MCR3U **Exam Review**

Unit 1

1. Given $f(x)=2x^2-3x+5$, determine each value.
   a) $f(2.4)$ b) $f(-3)$ c) $f(x+1)$

2. State the Domain and Range for each of the following relations, AND state whether the relation is a function or not.
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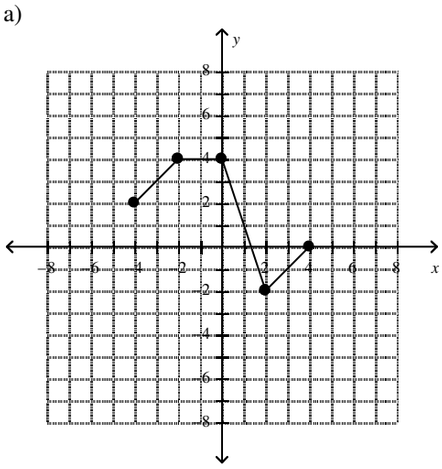
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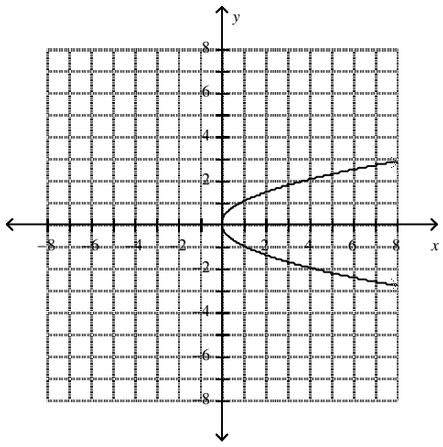
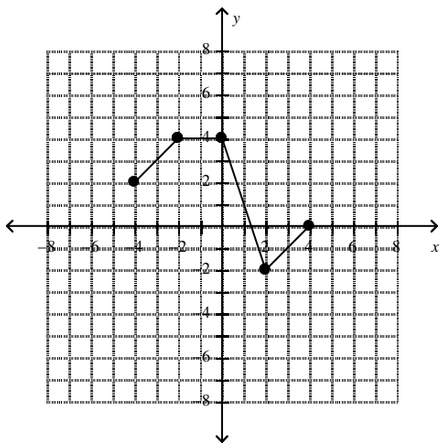
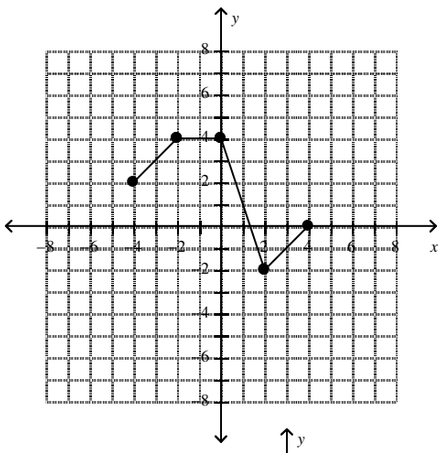

   c) $y=\sqrt{x-4}+5$ d) $y=-3(x+13)^2-22$

3. The graph of function f is given below. On the same grid plot
   a) $-f(2x)-3$ and b) $2f(x-4)$ c) $\frac{1}{2}f\left(\frac{-1}{2}x\right)-6$ and d) f^{-1}

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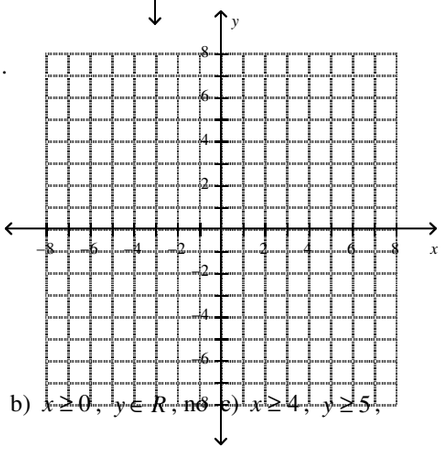

