

Renormalization Dynamics Yield A-Priori Estimates of Effective Coupling Constants from Black Hole Thermodynamics in a Continuously Parametrized Multiverse

Abstract.

Let Ω denote a nonempty class of universes, each element $\omega \in \Omega$ consisting of a complete specification of string-theoretic compactification data and the resulting four-dimensional effective dynamics. The present work constructs a measure-theoretic and probabilistic framework on Ω with the specific objective of obtaining *a priori* predictions for low-energy coupling constants evaluated at a canonically defined renormalization scale.

For each $\omega \in \Omega$, one specifies: (i) a compact internal manifold $X(\omega)$ of Calabi–Yau or related type; (ii) stabilized moduli fields, understood as equivalence classes of geometric and topological structures on $X(\omega)$; and (iii) additional discrete and continuous data such as flux assignments and brane configurations. These data determine, by dimensional reduction, a four-dimensional effective field theory $E(\omega)$ with a finite or countable set of coupling parameters.

A modulus is defined as any parameter in this internal geometric–algebraic configuration whose stabilized value uniquely selects a member of a family of effective theories. The collection Ω is endowed with a σ -algebra $\mathbb{F}$ such that each construction below is $\mathbb{F}$ -measurable, and with a reference measure μ_0 that encodes, for example, a counting measure on discrete flux sectors together with a suitable measure on continuous moduli spaces.

Associated to each ω is a cosmological solution $C(\omega)$ of the gravitational and matter dynamics of $E(\omega)$. This solution is assumed to exhibit an interval of cosmological times on which nontrivial structure and potential observers may exist. An *anthropic interval* $I(\omega) \subseteq \mathbb{R}$ is defined to be a maximal closed interval on which such conditions are satisfied. A bounded, nonnegative, integrable function $w(\cdot, \omega)$ supported on $I(\omega)$ is interpreted as a time-dependent observer weight.

On this basis, one defines an *anthropically averaged cosmological entropy* $S(\omega)$ as a functional depending on $C(\omega)$ and $w(\cdot, \omega)$, aggregating contributions from matter, radiation, black holes, and cosmological horizons. The precise decomposition of $S(\omega)$ is fixed in advance and is required to be $\mathbb{F}$ -measurable as a map $\omega \mapsto S(\omega)$.

A probability measure $\mathbb{P}$ on $(\Omega, \mathbb{F})$ is then defined by a Boltzmann-type weighting of the form $d\mathbb{P}/d\mu_0 \propto \exp\{\alpha S(\omega)\}$, with α a fixed nonnegative parameter chosen so that the associated partition function is finite. This construction yields a normalized probability space $(\Omega, \mathbb{F}, \mathbb{P})$ on which universes with greater anthropically averaged entropy acquire larger probability weight, in strict analogy with Gibbs measures on phase space.

For each ω , the effective theory $E(\omega)$ admits renormalization-group flows for its couplings, which are assumed to exist globally on a prescribed range of energy scales. The cosmological solution $C(\omega)$ determines a population of black holes in $I(\omega)$ with an associated mass distribution. A measurable selection procedure assigns to each ω a canonical black-hole mass $M^*(\omega)$.

Using black-hole thermodynamics, one associates to $M^*(\omega)$ a canonical Hawking temperature $T^*(\omega)$; this temperature is then fixed, by convention, as the renormalization scale $\mu_0(\omega)$. For each coupling index i , one obtains a low-energy coupling $g_i(\omega)$ by evolving the ultraviolet parameters of $E(\omega)$ along the renormalization-group flow down to $\mu_0(\omega)$. The map $\omega \mapsto g_i(\omega)$ is required to be $\mathbb{F}$ -measurable. Each g_i is thus a real-valued random variable on $(\Omega, \mathbb{F}, \mathbb{P})$.

The principal object of interest is the induced probability law of the vector of couplings at the Hawking scale, regarded as a random element in a finite- or countable-dimensional parameter space. The resulting probability distributions, expectations, and higher moments of these random variables constitute *a priori* statistical predictions for observable coupling strengths, derived from a composition of moduli-space data, anthropically averaged cosmological entropy, and a canonically fixed black-hole-based renormalization scale.