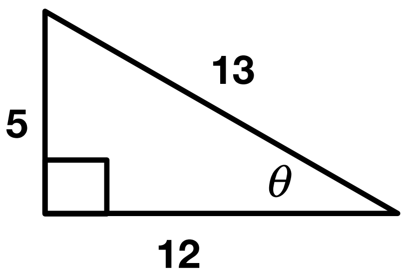
Solving Trigonometric Equations

1. How do I solve a trigonometric equation both analytically and graphically?

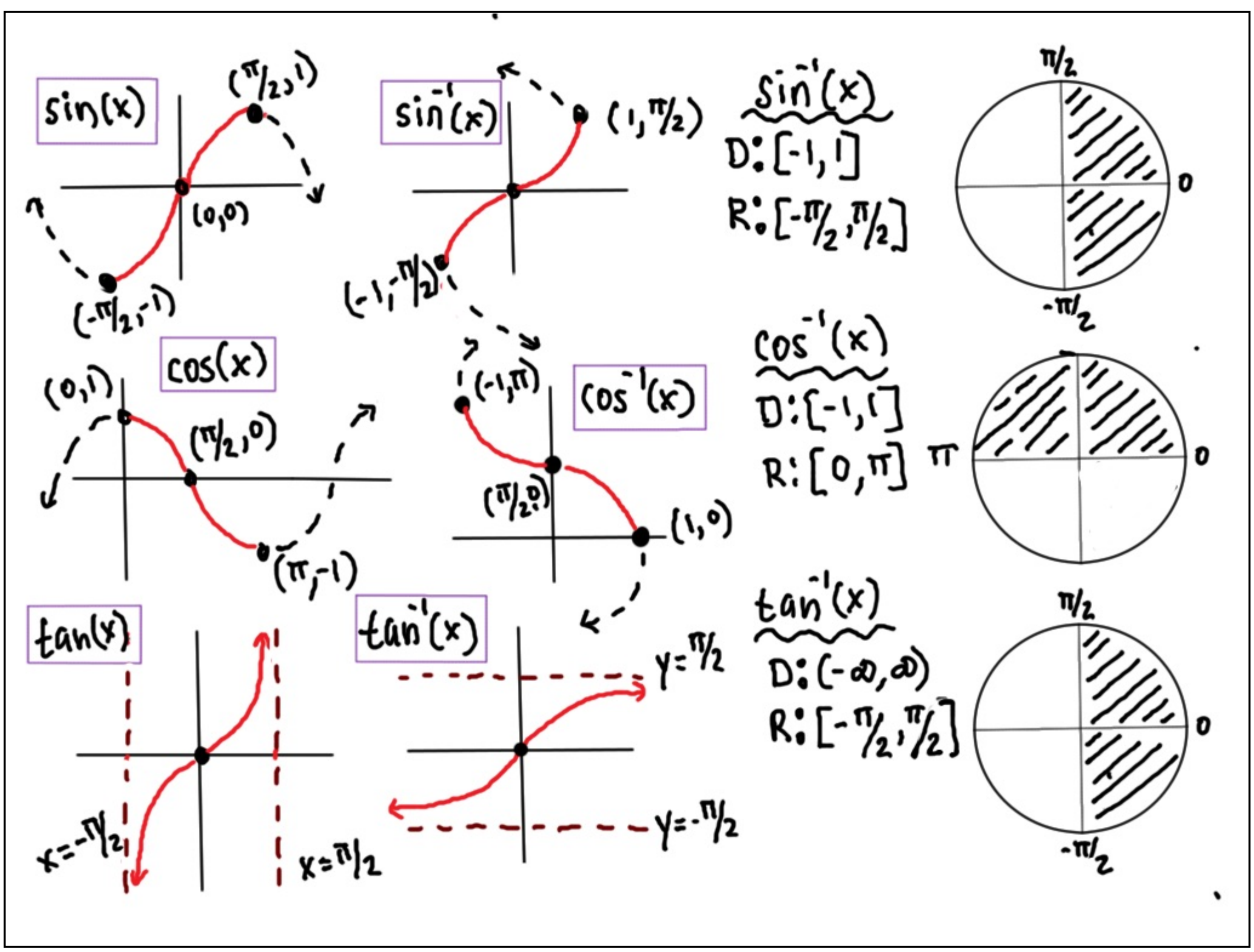
2. How do I use the inverse trig functions?

