

Math III	Using Patterns and Formulas	
Expand $(x-4)^2$ $(A+B)^2 = A^2 + 2AB + B^2$ $A = x \quad B = -4$ $(x)^2 + (2)(x)(-4) + (-4)^2$ $x^2 - 8x + 16$	Factor $2x^2 - 8$ $A^2 - B^2 = (A-B)(A+B)$ Rewrite $\rightarrow 2(x^2 - 4)$ $2(x^2 - 2^2)$ $A = x \quad B = 2$ $2(x-2)(x+2)$	Factor $3x^3 - 81$ $A^3 - B^3 \rightarrow (A-B)(A^2 + AB + B^2)$ Rewrite $\rightarrow 3(x^3 - 27)$ $3(x^3 - 3^3)$ $A = x \quad B = 3$ $3(x-3)(x^2 + 3x + 3^2)$ $3(x-3)(x^2 + 3x + 9)$
check: $(x-4)(x-4)$ $x^2 - 4x - 4x + 16$ $x^2 - 8x + 16 \checkmark$	check: $2(x-2)(x+2)$ $2(x^2 + 2x - 2x - 4)$ $2(x^2 - 4)$ $2x^2 - 8 \checkmark$	check: $3(x-3)(x^2 + 3x + 9)$ $3(x^3 + 3x^2 + 9x - 3x^2 - 9x - 27)$ $3(x^3 - 27)$ $3x^3 - 81 \checkmark$

Factor $x^3 - 8y^3$ $A^3 - B^3 \Rightarrow (A-B)(A^2 + AB + B^2)$ $x^3 - (2y)^3 \Rightarrow A=x \ B=2y$ $(x-2y)(x^2 + (x)(2y) + (2y)^2)$ $(x-2y)(x^2 + 2xy + 4y^2)$	Factor $x^3 + 1000$ $A^3 + B^3 \Rightarrow (A+B)(A^2 - AB + B^2)$ $x^3 + 10^3 \Rightarrow A=x \ B=10$ $(x+10)(x^2 - 10x + 100)$	Factor $64x^3 + 125$ $A^3 + B^3 \Rightarrow (A+B)(A^2 - AB + B^2)$ $(4x)^3 + (5)^3 \ A=4x \ B=5$ $(4x+5)((4x)^2 - (4x)(5) + (5)^2)$ $(4x+5)(16x^2 - 20x + 25)$
check: $(x-2y)(x^2 + 2xy + 4y^2)$ $x^3 + 2x^2y + 4xy^2 - 2x^2y - 4xy^2 - 8y^3$ $x^3 - 8y^3 \checkmark$	check $(x+10)(x^2 - 10x + 100)$ $x^3 - 10x^2 + 100x + 10x^2 - 100x + 1000$ $x^3 + 1000 \checkmark$	check $(4x+5)(16x^2 - 20x + 25)$ $64x^3 - 80x^2 + 100x + 80x^2 - 100x + 125$ $64x^3 + 125 \checkmark$

Expand $(2x+y)^2$

$$(A+B)^2 \rightarrow A^2 + 2AB + B^2$$

$$A=2x \quad B=y$$

$$(2x)^2 + 2(2x)(y) + (y)^2$$

$$4x^2 + 4xy + y^2$$

check:

$$(2x+y)(2x+y)$$

$$4x^2 + 2xy + 2xy + y^2$$

$$4x^2 + 4xy + y^2 \checkmark$$

Factor $\frac{1}{8}x^3 - \frac{1}{27}$

$$A^3 - B^3 \rightarrow (A-B)(A^2 + AB + B^2)$$

$$\left(\frac{1}{2}x\right)^3 - \left(\frac{1}{3}\right)^3 \rightarrow A=\frac{1}{2}x \quad B=\frac{1}{3}$$

$$\left(\frac{1}{2}x - \frac{1}{3}\right)\left(\frac{1}{4}x^2 + \frac{1}{6}x + \frac{1}{9}\right)$$

$$\left(\frac{1}{2}x - \frac{1}{3}\right)\left(\frac{1}{4}x^2 + \frac{1}{6}x + \frac{1}{9}\right)$$

check:

$$\left(\frac{1}{2}x - \frac{1}{3}\right)\left(\frac{1}{4}x^2 + \frac{1}{6}x + \frac{1}{9}\right)$$

$$\frac{1}{8}x^3 + \cancel{\frac{1}{12}x^2} + \cancel{\frac{1}{18}x} - \cancel{\frac{1}{12}x^2} - \cancel{\frac{1}{18}x} - \frac{1}{27}$$

$$\frac{1}{8}x^3 - \frac{1}{27} \checkmark$$

Factor $x^6 + 1$

$$A^3 + B^3 \rightarrow (A+B)(A^2 - AB + B^2)$$

$$(x^2)^3 + (1)^3 \rightarrow A=x^2 \quad B=1$$

$$(x^2+1)(x^4 - x^2 + 1)$$

$$(x^2+1)(x^4 - x^2 + 1)$$

check:

$$(x^2+1)(x^4 - x^2 + 1)$$

$$x^6 - \cancel{x^4} + \cancel{x^2} + \cancel{x^4} - \cancel{x^2} + 1$$

$$x^6 + 1 \checkmark$$

Factor $5x^3 + 40$ $A^3 + B^3 \rightarrow (A+B)(A^2 - AB + B^2)$ $5(x^3 + 8) \rightarrow 5(x^3 + (2)^3)$ $A = x \quad B = 2$ $5(x+2)(x^2 - 2x + (2)^2)$ $5(x+2)(x^2 - 2x + 4)$	Expand $(x^2 + y^3)^2$ $(A+B)^2 \rightarrow A^2 + 2AB + B^2$ $A = x^2 \quad B = y^3$ $(x^2)^2 + 2(x^2)(y^3) + (y^3)^2$ $x^4 + 2x^2y^3 + y^6$	Factor $(x^4 - 16)$ $A^2 - B^2 \rightarrow (A-B)(A+B)$ $(x^2)^2 - (4)^2 \rightarrow A = x^2 \quad B = 4$ $(x^2 - 4)(x^2 + 4)$ can factor again! $(x-2)(x+2)(x^2 + 4)$
check: $5(x+2)(x^2 - 2x + 4)$ $5(x^3 - \cancel{2x^2} + \cancel{4x} + \cancel{2x^2} - \cancel{4x} + 8)$ $5(x^3 + 8)$ $5x^3 + 40 \checkmark$	check: $(x^2 + y^3)(x^2 + y^3)$ $x^4 + x^2y^3 + x^2y^3 + y^6$ $x^4 + 2x^2y^3 + y^6 \checkmark$	check: $(x-2)(x+2)(x^2 + 4)$ $x^2 + \cancel{2x} - \cancel{2x} - 4$ $(x^2 - 4)(x^2 + 4)$ $x^4 + \cancel{4x^2} - \cancel{4x^2} - 16$ $x^4 - 16 \checkmark$