Unit 1 Answers

1. a) 9.32 b) 32 c) $2x^2+x+4$ **2**. a) $-4\le x\le 4$, $-2\le y\le 4$, yes b) $x\ge 0$, $y\in R$, no c) $x\ge 4$, $y\ge 5$, yes d) $x\in R$, $y\le -22$, yes

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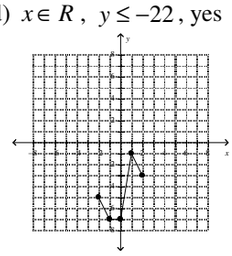
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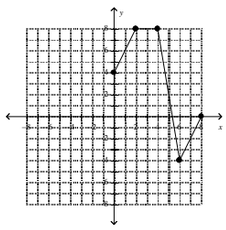
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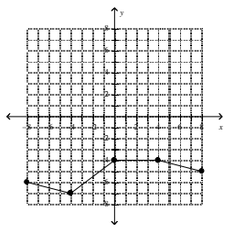
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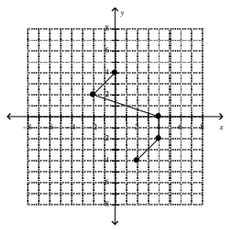

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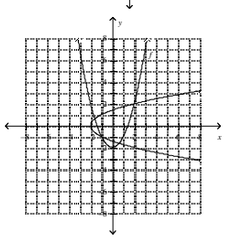
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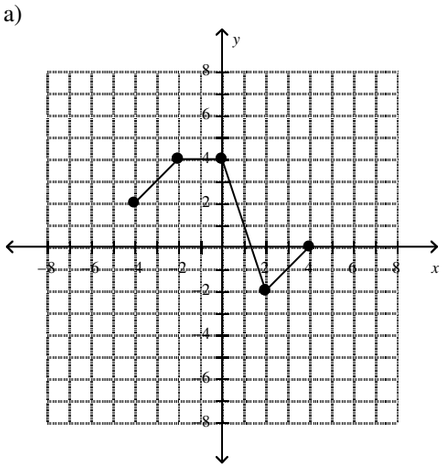
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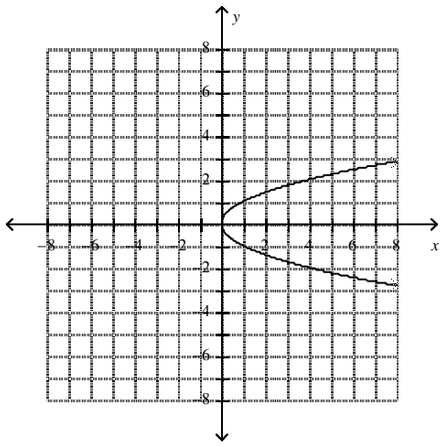
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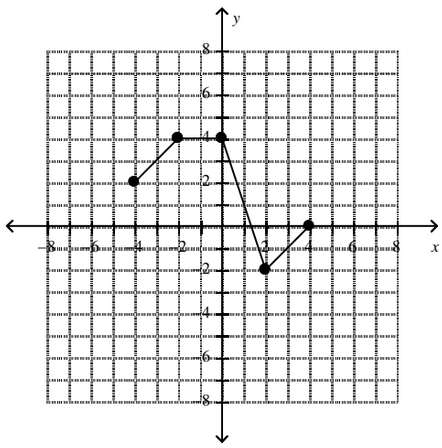
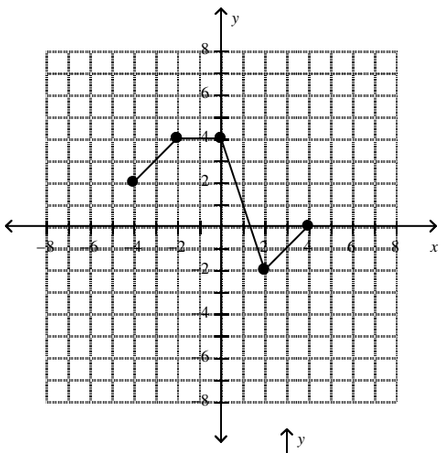

