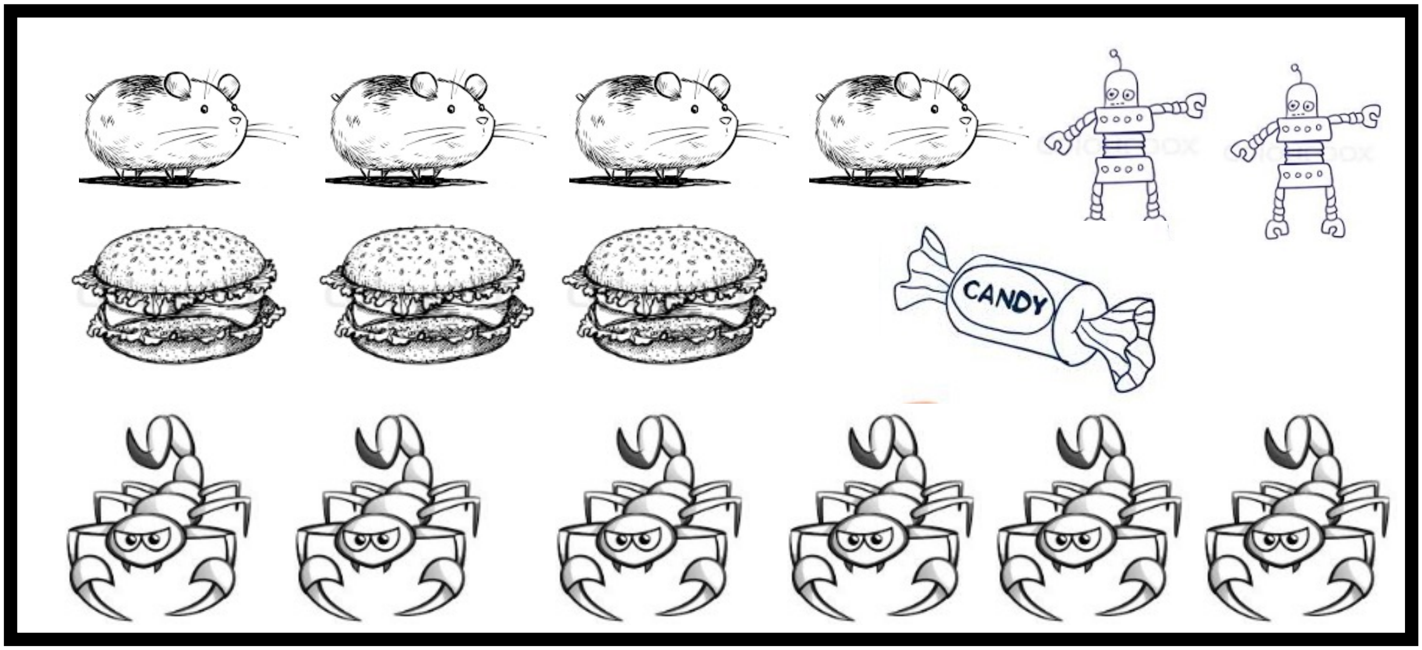
Independent and Dependent Conditional Probabilities



CONDITIONAL PROBABILITY OF INDEPENDENT EVENTS

Say you randomly choose an item from the box above. As we studied in the past units the probability of choosing a random item would be

$$P(\text{item}) = \frac{\text{item}}{\text{total number of ALL POSSIBLE items}}$$

$P(\text{Candy}) = \frac{1}{16} = 6.25\%$

$P(\text{Robot}) = \frac{2}{16} = \frac{1}{8} = 12.5\%$

$P(\text{Cheeseburger}) = \frac{3}{16} = 18.75\%$

$P(\text{Hamster}) = \frac{4}{16} = \frac{1}{4} = 25\%$

$P(\text{Scorpion}) = \frac{6}{16} = \frac{3}{8} = 37.5\%$

Scenario: picking two items and ***REPLACING*** them after each pick.
If we **REPLACE** item before we pick the next item, the picking of the items are ***independent.*** Why?

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Since the events are independent, we can use $P(A \cap B) = P(A) \bullet P(B)$ to calculate the following probabilities.

Find the following probabilities.

