

KNOWLEDGE (KU)		158
APPLICATION (AP)		120
THINKING (TI)		116
COMMUNICATION (CO)		Level

Name:

ANSWERS

UNIT 3 TEST: QUADRATIC FUNCTIONS

Knowledge & Understanding (KU)

1. Express each radical in simplest form.

a.) $\sqrt{68} = \sqrt{4 \times 17} = 2\sqrt{17}$

b.) $5\sqrt{98} = 5(\sqrt{2 \times 49}) = 5(7)(\sqrt{2}) = 35\sqrt{2}$

c.) $-3\sqrt{75} = -3(\sqrt{3 \times 25}) = -3(5)(\sqrt{3}) = -15\sqrt{3}$

2. Simplify.

a.) $3\sqrt{5} \times 2\sqrt{15} = (3)(2)\sqrt{5} \times \sqrt{15} = 6\sqrt{75} = 6(\sqrt{3 \times 25}) = 6(5)(\sqrt{3}) = 30\sqrt{3}$

b.) $\sqrt{12} + 2\sqrt{48} - 5\sqrt{27} = \sqrt{3 \times 4} + 2\sqrt{3 \times 16} - 5\sqrt{3 \times 9} = 2\sqrt{3} + 8\sqrt{3} - 15\sqrt{3} = -5\sqrt{3}$

c.) $(4 - \sqrt{3})(5 + 2\sqrt{3}) = 20 + 8\sqrt{3} - 5\sqrt{3} - 2\sqrt{3} \times 3 = 20 + 3\sqrt{3} - 2(3) = 20 - 6 + 3\sqrt{3} = 14 + 3\sqrt{3}$

3. Determine the equation of the inverse of $f(x) = 2(x-1)^2 - 3$. Sketch a graph of $f(x)$ and its inverse. Determine the domain and range of $f(x)$ and its inverse.

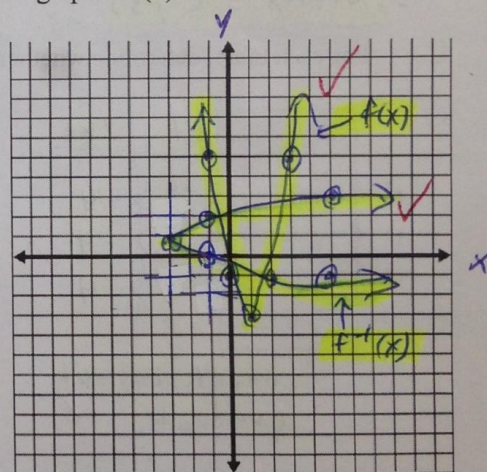
$$\begin{aligned} y &= 2(x-1)^2 - 3 \\ x &= 2(y-1)^2 - 3 \\ x+3 &= 2(y-1)^2 \\ \frac{x+3}{2} &= (y-1)^2 \\ \pm \sqrt{\frac{x+3}{2}} &= y-1 \\ 1 \pm \sqrt{\frac{x+3}{2}} &= y \end{aligned}$$

$\therefore f^{-1}(x) = 1 \pm \sqrt{\frac{x+3}{2}}$

x	y = x ²
-2	4
-1	1
0	0
1	1
2	4

x	f(x)
-2+1 = -1	2(4)-3 = 5
-1+1 = 0	2(1)-3 = -1
0+1 = 1	2(0)-3 = -3
1+1 = 2	2(1)-3 = -1
2+1 = 3	2(4)-3 = 5

x	f ⁻¹ (x)
5	-1
-1	0
-3	1
-1	2
5	3



$f(x)$ $D = \{x \in \mathbb{R}\}$
 $R = \{y \geq -3 \mid y \in \mathbb{R}\}$

$f^{-1}(x)$ $D = \{x \geq -3 \mid x \in \mathbb{R}\}$
 $R = \{y \in \mathbb{R}\}$

