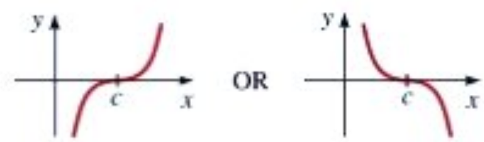
Episode VI: Revenge of the Polynomials

By the end of this lesson, I will be able to answer the following questions…

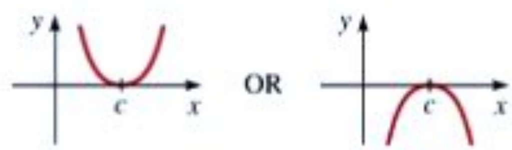
1. What are zeros of polynomials?
2. What are the techniques to find the zeros of a polynomial?
3. How do I find zeros of a polynomial using technology?
4. What is multiplicity?
5. how does multiplicity affect a polynomial graph?

Vocabulary

- **Zeros of a polynomial** - the "x" value(s) of polynomials that make the function zero. Also, can be considered the x-intercepts of the function.
- **Multiplicity** - when a function has multiply zeros at a single point, that will affect the graph in certain ways.
- **Tangency** - when a two graphs intersect and exactly one point.
- **Intermediate Value Theorem** - Used to prove the existence of certain values

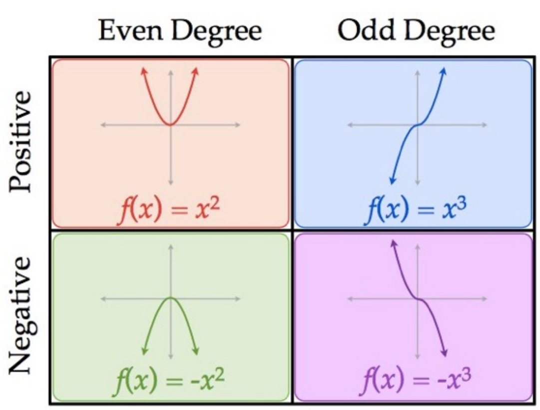


Graph behavior around x-intercept for odd multiplicities



Graph behavior around x-intercept for even multiplicities

Prerequisite Skills with Practice



$f(x) = -x^3 + 4x$

if $x \to \infty$ then $y \to$ _____

if $x \to -\infty$ then $y \to$ _____

$g(x) = 4x^4 + 4x + 1$

if $x \to \infty$ then $y \to$ _____

if $x \to -\infty$ then $y \to$ _____

$h(x) = -3x^4 + 4x + 1$

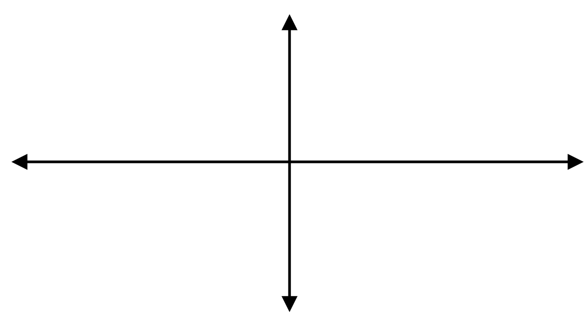
if $x \to \infty$ then $y \to$ _____

if $x \to -\infty$ then $y \to$ _____

$l(x) = 3x^3 + x$

if $x \to \infty$ then $y \to$ _____

if $x \to -\infty$ then $y \to$ _____



$$f(x) = x^4 + x^3 - 17x^2 - 21x + 36$$

Find the following using your graphing calc. Decimals ACCURATE TO THE THOUSANDTH.

Relative Min(s): ______________________________

Relative Max(s): ______________________________

x-intercept(s): ________________________

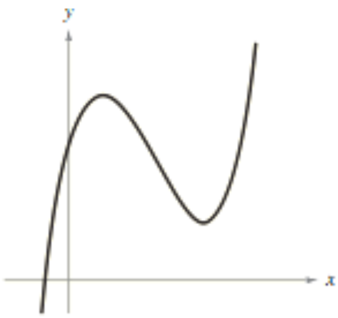
y-intercept: ______________________________

