MCR3U
January 23, 2018

KNOWLEDGE (KU)		/18
APPLICATION (AP)		/16
THINKING (TI)		/20
COMMUNICATION (CO)		level

Name: ANSWERS

TEST #7 SEQUENCES & SERIES

Knowledge & Understanding (KU)

1. What is the 33rd term of the sequence 18, 11, 4, −3, ...?

-7 -7 -7

$a = 18$
$d = -7$
$n = 33$
$t_{33} = ?$

$t_n = a + (n-1)d$
$t_{33} = 18 + (33-1)(-7)$ ✓
$t_{33} = -206$ ✓ ✓

Out of 2

2. For each arithmetic sequence, determine
   i) the general term ii) the recursive formula iii) t_{11}

a.) 31, 20, 9, ...

-11 -11

i.) $t_n = a + (n-1)d$

$a = 31$
$d = -11$

$t_n = 31 + (n-1)(-11)$
or
$t_n = -11n + 42$ ✓

ii.) $t_1 = 31$

$t_n = t_{n-1} + d$

$t_n = t_{n-1} - 11$ ✓

iii.)

$a = 31$
$d = -11$
$n = 11$
$t_{11} = ?$

$t_n = a + (n-1)d$
$t_{11} = 31 + (11-1)(-11)$ ✓
$t_{11} = -79$ ✓

Out of 4

3. Determine the sum of $-31 - 35 - 39 - \ldots - 403$.

-4 -4

$a = -31$
$d = -4$
$t_n = -403$
$n = ?$

$-403 = (-31) + (n-1)(-4)$ ✓
$-403 + 31 = -4n + 4$
$-403 + 31 - 4 = -4n$
$\frac{-376}{-4} = \frac{-4n}{-4}$
$94 = n$ ✓

$S_n = \frac{n[2a + (n-1)d]}{2}$

$S_{94} = \frac{94[2(-31) + (94-1)(-4)]}{2}$ ✓

$S_{94} = \frac{94(-62 - 372)}{2}$

$S_{94} = -20398$ ✓

Out of 5

4. Calculate the sum of each geometric series.

a.) 960 + 480 + 240 + ... + 15

×0.5 ×0.5

$a = 960$
$r = 0.5$
$n = ?$
$t_n = 15$

$\frac{15}{960} = \frac{960(\frac{1}{2})^{n-1}}{960}$ ✓

$\frac{1}{64} = (\frac{1}{2})^{n-1}$

$\frac{1}{(2)^6} = (\frac{1}{2})^{n-1}$ ✓

$(\frac{1}{2})^6 = (\frac{1}{2})^{n-1}$

$6 = n - 1$

$n = 7$ ✓

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_7 = \frac{(960)[(0.5)^7 - 1]}{0.5 - 1}$ ✓

$S_7 = \frac{-952.5}{-0.5}$

$S_7 = 1905$ ✓ ✓

Out of 5

16

5. Expand & Simplify $(a+6)^4$

0 | 1
1 | 1 1
2 | 1 2 1
3 | 1 3 3 1
4 | 1 4 6 4 1

$= 1(a)^4(6)^0 + 4(a)^3(6)^1 + 6(a)^2(6)^2 + 4(a)^1(6)^3 + 1(a)^0(6)^4$ ✓

$= a^4 + 24a^3 + 216a^2 + 864a + 1296$ ✓

Out of 2

2

Application (AP)

1. The 7th term of an arithmetic sequence is 53 and the 11th term is 97. Determine the 100th term.

$53 = a + (7-1)d$ | $97 = a + (11-1)d$

① $53 = a + 6d$ ✓ | ② $97 = a + 10d$ ✓

$53 = a + 6d$

$\ominus\ 97 = a + 10d$ ✓

$-44 = -4d$ ✓

$\frac{-44}{-4} = \frac{-4d}{-4}$ ✓

$11 = d$ ✓

sub $d = 11$ into ① ✓

$53 = a + 6(11)$ ✓

$a = 53 - 66$

$a = -13$ ✓

$t_{100} = -13 + (100-1)(11)$ ✓

$t_{100} = -13 + (99)(11)$

$t_{100} = 1076$ ✓

Out of 10

2. At a fish hatchery, fish hatch at different times even though the eggs were all fertilized at the same time. The number of fish that hatched on each of the first four days after fertilization was 2, 10, 50, and 250, respectively. If the pattern continues, calculate the total number of fish hatched during the first 10 days.