Out of 14

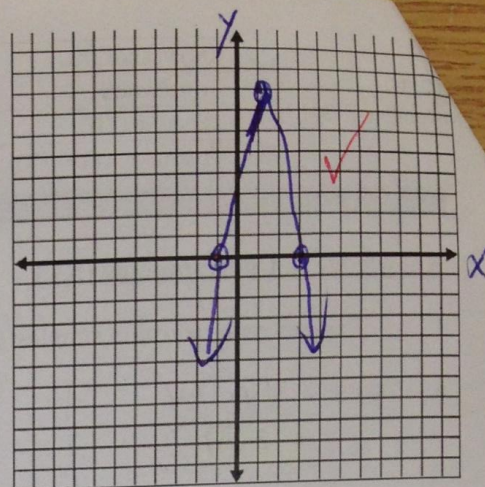
4. Consider the quadratic function $f(x) = -2(x-3)(x+1)$
- State the direction of opening and the zeros of the function.
 - Determine the coordinates of the vertex.
 - State the domain and range.
 - Graph the function.

a.) $a = -2$ opens down ✓

$$0 = -2(x-3)(x+1)$$

zeros $x=3$ ✓ $x=-1$ ✓

Out of 8



b.) x of vertex $\frac{3+(-1)}{2} = \frac{2}{2} = 1$ ✓

y of vertex $y = -2(1-3)(1+1)$
 $y = -2(-2)(2)$
 $y = 8$

$\therefore$ vertex $= (1, 8)$ ✓

c.) $D = \{x \in \mathbb{R}\}$ ✓

$R = \{y \leq 8 \mid y \in \mathbb{R}\}$ ✓

5. For each table, determine whether the function is linear, quadratic or neither.

a.)

x	y	1st	2nd
-2	-8	+6	-4
-1	-2	+2	-4
0	0	-2	-4
1	-2	-6	-4
2	-8		

2nd diff. constant
 $\therefore$ quadratic ✓

Out of 6

b.)

x	y	1st	2nd
-2	0	+1	+2
-1	1	+3	+2
0	4	+5	+2
1	9	+7	+2
2	16		

2nd diff. are constant
 $\therefore$ quadratic ✓

6. For each function, state the vertex, the equation of the axis of symmetry, and the domain and range.

a.) $f(x) = -3(x-2)^2 + 5$

vertex $= (2, 5)$ ✓

A.O.S. $\rightarrow x=2$ ✓

$D = \{x \in \mathbb{R}\}$ ✓

$R = \{y \leq 5 \mid y \in \mathbb{R}\}$ ✓

zeros -4 & 6 ✓

x of vertex $\frac{-4+6}{2} = \frac{2}{2} = 1$ ✓

y of vertex $y = 2(1+4)(1-6)$
 $y = 2(5)(-5)$
 $y = -50$ ✓

b.) $f(x) = 2(x+4)(x-6)$

vertex $= (1, -50)$ ✓

$D = \{x \in \mathbb{R}\}$ ✓

$R = \{y \geq -50 \mid y \in \mathbb{R}\}$ ✓

Out of 10

Application (AP)

1. When priced at \$40 each, a toy has annual sales of 5000 units. The manufacturer estimates that each \$1 increase in price will decrease sales by 100 units. It costs \$5 to produce each toy. Find the price that maximizes the profit. What is the maximum profit?

$$R(x) = (40 + x)(5000 - 100x) \quad \checkmark$$

$$R(x) = 200000 - 4000x + 5000x - 100x^2$$

$$R(x) = -100x^2 + 1000x + 200000 \quad \checkmark$$

$$C(x) = 5(5000 - 100x) \quad \checkmark$$

$$C(x) = -500x + 25000 \quad \checkmark$$

$$P(x) = R(x) - C(x)$$

$$P(x) = (-100x^2 + 1000x + 200000) - (-500x + 25000) \quad \checkmark$$

$$P(x) = -100x^2 + 1500x + 175000 \quad \checkmark$$

$$P(x) = -100x^2 + 1500x + 175000 \quad \checkmark$$

$$P(x) = -100(x^2 - 15x) + 175000$$

out of 12

Let x be the # of price increases $\checkmark$

$$c = \left(\frac{b}{2}\right)^2 \quad P(x) = -100(x^2 - 15x + 56.25 - 56.25) + 175000$$

