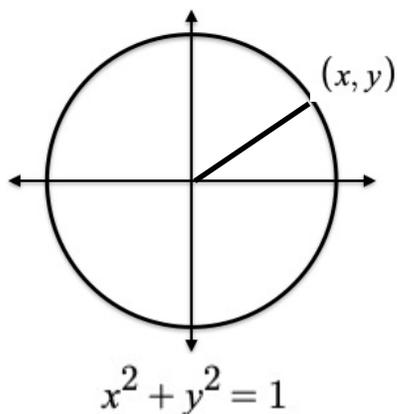


## Exploring Basic Trigonometric Identities.

## Basic Definitions



$\sin \theta =$

$\cos \theta =$

$\tan \theta =$

$\csc \theta =$

$\sec \theta =$

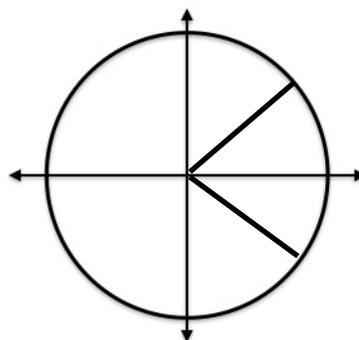
$\cot \theta =$

## Even/Odd Formulas

$\sin(-\theta) = -\sin \theta$        $\csc(-\theta) = -\csc \theta$

$\cos(-\theta) = \cos \theta$        $\sec(-\theta) = \sec \theta$

$\tan(-\theta) = -\tan \theta$        $\cot(-\theta) = -\cot \theta$



## Pythagorean Identities

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\tan^2(\theta) + 1 = \sec^2(\theta)$

$1 + \cot^2(\theta) = \csc^2(\theta)$

## Tangent Identities

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$        $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$

## Doing an Identity

$$\frac{\sec^2 \theta - 1}{\sec^2 \theta} = \sin^2 \theta$$

1. Work the side with more to do.
2. Work one side ONLY.
3. If you are stuck, change everything sine and cosine.
4. Try SOMETHING! You never know.....

$$\frac{1}{1 - \sin\theta} + \frac{1}{1 + \sin\theta} = 2\sec^2\theta$$

### Doing an Identity

1. Work the side with more to do.
2. Work one side ONLY.
3. If you are stuck, change everything sine and cosine.
4. Try SOMETHING! You never know.....

$$\tan\theta + \cot\theta = \sec\theta \csc\theta$$

### Doing an Identity

1. Work the side with more to do.
2. Work one side ONLY.
3. If you are stuck, change everything sine and cosine.
4. Try SOMETHING! You never know.....

$$\frac{\sin^2 x + 4 \sin x + 3}{\cos^2 x} = \frac{3 + \sin x}{1 - \sin x}$$

### Doing an Identity

1. Work the side with more to do.
2. Work one side ONLY.
3. If you are stuck, change everything sine and cosine.
4. Try SOMETHING! You never know.....

## Sum/Difference Trig Formulas

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \sin(\beta)\cos(\alpha)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

$$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$$

**Find the exact value of the following**

$$\cos(75^\circ)$$

$$\tan\left(\frac{\pi}{12}\right)$$

**Evaluate**  $\sin(\alpha + \beta)$

$$\sin \alpha = \frac{4}{5} \text{ First quad}$$

$$\cos \beta = -\frac{12}{13} \text{ Second quad}$$

**Using Sum Form Identities**

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x.$$

$$\frac{\cos(\alpha - \beta)}{\cos \alpha \cos \beta} = 1 + \tan \alpha \tan \beta.$$

## Double Angle Trig Formulas

$$\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\tan(2\theta) = \frac{2 \tan(\theta)}{1 - \tan^2(\theta)}$$

proof....

$$\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$$

proof....

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

proof....

$$\tan(2\theta) = \frac{2 \tan(\theta)}{1 - \tan^2(\theta)}$$

Use the following to find  $\sin 2\theta$ ,  $\cos 2\theta$ , and  $\tan 2\theta$ .

$$\cos \theta = \frac{5}{13}, \quad \frac{3\pi}{2} < \theta < 2\pi$$

$$\csc 2\theta = \frac{\csc \theta}{2 \cos \theta}$$

**Identities with Double Angles**

$$(\sin x + \cos x)^2 = 1 + \sin 2x$$

## Half Angle Trig Formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = \frac{1 - \cos\theta}{\sin\theta} = \frac{\sin\theta}{1 + \cos\theta}$$

Note - positive or negative sign for

$$\sin\left(\frac{\theta}{2}\right) \text{ and } \cos\left(\frac{\theta}{2}\right)$$

depends where the original angle is located.

**proof....**

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

**proof....**

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

**proof....**

$$\tan\left(\frac{\theta}{2}\right) = \frac{1 - \cos\theta}{\sin\theta}$$

**Evaluate**

$$\sin\left(\frac{\pi}{8}\right)$$

**Evaluate**

$$\tan\left(\frac{5\pi}{8}\right)$$

**Simplify**

$$\sin^2\left(\frac{\theta}{2}\right) \cos^2\left(\frac{\theta}{2}\right)$$