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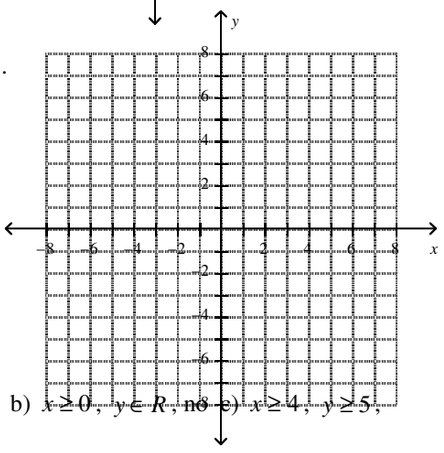

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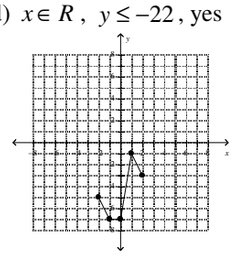
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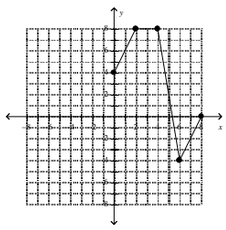
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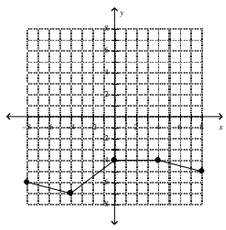
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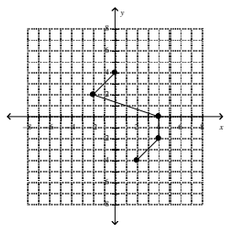

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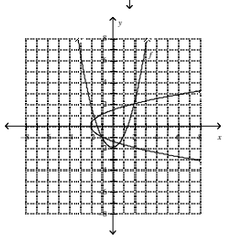

b) $y=\pm\sqrt{x+2}$

Unit 2

1. Expand, then simplify:
   a) $(3p-5)(p+2)^2$ b) $(5x-1)^2+4(2x-1)(9x+1)$

2. Factor each of the following:
   a) $8y^2-12y^5$ b) $p^2-6p-16$ c) $y^2+2xy-15x^2$ d) $6x^2+5x-4$
   e) $27x^4-75$ f) n^4-5n^2+4 g) $6y^2-4y-10$

3. Simplify. State any restrictions on the variables.
   a) $\dfrac{y^2-5y-24}{y^2-10y+16}$ b) $\dfrac{3x-2}{6x}-\dfrac{4-x}{x-1}$

4. Simplify. There is no need to state the restrictions.
   $\dfrac{3x-5y}{9x^2-25y^2}\times\dfrac{3x+5y}{9x-7y}$

5. Simplify. There is no need to state the restrictions.
   a) $\dfrac{4x}{2x^2+8x+6}-\dfrac{3x}{3x^2+9x+6}$ b) $\dfrac{2x-5}{8x^2-14x-15}+\dfrac{3x+1}{9x^2-3x-2}$

Answers

1. a) $3p^3+7p^2-8p-20$ b) $97x^2-38x-3$ **2**. a) $4y^2\left(2-3y^3\right)$ b) $(p-8)(p+2)$ c) $(y+5x)(y-3x)$
2. d) $(2x-1)(3x+4)$ e) $3\left(3x^2-5\right)\left(3x^2+5\right)$ f) $(n-1)(n+1)(n-2)(n+2)$ g) $2(y+1)(3y-5)$

3. a) $\dfrac{y+3}{y-2}, y\neq 2,8$ b) $\dfrac{9x^2-29x+2}{6x(x-1)}, x\neq 0,1$ **4**. $\dfrac{1}{9x-7y}$ **5**. a) $\dfrac{x}{(x+2)(x+3)}$ b) $\dfrac{7x+1}{(4x+3)(3x-2)}$

Unit 3

1. Write $y=2x^2-12x+3$ in the form $y=a(x-p)^2+q$.

2. For the function in number 1 above, state whether it has a maximum or minimum value, then state what that maximum or minimum value is.