$$c = \left(\frac{15}{2}\right)^2 \quad P(x) = -100(x - 7.5)^2 + 175000 + 5625 \quad \checkmark$$

$$c = (-7.5)^2 \quad P(x) = -100(x - 7.5)^2 + 180625 \quad \checkmark$$

$$c = 56.25$$

$(7.5, 180625) \rightarrow$ vertex
 $x \quad P(x)$

$$\text{Price} = 40 + x$$

$$= 40 + 7.5$$

$$= 47.50 \quad \checkmark$$

A price of \$47.50 will give a max profit of \$180625 $\checkmark$

2. A daredevil jumps off the CN Tower and falls freely for several seconds before releasing his parachute. His height, $h(t)$, in metres, t seconds after jumping can be modelled by

① $h_1(t) = -4.9t^2 + t + 360$ before he released his parachute; and

② $h_2(t) = -4t + 142$ after he released his parachute.

How long after jumping did the daredevil release his parachute?

Sub ① into ② $\checkmark$

$$-4.9t^2 + t + 360 = -4t + 142 \quad \checkmark$$

$$-4.9t^2 + t + 4t + 360 - 142 = 0$$

$$-4.9t^2 + 5t + 218 = 0 \quad \checkmark$$

$$a = -4.9 \quad b = 5 \quad c = 218 \quad \checkmark$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-4.9)(218)}}{2(-4.9)}$$

$$t = \frac{-5 \pm \sqrt{25 + 4272.8}}{-9.8} \quad \checkmark$$

$$t = \frac{-5 \pm \sqrt{4297.8}}{-9.8}$$

$$t = \frac{-5 \pm 65.6}{-9.8}$$

$$t_1 = \frac{-5 + 65.6}{-9.8}$$

$$t_2 = \frac{-5 - 65.6}{-9.8}$$

$$t_1 = -6.15 \quad \checkmark$$

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$$t_2 = 7.25 \quad \checkmark$$

out of 8

The daredevil should open his parachute after about 7.25 $\checkmark$

Thinking & Inquiry (TI)

1. If $f(x) = x^2 - 6x + 14$ and $g(x) = -x^2 - 20x - k$, determine the value of k so that there is exactly one point of intersection between the two parabolas.

Sub ① into ② ✓

$$x^2 - 6x + 14 = -x^2 - 20x - k$$

$$x^2 + x^2 - 6x + 20x + 14 + k = 0$$

$$2x^2 + 14x + 14 + k = 0$$

$a = 2 \quad b = 14 \quad c = 14 + k$ ✓

$b^2 - 4ac = 0$

$(14)^2 - 4(2)(14 + k) = 0$

$196 - 8(14 + k) = 0$ ✓

$196 - 112 - 8k = 0$

$\frac{84}{8} = \frac{8k}{8}$

$\frac{21}{2} = k$ ✓

or

$10.5 = k$

Out of 6

2. Determine the equation of the quadratic function $f(x) = ax^2 - 6x - 7$ if $f(2) = 3$.

$y = ax^2 - 6x - 7$ ✓

$y = 3$ & $x = 2$ ✓

$(3) = a(2)^2 - 6(2) - 7$

$3 = 4a - 12 - 7$ ✓

$3 = 4a - 19$

$3 + 19 = 4a$

$\frac{22}{4} = \frac{4a}{4}$

$\frac{11}{2} = a$

or

$5.5 = a$

Out of 4

3. Determine the equation in standard form of the parabola shown.

$h = 4$

$y = a(x - h)^2 + k$

$k = 3$

$(2) = a(5 - 4)^2 + (3)$

$x = 5$ ✓

$2 = a(1)^2 + 3$ ✓

$y = 2$

$2 - 3 = a$

$a = -1$

$-1 = a$ ✓

$y = -1(x - 4)^2 + 3$ ✓

$y = -1(x - 4)(x - 4) + 3$

$y = -1(x^2 - 4x - 4x + 16) + 3$

$y = -1(x^2 - 8x + 16) + 3$ ✓

$y = -x^2 + 8x - 16 + 3$

$y = -x^2 + 8x - 13$ ✓

Out of 6

