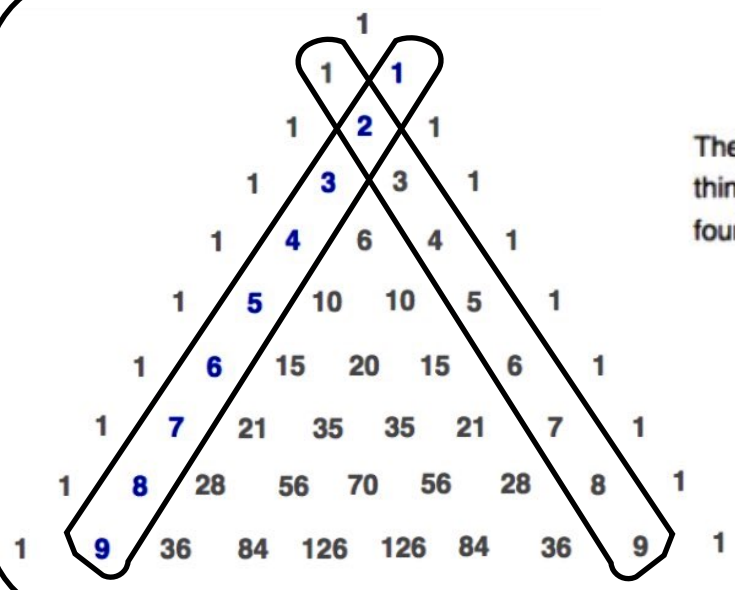
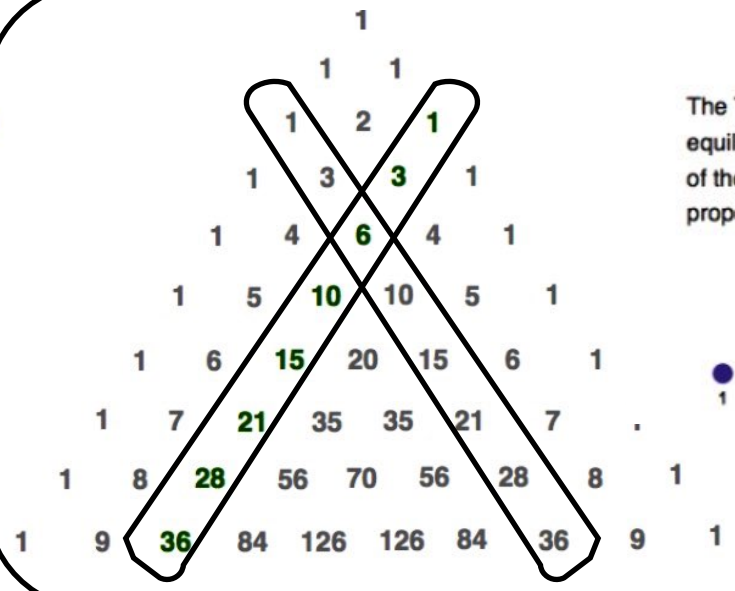
Pascal Triangle Interesting Patterns



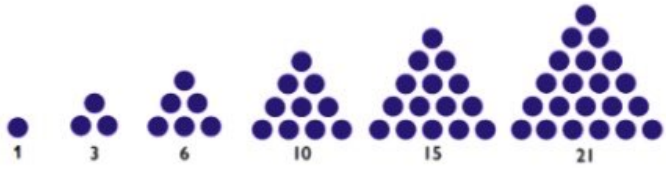
Natural Numbers

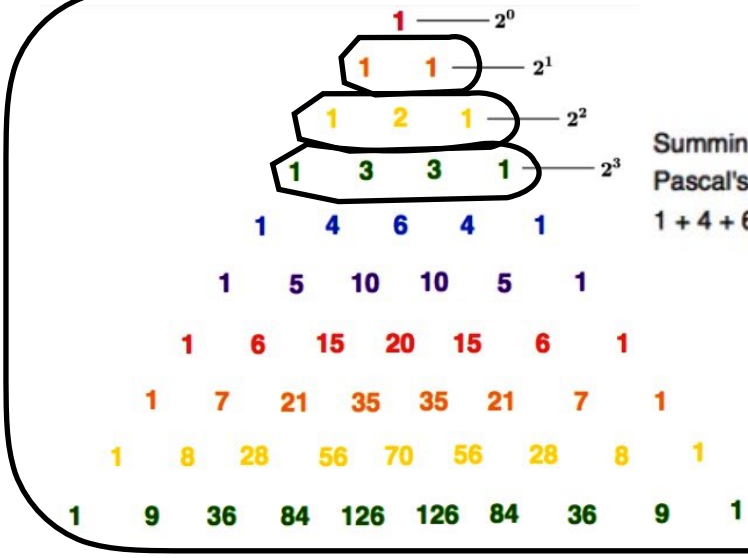
The Natural numbers are used for counting and ordering things. Countless other number series are based on the foundation of the Natural numbers.