3. WHEN do I use the inverse trig functions?

Review: Inverse Trig Functions



$$\arcsin(x) = \sin^{-1}(x) = \theta$$
$$\arccos(x) = \cos^{-1}(x) = \theta$$
$$\arctan(x) = \tan^{-1}(x) = \theta$$



Precalc — Solving Trigonometric Equations

Know when to use what technique and when there is NO SOLUTION

$\sin\theta = -\frac{1}{2}$

$\sin\theta = -\frac{3}{5}$

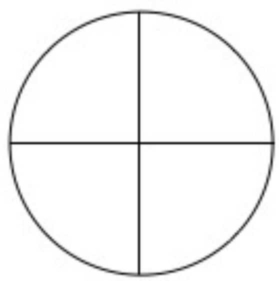
$\sin\theta = \frac{3}{2}$

$\tan\theta = \frac{3}{2}$

$\sec\theta = \frac{1}{2}$

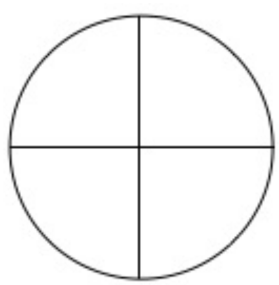
Use algebra and a graphing calculator all to find ALL the solutions of the following.

$\sin x + \sqrt{2} = -\sin x$



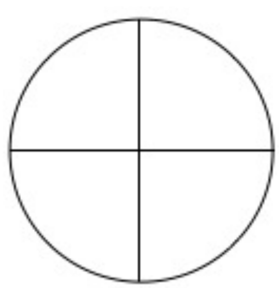
Sketch of graphical solution from calc

$3\tan^2 x - 1 = 0$



Sketch of graphical solution from calc

$\cot x \cos^2 x = 2\cot x$

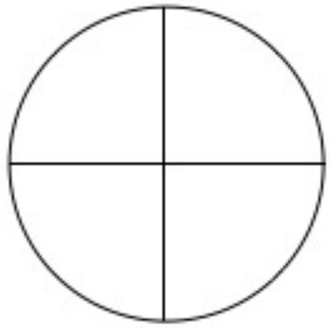


Sketch of graphical solution from calc

Precalc Solving Trigonometric Equations

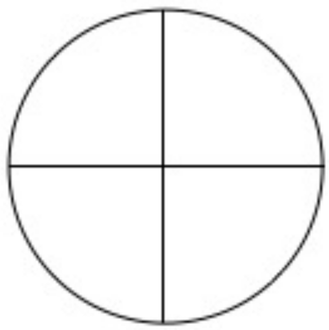
Use algebra and a graphing calculator all to find the solutions over the interval $[0, 2\pi)$

$2\sin^2 x - \sin x - 1 = 0$



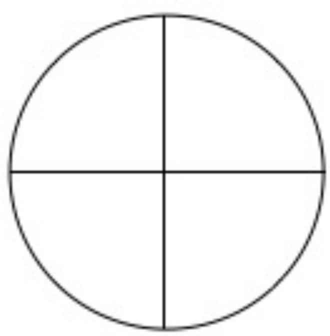
Sketch of graphical solution from calc

$2\sin^2 x + 3\cos x - 3 = 0$



Sketch of graphical solution from calc

$2\cos x + \sin 2x = 0$

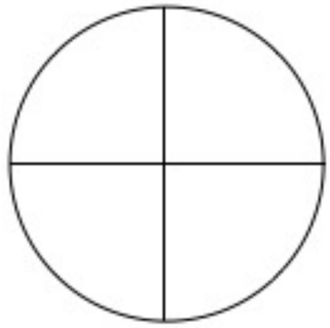


Sketch of graphical solution from calc

Solving multiple angle equations (not using identities)