3. Use partial factoring to determine the vertex of $y=3x^2+5x-1$

4. Solve each equation.
   a) $7x^2-21x=0$ b) $8x^2-10x-3=0$ c) $2x^2+12x-4=0$ d) $\dfrac{1}{2}x^2=\dfrac{x}{4}+1$

5. Determine the roots of each equation to 3 decimal places.
   a) $5x^2+5x-2=0$ b) $3w^2-8w+3=0$

6. A rectangular swimming pool measuring 10 m by 4 m is surrounded by a deck of uniform width. The combined area of the deck and the pool is $135m^2$. What is the width of the deck?

7. For the quadratic relation $y=-3x^2-12x+7$ use partial factoring to find two points which are equidistant from the axis of symmetry.

8. Find the equation (in form $y=ax^2+bx+c$) of the quadratic relation with roots -6 and -4, and through the point $(4,40)$.

9. Find the coordinates of the points of intersection of
   a) $y=-(x+1)^2+4$ and $y=-4x+4$
   b) $y=-(x+1)^2+4$ and $y=3x+13$

10. Simplify.
    a) $3\sqrt{2}-5\sqrt{8}$ b) $\sqrt{48}-5\sqrt{3}$ c) $2\sqrt{45}-\sqrt{80}+3\sqrt{20}$

11. Simplify.
    a) $\left(5\sqrt{2}\right)^2-\left(6\sqrt{3}\right)^2$ b) $3\sqrt{5}\left(\sqrt{8}+2\sqrt{18}\right)$ c) $\left(\sqrt{12}-3\sqrt{5}\right)\left(\sqrt{12}+3\sqrt{5}\right)$ d) $\dfrac{2-\sqrt{2}}{3+\sqrt{2}}$

Answers

1. a) $y = 2(x-3)^2 - 15$ **2**. a) minimum, -15 **3**. $\left(\frac{-5}{6}, \frac{-37}{12}\right)$

4. a) 0,3 b) $\frac{3}{2}, \frac{-1}{4}$ c) $-3 \pm \sqrt{11}$ d) $\frac{1 \pm \sqrt{33}}{4}$ **5**. a) $-1.306, 0.306$ b) $0.451, 2.215$ **6**. 2.5 m

7. $(0,7)(-4,7)$ **8**. $y = \frac{1}{2}x^2 + 5x + 12$ **9**. a) $(1,0)$ b) no intersection. **10**. a) $-7\sqrt{2}$ b) $-\sqrt{3}$ c) $8\sqrt{5}$

11. a) -58 b) $24\sqrt{10}$ c) -33 d) $\frac{8 - 5\sqrt{2}}{7}$

Unit 4

1. What is the asymptote of the function $y = b^x$, where $b > 0$?
2. For what values of b will the function $y = b^x$ be always increasing?
3. Simplify (ideally without using a calculator!)

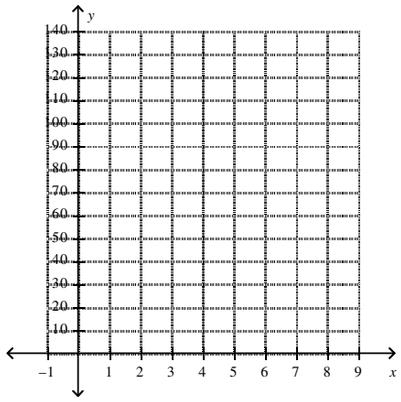
   a) $4^{\frac{3}{2}}$ b) $125^{\frac{-2}{3}}$ c) $\left(16^{\frac{-3}{5}}\right)^{\frac{-10}{3}}$

4. Simplify

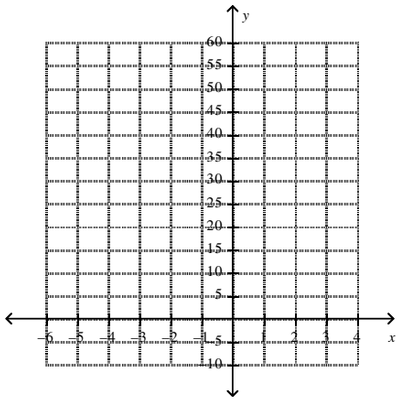
   a) $(2w)^4 \div (2w^2)^{-3}$ b) $\left(\frac{x^4}{49}\right)\left(\frac{49^{\frac{1}{2}}}{x^{\frac{3}{2}}}\right)^3$