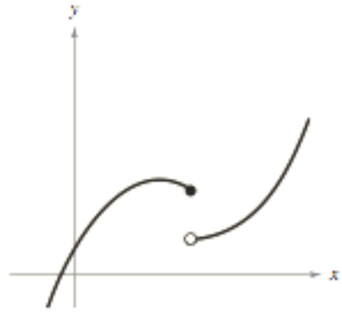
Increasing Intervals: ______________________________

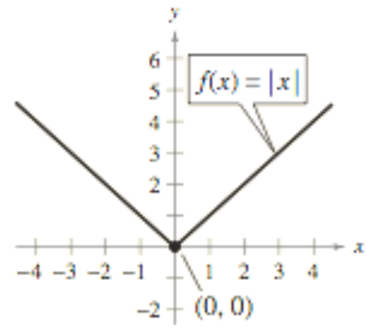
Decreasing Intervals: ______________________________

Properties of Polynomial Graphs

They are always *continuous*, that is – they have no breaks
They are smooth and rounded – no sharp turns
They have predicable end behavior.







Scenario: The Polynomial Can Be Completely Factored

$f(x) = -2x^4 - x^3 + 45x^2$

The ***fully factored form*** of f(x) is: ______________________________________

The ***zeros*** are:____

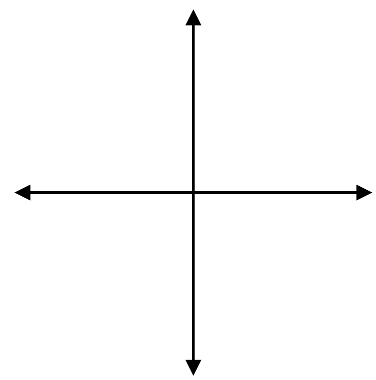
The ***x-intercepts*** are: ____

The ***y-intercept*** of the polynomial is: ___________

The ***end behavior*** of the polynomial is…

if $x \to \infty$ then $y \to$ ______

if $x \to -\infty$ then $y \to$ ______



Scenario: The Polynomial is Already Factored

$f(x) = 2(x+3)(x-1)^3(x+5)^2$

The ***fully factored form*** of f(x) is: ______________________________________

The ***zeros*** are:____

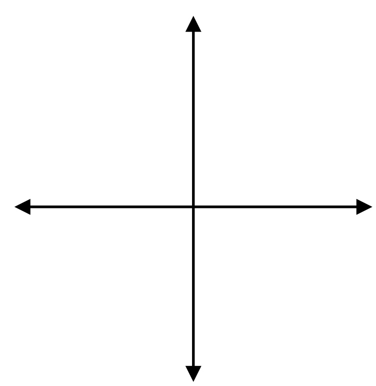
The ***x-intercepts*** are: ____

The ***y-intercept*** of the polynomial is: ___________

The ***end behavior*** of the polynomial is…

if $x \to \infty$ then $y \to$ ______

if $x \to -\infty$ then $y \to$ ______



Scenario: The Polynomial Has a Given Factor(s)

$f\left(x\right)=4x^4-4x^3-89x^2+9x+180$ given $\left(x-5\right)$ is a factor.

The ***fully factored form*** of f(x) is: ____

The ***zeros*** are:____

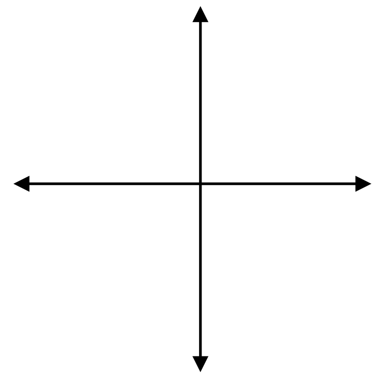
The ***x-intercepts*** are: ____

The ***y-intercept*** of the polynomial is: ___________

The ***end behavior*** of the polynomial is…

if $x\to\infty$ then $y\to$ ______

if $x\to-\infty$ then $y\to$ ______



Scenario: The Polynomial Has NO Given Factors and Cannot Be Factored by Conventional Means.

Using Rational Root Theorem (last resort as it's laborious and no guarantee it will work.)