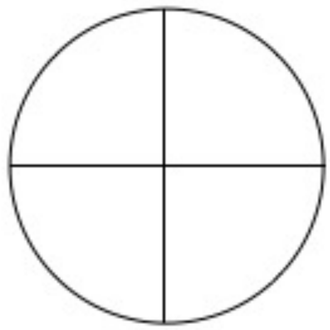
Solve the equations below for ALL solutions and then over the interval $[0,2\pi)$

$2\cos(3x)-1=0$



Sketch of graphical solution from calc

$3\tan\left(\frac{x}{3}\right)+3=0$



Sketch of graphical solution from calc

Solving equations that cannot be solving algebraically

Solve the equations below for solutions over the interval $[0,2\pi)$

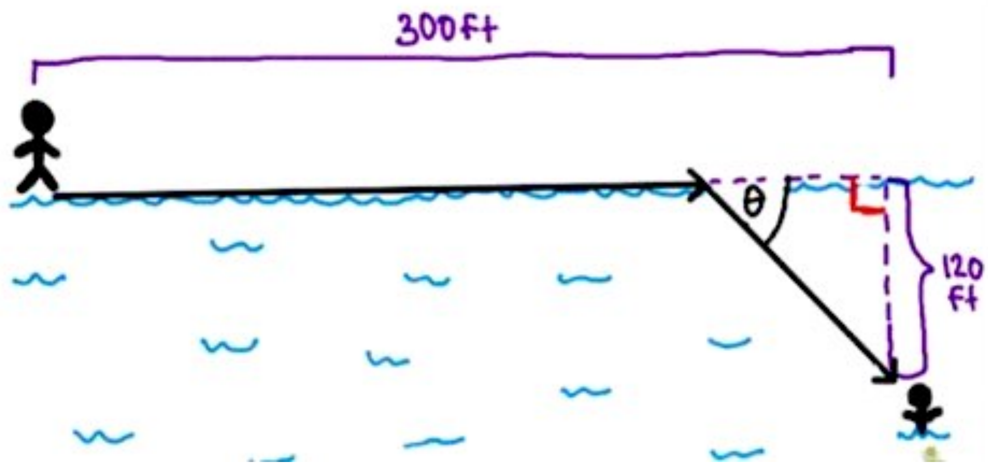
$\sin(x)+\frac{1}{2}\cos(x)=\sqrt{x+\frac{3}{2}}-2$

$\tan(x)-x=-(x-1)^2+1$

Sketch of graphical solution from calc

Sketch of graphical solution from calc

You are a lifeguard at the beach in HamsterVille, USA. One day, as you are sitting on your lifeguard chair next to Lake Squggly, you see a swimmer being attacked by a killer goldfish. The swimmer appears to be roughly 120 feet out to sea (on a straight line between the swimmer and the shore), and the lifeguard station is roughly 300 feet down the beach from the nearest point on shore to the swimmer. You can run at 13 feet per second along the beach, and you can swim at 5 feet per second. Given that you want to reach the swimmer as quickly as possible, how far down the beach do you run, and how far do you swim? What angle you should take to make the best approach?



You are an employee at the Hamster Water Company. you are trying to design a new canal from HamsterWille and San Hamsico. The canal will be above ground, for some reason, and built using bendable plastic sheets. The plastic sheets are ten feet wide, and come in sections 39 feet long. Sensibly enough, you'll assemble them into a quasi-U shape, with one section as the bottom of the canal, and two sections as sides. Since you want to maximize the volume of delicious, life-providing water that this canal carries, what should the angle between the two side pieces and the bottom piece be? (At some point in the answer, trig identities may be