5. For each of the following relations, state the equation of the horizontal asymptote (HA), and graph the relation labelling as many ordered pairs as appropriate:

a) $y = 3^{x-1}$ HA: _____



b) $y = \left(\frac{1}{2}\right)^x + 8$ HA: _____



6. Solve each equation.

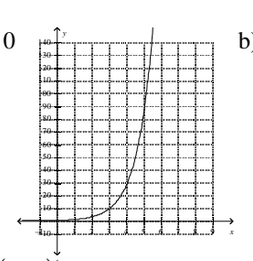
   a) $3^{2x+11} = 27^{x-5}$ b) $16^{3-x} = \left(\frac{1}{2}\right)^{2x+4}$ c) $14(8)^x = 7$

7. There are 300 fans in the Stompin' Tom fan club and the number of fans grows by 12% each year.
   a) Write an equation relating *N*, the number of fans, and *t*, the number of years elapsed. (Currently $t = 0$).
   b) Determine the number of fans in the club after 8 years.

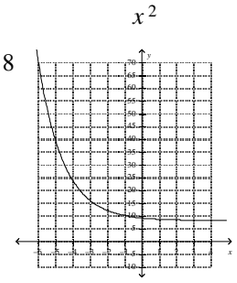
Answers

1. $y = 0$ **2**. $b > 1$ **3**. a) 8 b) $\frac{1}{25}$ c) 256 **4**. a) $128w^{10}$ b) $\frac{7}{x^{\frac{1}{2}}}$

5. a) HA: $y = 0$



b) HA: $y = 8$



6. a) 26 b) 8 c) $\frac{-1}{3}$

7. a) $N = 300(1.12)^t$ b) 743

Unit 5

1. Solve ΔABC in which $AB = 22.3$ cm, $BC = 26.8$ cm, and $\angle ABC = 113°$.
2. The point $Q(-2,-5)$ lies on the terminal arm of an angle θ in standard position. Determine the values of $\sin\theta$, $\cos\theta$, $\tan\theta$.
3. Determine the acute angle between the line $2x + 3y = 6$ and the y-axis.
4. Solve for θ to the nearest degree.
   a) $\sin\theta = -0.5150$ b) $\tan\theta = 1.6$
5. Without a calculator evaluate $\sin 90° + \cos 180° + \sin 70° - \cos 20°$
6. State the exact value of $\sin 240°$.
7. Solve ΔABC in which
   a) $\angle A = 55°$, $a = 7.1$, $b = 9.6$. b) $\angle A = 44°$, $a = 9.3$, $b = 12.3$.
8. Two planes flying at the same altitude are 3000.0 m apart when they spot a raft on the sea below them. The angles of depression to the raft are $47°$ and $38°$. Find the distance from the raft to the closest plane.
9. Evaluate $\sec 97°$ to four decimal places.
10. What is the exact value of $\cot 330°$?
11. Given that $\csc\alpha = \frac{17}{15}$, find 2 values of $\cos\alpha$.

Answers

1. $AC = 41.0$ cm, $\angle BAC = 37°$, $\angle ACB = 30°$ **2.** $\frac{-5}{\sqrt{29}}, \frac{-2}{\sqrt{29}}, \frac{5}{2}$ **3.** 56.3°. **4.** a) 211°,329° b) 58°,238°

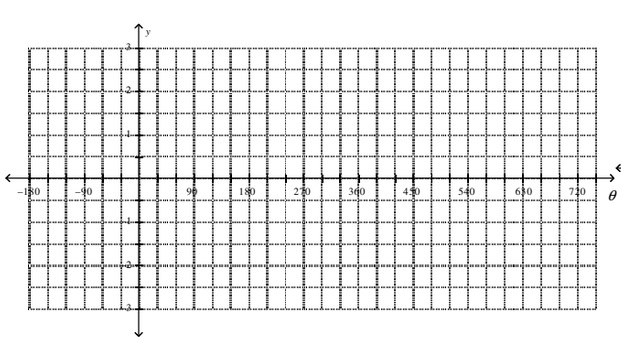
5. 0 **6.** $\frac{-\sqrt{3}}{2}$ **7.** a) no solution b) $\angle B = 67°, \angle C = 69°, c = 12.5$ OR $\angle B = 113°, \angle C = 23°, c = 5.2$

