

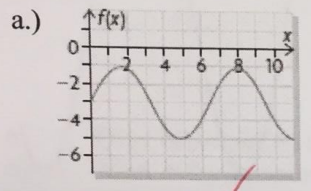
KNOWLEDGE (KU)		136
APPLICATION (AP)		13
THINKING (TI)		17
COMMUNICATION (CO)		level

Name: ANSWERS

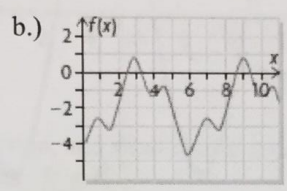
TEST #6 SINUSOIDAL FUNCTIONS

Knowledge & Understanding (KU)

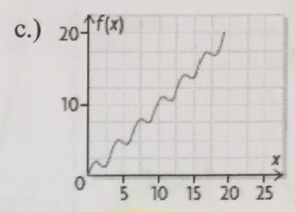
1. Determine whether the graph represents a periodic function. If it does, determine whether it represents a sinusoidal function.



periodic & sinusoidal ✓



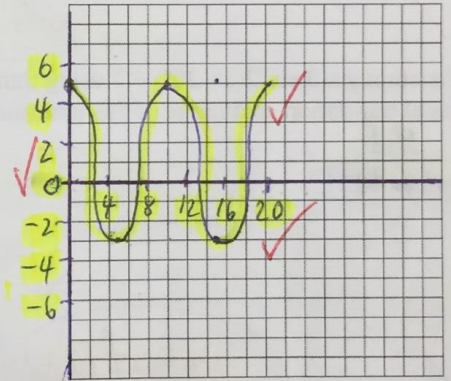
periodic only ✓
Out of 4



not periodic ✓

2. Sketch a graph of a periodic function whose period is 10 and whose range is $\{y \in \mathbb{R} \mid -3 \leq y \leq 5\}$.

Out of 3



3. Determine the equation of the cosine function whose graph has the following features.

	Amplitude	Period	Equation of the Axis	Horizontal Translation
a)	2	40°	$y = 0$	7°
b)	0.5	720°	$y = -3$	-56°

a.) $k = \frac{360^\circ}{40^\circ}$

- $k = 9$
- $a = 2$
- $c = 0$
- $d = 7$

$y = 2 \cos(9(x - 7^\circ))$

Out of 7

b.) $k = \frac{360^\circ}{720^\circ}$

- $k = 0.5$
- $a = 0.5$
- $c = -3$
- $k = -56^\circ$

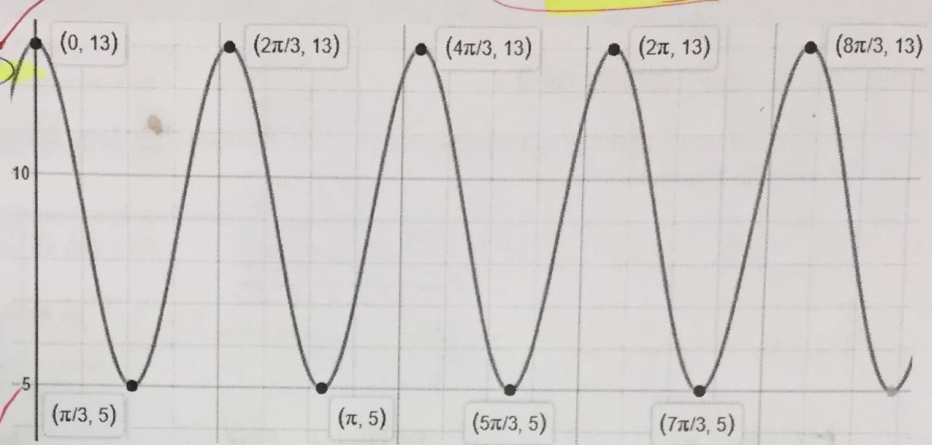
$y = 0.5 \cos(0.5(x + 56^\circ)) - 3$

4. a.) Given the graph of $y = 4\cos(3x) + 9$ below, determine the period, equation of axis, amplitude, and the range of the function.
 b.) Is the function Sinusoidal?
 c.) Calculate $h(45)$.
 d.) Determine values of x , $0^\circ \leq x \leq 360^\circ$, for $h(x) = 5$.

E.O.A.