$\times 5 \quad \times 5$

2, 10, 50, ~, 250

$n = 10$
$a = 2$
$r = 5$
$S_{10} = ?$

$S_{10} = \frac{a(r^n - 1)}{r-1}$

$S_{10} = \frac{(2)[(5)^{10} - 1]}{5-1}$ ✓

$S_{10} = 4882812$ ✓

∴ there are 4882812 fish after 10 days ✓

Out of 3

3. In an amphitheatre, seats are arranged in 50 semicircular rows facing a domed stage. The first row contains 23 seats, and each row contains 4 more seats than the previous row. How many seats are in the amphitheatre?

$S_{50} = ?$
$a = 23$
$n = 50$
$d = 4$

$S_n = \frac{n[2a + (n-1)d]}{2}$

$S_{50} = \frac{(50)[2(23) + (50-1)(4)]}{2}$ ✓

$S_{50} = 6050$ ✓

∴ there are 6050 seats in the amphitheatre ✓

Out of 3

16

Thinking & Inquiry (TI)

1. A hockey arena has 10920 seats. The first row of seats around the rink has 220 seats. The number of seats in each subsequent row increases by 16.

   a.) How many rows of seats does the arena have?
   b.) The arena's owners would like to expand the arena by adding four more rows of seats. What will be the new capacity of the arena?

a.) $S_n = \frac{n[2a+(n-1)d]}{2}$

$\frac{10920}{1} = \frac{n(2(220)+(n-1)(16))}{2}$ ✓

$21840 = 16n^2 + 424n$ ✓

$\frac{0}{4} = \frac{16n^2}{4} + \frac{424n}{4} - \frac{21840}{4}$ ✓

$0 = 4n^2 + 106n - 5460$ ✓

$a=4 \quad b=106 \quad c=-5460$ ✓

$n = \frac{-106 \pm \sqrt{(106)^2 - 4(4)(-5460)}}{2(4)}$ ✓

$n = \frac{-106 \pm \sqrt{98596}}{8}$

$n = \frac{-106 \pm 314}{8}$

$n_1 = \frac{-106-314}{8} \quad n_2 = \frac{-106+314}{8}$

$n_1 = -52.5$ ✓ ignore $\quad n_2 = 26$ ✓

∴ there are 26 rows in the arena ✓

b.) $S_n = \frac{n[2a+(n-1)d]}{2}$

$S_{30} = \frac{(30)[2(220)+(30-1)(16)]}{2}$ ✓

$S_{30} = 13560$ ✓

∴ the new arena has 13560 seats

Out of 10

2. A farmer has to plant seeds in a triangular field. In the 17th row, there are 53 seeds and in the 28th row there are 86 seeds. If the number of seeds planted in each row follows an arithmetic sequence, how many seeds are planted in the 35th row?

$53 = a+(17-1)d$ | $86 = a+(28-1)d$

① $53 = a+16d$ ✓ | ② $86 = a+27d$ ✓

$53 = a+16d$ ✓

$\ominus \quad 86 = a+27d$ ✓

$\frac{-33}{-11} = \frac{-11d}{-11}$ ✓

$d = 3$ ✓

Sub $d=3$ into ① ✓

$53 = a+16(3)$ ✓

$a = 53-48$

$a = 5$ ✓

$t_{35} = 5+(35-1)(3)$ ✓

$t_{35} = 107$ ✓

∴ there are 107 seeds in row 35.

Out of 10

20