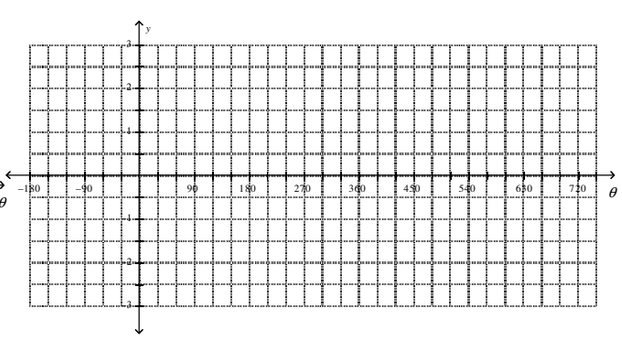
8. 1854.0 m **9.** -8.2055 **10.** $-\sqrt{3}$ **11.** $\frac{\pm 8}{17}$

Unit 6

1. For each of the following, complete the table and sketch for at least one cycle:

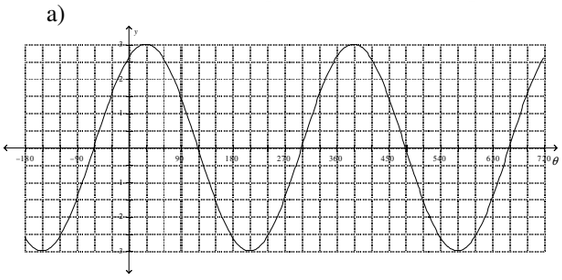
Function	**Amplitude (*a*)**	**Period**	**Phase Shift (*ps*)**	**Vertical Shift (*vs*)**
a) $y = -2\cos\left(\frac{1}{2}\theta\right) + 1$				
b) $y = \sin(2\theta + 60°)$				





2. Write the sinusoidal equation which best represents each of the following:

a)



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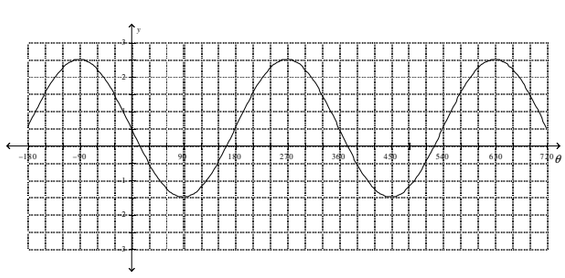


3. Because of the tide, the depth of the water in a harbour is modelled by the equation $d = -2.5\cos 30t + 7.5$, where *d* represents the depth of the water in metres and *t* represents the number of hours after midnight. (i.e. t = 0 means midnight, t = 1 means 1 A.M., and so on.)

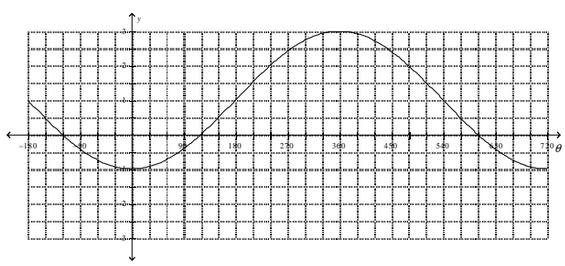
   a) Sketch the graph for one cycle, showing five ordered pairs.
   b) What is the depth of the water (to the nearest centimeter) at 7:30 a.m.?
   c) At what time will the depth of the water reach 9 metres for the first time?