Out of 10

a.) period $\frac{360}{k} = \frac{360}{3} = 120^\circ$
E.O.A. $y = 9$
amplitude $\frac{\text{max} - \text{min}}{2} = \frac{13 - 5}{2} = 4$



range $\{5 \leq y \leq 13 \mid y \in \mathbb{R}\}$

- b.) yes it is sinusoidal
 c.) $y = 4\cos(3(45)) + 9$
 $y = 6.17$

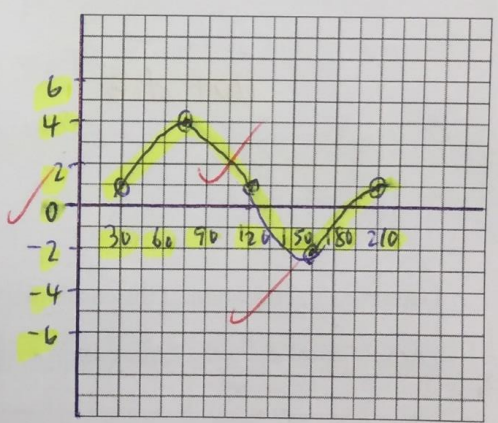
d.) $\frac{\pi}{3}$ or 60° , π or 180° , $\frac{5\pi}{3}$ or 300°

5. Graph the function $y = 3\sin(2(x - 30^\circ)) + 1$ using transformations. State the amplitude, period, equation of axis, horizontal translation, and range of this sinusoidal function.

E.O.A.

H.T.
 $a = 3 \rightarrow$ amplitude
period $= \frac{360}{k} = \frac{360}{2} = 180^\circ$
E.O.A. $y = 1$
H.T. $= 30^\circ$
range $\{-2 \leq y \leq 4 \mid y \in \mathbb{R}\}$

Out of 12

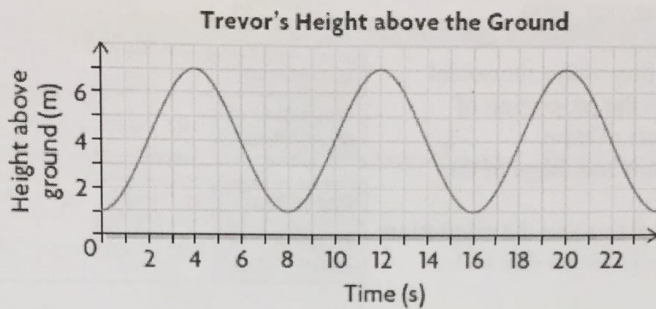


x	$y = \sin x$
0°	0
90°	1
180°	0
270°	-1
360°	0

x	$y = 3\sin(2(x - 30^\circ)) + 1$
$\frac{0}{2} + 30 = 30^\circ$	$3(0) + 1 = 1$
$\frac{90}{2} + 30 = 75^\circ$	$3(1) + 1 = 4$
$\frac{180}{2} + 30 = 120^\circ$	$3(0) + 1 = 1$
$\frac{270}{2} + 30 = 165^\circ$	$3(-1) + 1 = -2$
$\frac{360}{2} + 30 = 210^\circ$	$3(0) + 1 = 1$

Application (AP)

1. While riding on a Ferris wheel, Trevor's height above the ground in terms of time can be represented by the graph shown.



- What is the period of this function, and what does it represent?
- What is the equation of the axis? E.O.A
- What is the amplitude?
- What is the range of the function?
- After 24 s, when will Trevor be at the lowest height again?
- At what times is Trevor at the top of the wheel?
- When will his height be 4 m between 24 s and 30 s?

a.) period = 8 s → represents 1 time around the wheel ✓

b.) E.O.A. $y = \frac{\max + \min}{2}$

$$y = \frac{7+1}{2}$$

$$y = 4$$
 ✓

f.) 4s, 12s, 20s, ... etc

e.) 32s ✓

c.) amplitude $\frac{\max - \min}{2}$

$$= \frac{7-1}{2}$$

$$= 3$$
 ✓

g.) 26s ✓

Out of 8

d.) range $(1 \leq y \leq 7 | y \in \mathbb{R})$ ✓

2. Determine the coordinates of the point $P(x,y)$ resulting from a rotation of 70° centred at the origin and starting from the point $(3, 0)$. (Hint: use $\sin A = \frac{y}{r}$ and $\cos A = \frac{x}{r}$ where A is the angle)