Pick One is a Scorpion.
Pick Two is a Cheeseburger

Pick One is a Hamster
Pick Two is a Hamster

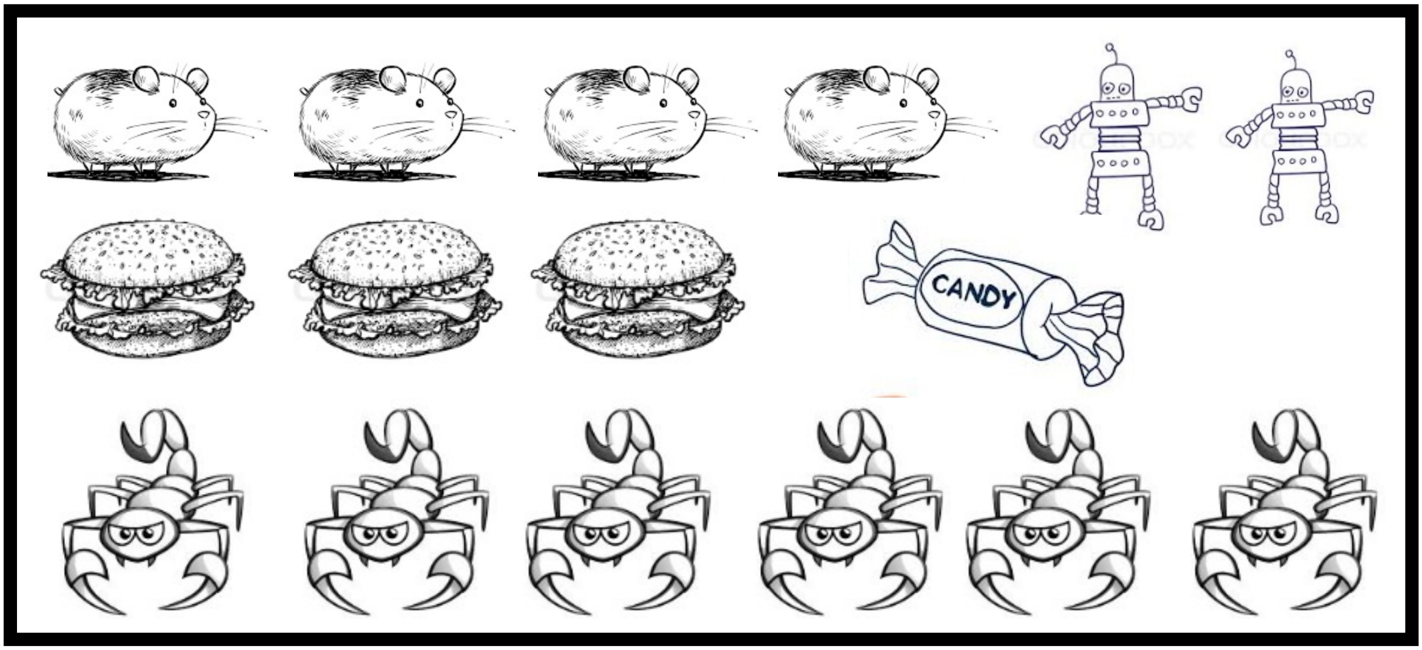
Pick One You Can Eat (realistically)
Pick Two Is an Animal

Pick One is a Candy.
Pick Two is a Robot.

Pick One is a candy.
Pick Two is a Not a Candy.

Pick One is a candy.
Pick Two is a Robot
Pick Three is Hamster

Independent and Dependent Conditional Probabilities



CONDITIONAL PROBABILITY OF DEPENDENT EVENTS

In the first example, we replaced the item we chose randomly. In this example we will ***NOT REPLACE*** the first item chosen before randomly choosing a second.

$P(\text{Candy}) = \frac{1}{16} = 6.25\%$

$P(\text{Robot}) = \frac{2}{16} = \frac{1}{8} = 12.5\%$

$P(\text{Cheeseburger}) = \frac{3}{16} = 18.75\%$

$P(\text{Hamster}) = \frac{4}{16} = \frac{1}{4} = 25\%$

$P(\text{Scorpion}) = \frac{6}{16} = \frac{3}{8} = 37.5\%$

Since we are not replacing the first item before picking the second, the picking of the two items are ***dependent.*** Why?

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To find out what to do, we start with the conditional probability model....

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

....Multiply each side by P(B)....

$$P(B) \cdot P(A|B) = P(B) \cdot \frac{P(A \cap B)}{P(B)}$$

...and after the P(B)'s divide out on the right we have....

$$P(B) \cdot P(A|B) = P(A \cap B)$$

Here's what it means in context to an example.

Pick one: Cheeseburger
The Cheeseburger is NOT REPLACED.
Pick two: A Scorpion

$$P(B) \cdot P(A|B) = P(A \cap B)$$

$P(\text{Cheeseburger})$ $\quad P(\text{Scorpion}|\text{Cheeseburger})$ $\quad P(\text{Scorpion and Cheeseburger})$

Independent and Dependent Conditional Probabilities

<u>Here are all the scenarios we calculated in part one (the independent part.) Now calculate them using you DO NOT REPLACE the item from a previous pick</u>

<u>Find the following probabilities</u>

Pick One is a Scorpion.
Pick Two is a Cheeseburger

Pick One is a Hamster
Pick Two is a Hamster

Pick One You Can Eat (realistically)
Pick Two Is an Animal

Pick One is a Candy.
Pick Two is a Robot.

Pick One is a candy.
Pick Two is a Not a Candy.

Pick One is a candy.
Pick Two is a Robot
Pick Three is Hamster

Compare your probabilities that you have calculated above to the probabilities in part one. Is there a noticeable difference between the probability of an Independent and Dependent event? State your observations below.

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Extra Example One

Your teacher passes around a box with 10 red pencils, 8 pink pencils, and 13 green pencils. If you and the two people in your group are the first to randomly select a pencil, what is the probability that all three of you select pink pencils?

What is the probability that and your group members DO NOT randomly select a green pencil?

Extra Example Two

Harper keeps her textbooks in her locker. She has 1 math book, 2 French books, 2 history books, 1 biology book, and 3 English books. Harper grabs 2 textbooks at random.

What is the probability that both books are history books?

What is the probability that both books are English books?

What is the probability that 1 book is a math book, and the other is a biology book?

What would Harper have to do to make the book choosing independent?