4. The population of frogs in a particular swamp in Ontario follows a yearly cycle. This cycle can be modelled as a sinusoidal function. The maximum population of 300 frogs occurs at the beginning of October. The minimum population of 60 frogs occurs 6 months later, at the beginning of April. Determine the equation of this function relating the population, *P*, of frogs over time, *t*, in months. ($t = 0$ represents October, $t = 1$ represents November, and so on.)

Answers

1. a) 2, 720°, 0, +1 b) 1, 180°, −30°, 0

1. a)



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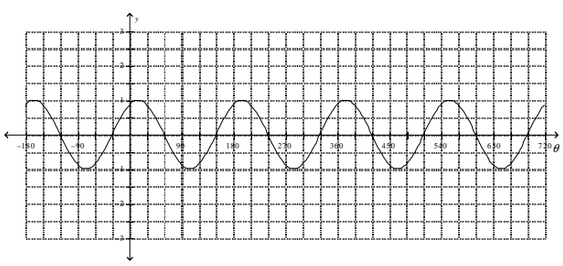
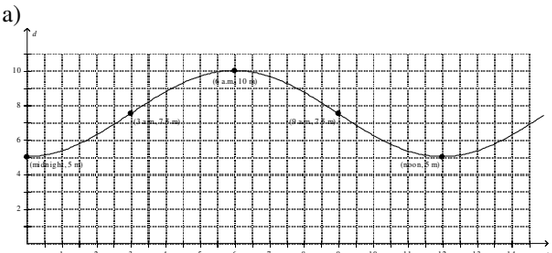


2. a) $y = 3\cos(\theta - 30°)$ b) $y = -2\sin\theta + \frac{1}{2}$

3. a)



3. b) 9.27 m c) 4:14 a.m.

4. $P = 120\cos 30t + 180$

Unit 7

1. Find t_8 for each of the following sequences.
   a) 384,192,96,48,... b) −34,−28,−22,−16,...
2. The second term of a geometric sequence is 6 and the third term is 9. List the first 5 terms of this sequence.
3. Determine the first term, the common difference and the general term for the arithmetic sequence where $t_{20} = 68$ and $t_{14} = 44$.
4. Write a recursive formula for the sequence 11,9,7,5,... .
5. Evaluate each of the following:
   a) $S_{15} = 3 + 7 + 11 + ...$ b) $15 + 11 + 7 + ... - 37$ c) $S_8 = 6 - 12 + 24 - 48 + ...$
   d) $7 + 14 + 28 + ... + 3584$ e) $1 + \frac{5}{2} + \frac{25}{4} + ... + \frac{15625}{64}$
6. Emily hits a nail with her hammer and the nail goes 30 mm into the board. On each successive hit the nail goes in 40% as far as it did on the previous hit.
   a) How far will the nail go in on the sixth hit?
   b) After the sixth hit, what is the total distance (to the nearest mm) that the nail has entered into the board?
7. A hockey arena has a total seating capacity of 15,690. The first row of seats around the rink has 262 seats. The number of seats in each subsequent row increases by 18. How many rows of seats are in the arena?
8. In an arithmetic series, the 12^{th} term is 15 and the sum of the first 15 terms is 105. What is the sum of the first three terms of the series?

Answers

1. a) 3 b) 8 **2**. 4, 6, 9, 13.5, 20.25 **3**. -8, 4, $4n - 12$ **4**. $t_n = t_{n-1} - 2$, $t_1 = 11$. **5**. a) 465 b) -154 c) -510

5. d) 7161 e) $\frac{19749}{64}$ **6**. a) 0.3072 mm b) 50 mm. **7**. 30 **8**. -15

3. Because of the tide, the depth of the water in a harbour is modelled by the equation $d = -2.5\cos 30t + 7.5$, where *d* represents the depth of the water in metres and *t* represents the number of hours after midnight. (i.e. t = 0 means midnight, t = 1 means 1 A.M., and so on.)