Triangular Numbers

The Triangular sequence gives the number of objects that form an equilateral triangle. Each Triangular number represents a finite sum of the natural numbers. The Triangular numbers have many fascinating properties related to squares, pi, the golden ratio, and cubes.

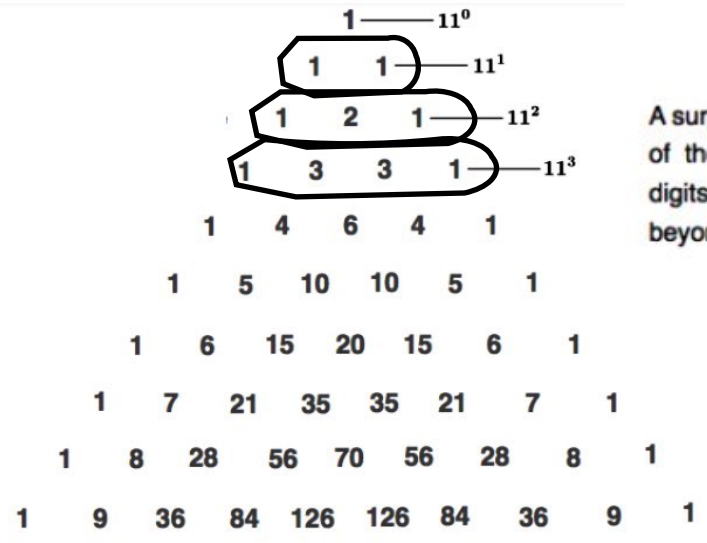




Powers of 2

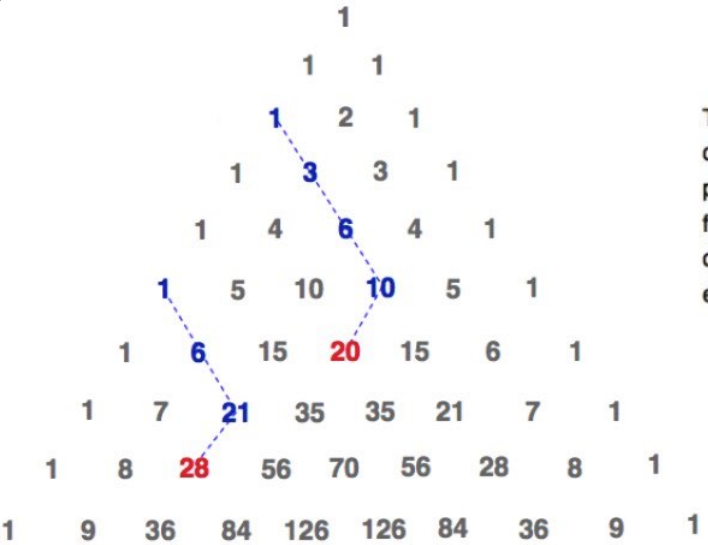
Summing the numbers of each individual horizontal row of Pascal's Triangle reveals the powers of "2". For example:
$1 + 4 + 6 + 4 + 1 = 2^4$

Pascal Triangle Interesting Patterns



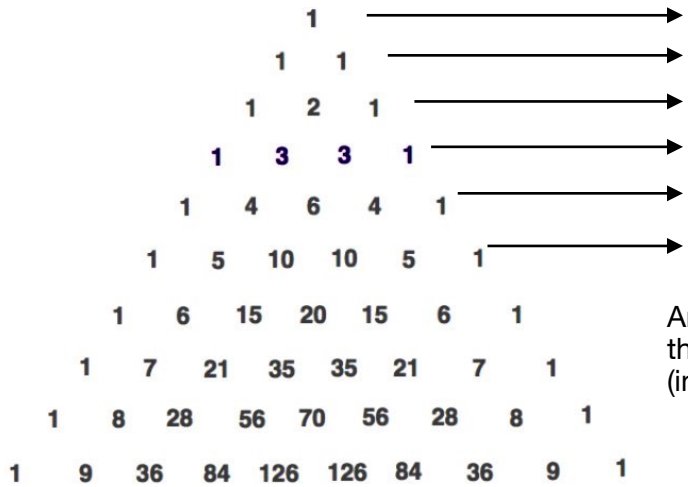
Powers of 11

A surprising property of Pascal's Triangle is the existence of the Powers of 11. The horizontal row numbers are the digits of the Powers of 11 up to the 5th row. The 5th row and beyond requires some carry over of digits. Example, $121 = 11^2$