$f\left(x\right)=x^3+4x^2-11x-30$

The ***fully factored form*** of f(x) is: ____

The ***zeros*** are:____

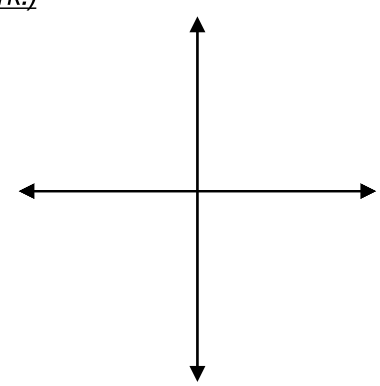
The ***x-intercepts*** are: ____

The ***y-intercept*** of the polynomial is: ___________

The ***end behavior*** of the polynomial is…

if $x\to\infty$ then $y\to$ ______

if $x\to-\infty$ then $y\to$ ______



Using Technology

$f\left(x\right)=3x^3-7x^2-103x+35$

The ***fully factored form*** of f(x) is: ____

The ***zeros*** are:____

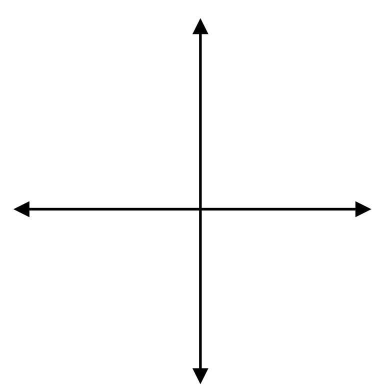
The ***x-intercepts*** are: ____

The ***y-intercept*** of the polynomial is: ___________

The ***end behavior*** of the polynomial is…

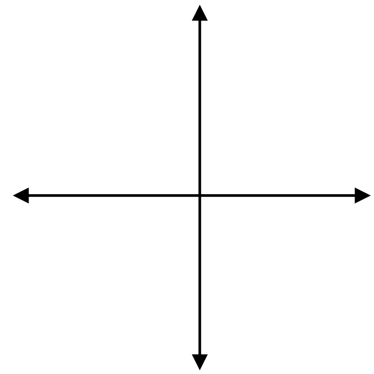
if $x\to\infty$ then $y\to$ ______

if $x\to-\infty$ then $y\to$ ______



Scenario: The Polynomial Has Irrational Roots and All Real Coefficients

$f(x) = x^3 + 7x^2 + 11x - 3$



The ***fully factored form*** of f(x) is: ______________________________

The ***zeros*** are: ______________________________

The ***x-intercepts*** are: ______________________________

The ***y-intercept*** of the polynomial is: __________

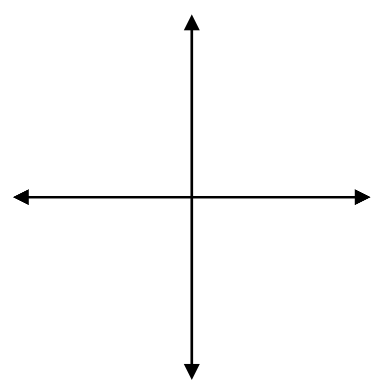
The ***end behavior*** of the polynomial is…

if $x \to \infty$ then $y \to$ ______

if $x \to -\infty$ then $y \to$ ______

Scenario: The Polynomial Has Non-Real Roots and All Real Coefficients

$f(x) = x^4 - 6x^2 - 8x + 24$



The ***fully factored form*** of f(x) is:

____

The ***zeros*** are: ______________________________

The ***x-intercepts*** are: ______________________________

The ***y-intercept*** of the polynomial is: __________

The ***end behavior*** of the polynomial is…

if $x \to \infty$ then $y \to$ ______

if $x \to -\infty$ then $y \to$ ______

Using the **Intermediate Value Theorem** to prove existence of zeros.

"Since f(x) is continuous over (a, b) and $f(a) < f(c) < f(b)$ there exists AT LEAST one value "c" such that f(c) = ***the output you are trying prove exists on the interval*** by the *Intermediate Value Theorem*."

Find three intervals of length 1 in which the polynomial below is guaranteed to have a zero over the interval (-1,3).

$$f(x) = 12x^3 - 32x^2 + 3x + 5$$

$f(-1) =$ _____

$f(0) =$ _____

Findings: ______________________________

____

____

____

____

____

$f(0) =$ _____

$f(1) =$ _____

Findings: ______________________________

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$f(1) =$ _____

$f(2) =$ _____

Findings: ______________________________

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$f(2) =$ _____

$f(3) =$ _____

Findings: ______________________________

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