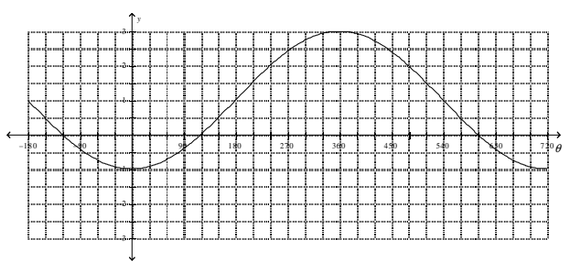
   a) Sketch the graph for one cycle, showing five ordered pairs.
   b) What is the depth of the water (to the nearest centimeter) at 7:30 a.m.?
   c) At what time will the depth of the water reach 9 metres for the first time?

4. The population of frogs in a particular swamp in Ontario follows a yearly cycle. This cycle can be modelled as a sinusoidal function. The maximum population of 300 frogs occurs at the beginning of October. The minimum population of 60 frogs occurs 6 months later, at the beginning of April. Determine the equation of this function relating the population, *P*, of frogs over time, *t*, in months. ($t = 0$ represents October, $t = 1$ represents November, and so on.)

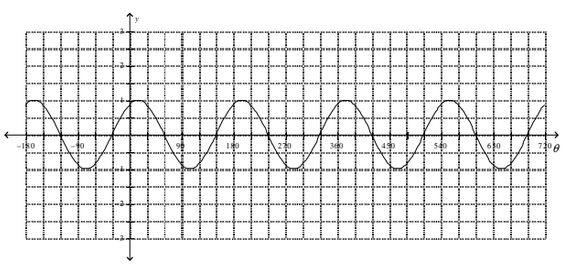
Answers

1. a) 2, 720°, 0, +1 b) 1, 180°, −30°, 0

1. a)

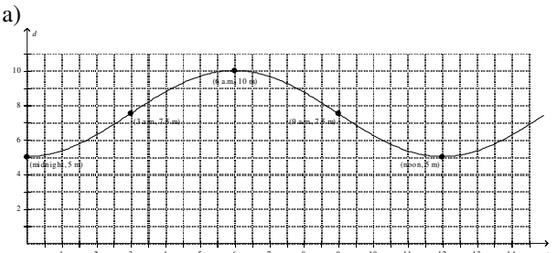


b)



2. a) $y = 3\cos(\theta - 30°)$ b) $y = -2\sin\theta + \frac{1}{2}$

3. a)



3. b) 9.27 m c) 4:14 a.m.

4. $P = 120\cos 30t + 180$

Unit 7

1. Find t_8 for each of the following sequences.
   a) 384,192,96,48,... b) −34,−28,−22,−16,...
2. The second term of a geometric sequence is 6 and the third term is 9. List the first 5 terms of this sequence.
3. Determine the first term, the common difference and the general term for the arithmetic sequence where $t_{20} = 68$ and $t_{14} = 44$.
4. Write a recursive formula for the sequence 11,9,7,5,... .
5. Evaluate each of the following:
   a) $S_{15} = 3 + 7 + 11 + ...$ b) $15 + 11 + 7 + ... - 37$ c) $S_8 = 6 - 12 + 24 - 48 + ...$
   d) $7 + 14 + 28 + ... + 3584$ e) $1 + \frac{5}{2} + \frac{25}{4} + ... + \frac{15625}{64}$
6. Emily hits a nail with her hammer and the nail goes 30 mm into the board. On each successive hit the nail goes in 40% as far as it did on the previous hit.
   a) How far will the nail go in on the sixth hit?
   b) After the sixth hit, what is the total distance (to the nearest mm) that the nail has entered into the board?
7. A hockey arena has a total seating capacity of 15,690. The first row of seats around the rink has 262 seats. The number of seats in each subsequent row increases by 18. How many rows of seats are in the arena?
8. In an arithmetic series, the 12^{th} term is 15 and the sum of the first 15 terms is 105. What is the sum of the first three terms of the series?

Answers

1. a) 3 b) 8 **2**. 4, 6, 9, 13.5, 20.25 **3**. -8, 4, $4n - 12$ **4**. $t_n = t_{n-1} - 2$, $t_1 = 11$. **5**. a) 465 b) -154 c) -510

5. d) 7161 e) $\frac{19749}{64}$ **6**. a) 0.3072 mm b) 50 mm. **7**. 30 **8**. -15