Hockey Stick

The Hockey Stick pattern can be discovered by starting at "1" on either side and traversing down a diagonal column. At a point of your choosing, change the direction of the diagonal for the last number. The number in red (see left) is the sum of all the previous numbers in the diagonal column. For example: 1 + 3 + 6 + 10 = 20



$(a+b)^0 = 1$

$(a+b)^1 = 1a + 1b$

$(a+b)^2 = 1a^2 + 2ab + 1b^2$

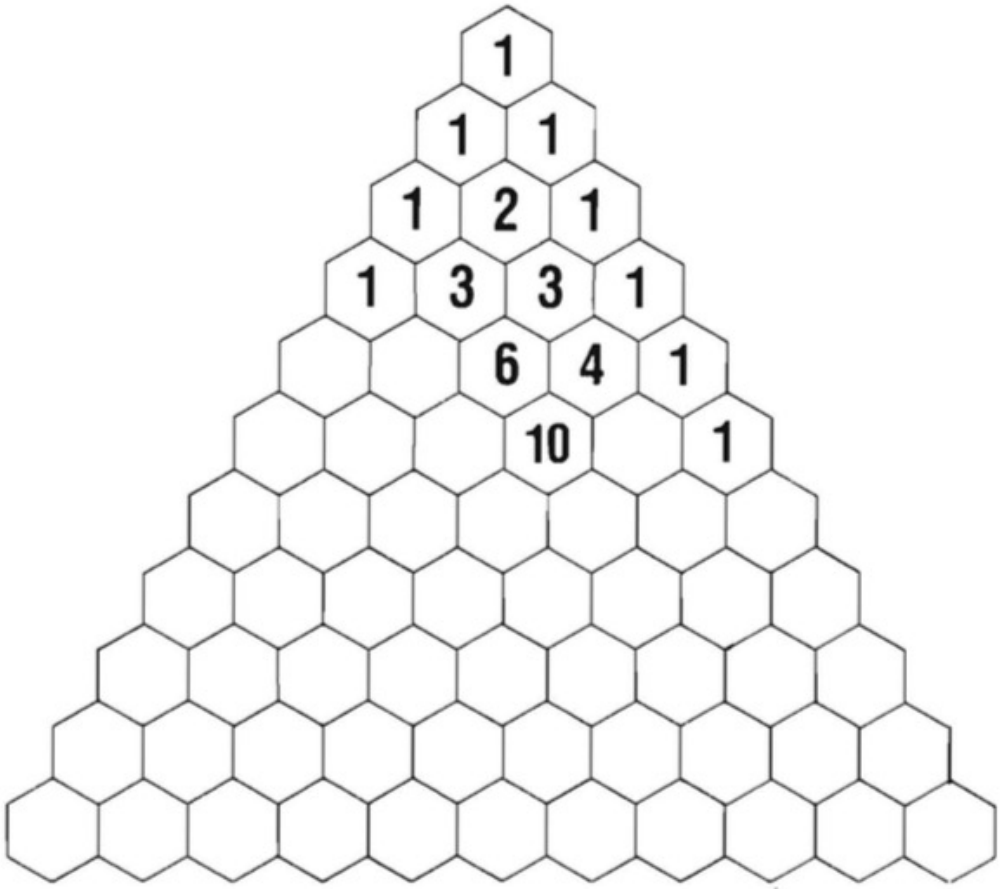
$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$

$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

$(a+b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$

And so forth. This pattern is what we will be using in this section. It will help us expand binomials quickly (in theory.)

How do I build Pascal's Triangle?



How do I use it to help expand binomials?

Observe the pattern below and make a prediction of the final line

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = a^2+2ab+b^2$

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$

$(a+b)^5 = a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$

$(a+b)^6 =$ _____

Putting it all together

Expand $(2x+3)^4$

1. Number of terms?
2. Which row to use of Pascal's Triangle?
3. Ascending and descending orders of "a" and "b"
4. Plug in "a" and "b"
5. Simplify

Expand $(3x+y)^5$

1. Number of terms?
2. Which row to use of Pascal's Triangle?
3. Ascending and descending orders of "a" and "b"
4. Plug in "a" and "b"
5. Simplify

Expand $(x-4y)^3$

1. Number of terms?
2. Which row to use of Pascal's Triangle?
3. Ascending and descending orders of "a" and "b"
4. Plug in "a" and "b"
5. Simplify

Expand $(2x^2-y)^4$

1. Number of terms?
2. Which row to use of Pascal's Triangle?
3. Ascending and descending orders of "a" and "b"
4. Plug in "a" and "b"
5. Simplify