$$\begin{aligned} A &= 70^\circ \\ r &= 3 \end{aligned}$$

$$\sin A = \frac{y}{r}$$

$$y = r \sin A$$

$$y = 3 \sin 70^\circ$$
 ✓

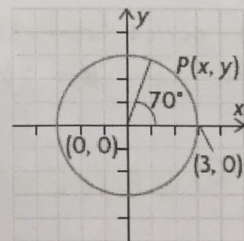
$$y = 2.81$$
 ✓

$$\cos A = \frac{x}{r}$$

$$x = r \cos A$$

$$x = 3 \cos 70^\circ$$
 ✓

$$x = 1.03$$
 ✓



Out of 5

$$P(x,y) = (1.03, 2.81)$$
 ✓

Thinking & Inquiry (TI)

1. The Moon is always half illuminated by the Sun. How much of the Moon we see depends on where it is in its orbit around Earth. The table shows the proportion of the Moon that was visible from Southern Ontario on days 1 to 74 in the year 2006.

Out of 9

Day of Year	1	4	7	10	14	20	24	29	34
Proportion of Moon Visible	0.02	0.22	0.55	0.83	1.00	0.73	0.34	0.00	0.28

- a) Determine the equation of the sinusoidal function that models the proportion of visible Moon in terms of time.
- b) Determine the domain and range of the function.
- c) Use the equation to determine the proportion of the Moon that is visible on day 110.

Day of Year	41	44	48	53	56	59	63	70	74
Proportion of Moon Visible	0.92	1.00	0.86	0.41	0.12	0.00	0.23	0.88	1.00

max = 1 & min = 0

a.) period = $44 - 14 = 30$
 $\therefore k = \frac{360^\circ}{30}$
 $k = 12$ ✓

amplitude = $\frac{\text{max} - \text{min}}{2}$
 $= \frac{1 - 0}{2} = 0.5$ ✓
 $\therefore a = 0.5$ ✓

G.O.A. $\frac{\text{max} + \text{min}}{2}$
 $= \frac{1 + 0}{2} = 0.5$ ✓
 $\therefore c = 0.5$ ✓

$d = 14$ ✓
 $\therefore y = 0.5 \cos(12(x - 14)^\circ) + 0.5$ ✓

b.) domain $\{0 \leq x \leq 365 \mid x \in \mathbb{R}\}$ ✓
 range $\{0 \leq y \leq 1 \mid y \in \mathbb{R}\}$ ✓

c.) $y = 0.5 \cos(12(110 - 14)^\circ) + 0.5$
 $y = 0.65$ ✓
 $\therefore 65\%$ ✓

2. Determine the equation of a cosine function that would have the range $\{y \in \mathbb{R} \mid -1 \leq y \leq 7\}$ and a period of 720° .

max = 7
min = -1

$k = \frac{360^\circ}{\text{period}}$
 $k = \frac{360^\circ}{720}$
 $k = 0.5$ ✓

amplitude = $\frac{\text{max} - \text{min}}{2}$
 $= \frac{7 - (-1)}{2} = 4$ ✓
 $\therefore a = 4$ ✓

G.O.A. $y = \frac{\text{max} + \text{min}}{2}$
 $y = \frac{7 - 1}{2} = 3$ ✓
 $\therefore c = 3$ ✓

$\therefore y = 4 \cos(0.5x) + 3$ ✓

Out of 3

3. Describe a procedure for writing the equation of a sinusoidal function based on a given graph.

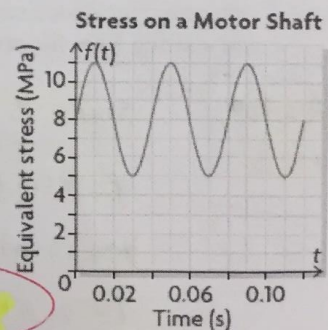
1.) find amplitude (a) $\rightarrow \frac{\text{max} - \text{min}}{2}$ ✓

2.) find period from the graph and then use $k = \frac{360^\circ}{\text{period}}$ ✓

3.) find G.O.A (c) $\rightarrow \frac{y = \text{max} + \text{min}}{2}$ ✓

4.) find the peak (max) to find horizontal translation (d) ✓

5.) Equation will be in the form $y = a \cos(k(x - d)) + c$ ✓



Out of 5