Functions: Episode IV

By the end of this lesson, I will be able to answer the following questions…

1. How do I perform **arithmetic combinations of functions** and how are they are represented graphically?
2. How do I **build composite functions** and determine their domain?
3. How do I **build an inverse function algebraically** from an original function?
4. What are the characteristic of **inverse functions**?
5. What is a **one-to-one function**?

Vocabulary

1. **Sum:** $(f+g)(x)=f(x)+g(x)$
2. **Difference:** $(f-g)(x)=f(x)-g(x)$
3. **Product:** $(fg)(x)=f(x)\cdot g(x)$
4. **Quotient:** $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$
5. **Composite:** $(f\circ g)(x)=f(g(x))$

6. Inverse function notation:

If f(x) and g(x) are inverses, g(x) can be renamed

$$f^{-1}\left(x\right)$$

7. One-to-one function: When the inverse of a function is a function also.

Prerequisite Skills with Practice

Calculator exercise introducing the storage button and the variable button.

Put the following equations in terms of x:

$$y=\frac{2x-4}{5x+1}$$

$$y=-\frac{(x-3)^3}{2}+10$$

Understanding Function Notation

Given the following functions, perform the indicated operation.

$f(x) = 2x - 1$

$g(x) = 6x^2 + x - 2$

$h(x) = \sqrt{x}$

$f(3) - g(-2) =$

$\left(\frac{g}{f}\right)(x) =$

$3h(16x^4) =$

$(g \circ f)(x) =$

$2g(t^2 - 1) =$

Composition of functions: Plugging functions into other functions.

$f(x) = x^2 - 1$

$g(x) = 2x - 1$

Given the functions on the the left, find $(g \circ f)(x)$ and $(f \circ g)(x)$
Then evaluate the functions at 1,2 & 3 your graphing calculator.

x	$(f \circ g)(x)$
1	
2	
3	

x	$(g \circ f)(x)$
1	
2	
3	

Composition of functions: A simple application

A stone is thrown into a pond. A circular ripple is spreading over the pond in such a way that the radius is increasing at the rate of 5.3 feet per second. Find a function, r(t), for the radius in terms of "t". Find a Function, A(r), for the area of the ripple in terms of "r".
Find $(A \circ r)(t)$

Domains and Composite Functions

Given the following functions, find the DOMAIN of each.

$f(x) = \sqrt{x}$

$g(x) = \dfrac{1}{x}$

$h(x) = 3x^2 - 10x - 8$

$l(x) = x^2 - 16$

*Consider the Domain of the function being input. Then consider the Domain of the simplified build. The the restricted elements both conditions above make the final composite domain.

$(g \circ h)(x)$

$(l \circ f)(x)$

$(g \circ g)(x)$

$(f \circ l)(x)$

Using Properties of Inverses to Verify Inverses

Definition of inverse functions.

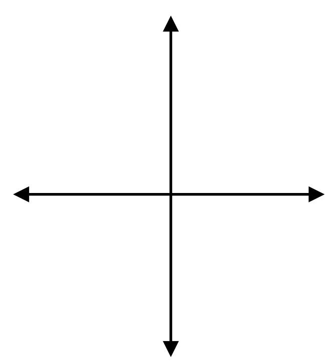
Suppose f(x) and g(x) are inverse functions. The following would hold true….

1.f[g(x)]=x and g[f(x)]=x

2. The Domain of f(x) becomes the Range of g(x) and Range of f(x) becomes the Domain g(x)

3. Graphs of f(x) and g(x) reflect about the y = x axis.

Verify that $f(x) = 2x^3 - 1$ and $g(x) = \sqrt[3]{\dfrac{x+1}{2}}$ are inverses.



Finding Inverse Algebraically

1. Switch x and y.
2. Solve for y.

$f(x) = -\frac{2}{3}x + 4$

$g(x) = \sqrt{x+2} - 3$

Other things to consider…

- One-to-one?
- Restricted domain?
- Inverse can't be found by conventional means?

$h(x) = \frac{x^2}{4} + 1$

$l(x) = \frac{x}{x-4} + 6$

CHALLANGE!

$m(x) = 2x^3 - x + 3$

Interpreting inverse values/regular values from a graph.

$f^{-1}(2) =$

$g^{-1}(-1) =$

$(f \circ g)(-1) =$

$(f^{-1} \circ f)(3) =$

$(f \circ g)(-2) =$