MPM2D
October 20, 2016

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COMMUNICATION		

Name: ANSWERS

UNIT 2: ANALYTIC GEOMETRY

KNOWLEDGE

MULTIPLE CHOICE out of 5

Questions #1 to 5 are multiple choice. Circle the letter that best completes the statement or answers the question.

1. Line segment AB has endpoints $A(5, -3)$ and $B(-7, -5)$. What are the coordinates of the midpoint of segment AB?

A
a. $(-1, -4)$
~~b.~~ $(1, -4)$
~~c.~~ $(1, -6)$
~~d.~~ $(-1, 6)$

2. Which one of the following equations represents a circle with its centre at $(0, 0)$ and a radius of 4?

C
~~a.~~ $\frac{x}{2}+\frac{y}{2}=4$
~~b.~~ $x^2+y=16$
c. $x^2+y^2=16$
~~d.~~ $x^2+y^2=4$

3. Which one of the following points does not lie within the circle represented by the equation $2x^2+2y^2=9$?

B
~~a.~~ $(2, 0)$
b. $(-2, -1)$
~~c.~~ $(1.5, 1.5)$
~~d.~~ $(-1, 0.5)$

4. Which of the following lines is the right bisector of the segment joining the points $(4, -2)$ and $(-4, -2)$?

B
~~a.~~ $y=-2$
b. $x=0$
~~c.~~ $y=4x-2$
~~d.~~ $y=x$

5. What is the length of the line segment that has endpoints at $(3, -2)$ and $(1, -7)$?

D
~~a.~~ 13
~~b.~~ $\sqrt{97}$
~~c.~~ 7
d. $\sqrt{29}$

MATCHING out of 6

6. *Match the words with the corresponding definition.*
~~a.~~ a point that divides a line segment into two equal line segments
~~b.~~ a segment from a vertex of a triangle meeting the opposite side in a right angle
~~c.~~ a segment from a vertex of a triangle to the midpoint of the opposite side
~~d.~~ a segment from a vertex of a triangle to another vertex of the same triangle
~~e.~~ a segment through the midpoint of a side of a triangle, perpendicular to that side
~~f.~~ a series of points that satisfy an equation of the form $x^2+y^2=r^2$
~~g.~~ a series of points that satisfy an equation of the form $y=mx+b$

a midpoint
b altitude
c median
e perpendicular bisector
g line
f circle

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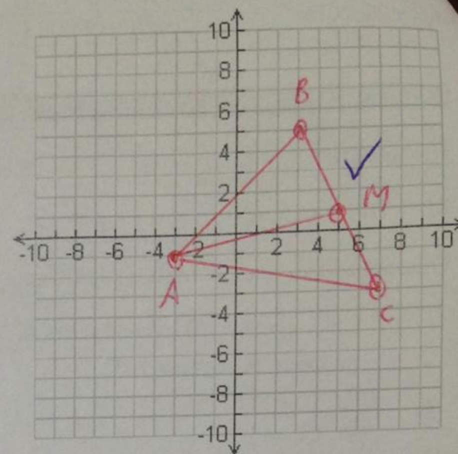
SHORT ANSWER

7. A triangle has vertices at $A(-3, -1)$, $B(3, 5)$, and $C(7, -3)$. Determine an equation for the **median** from vertex A.



$M_{BC} = \left(\frac{3+7}{2}, \frac{5+(-3)}{2}\right)$ ✓

$M_{BC} = (5, 1)$ ✓

$m_{AM} = \frac{1-(-1)}{5-(-3)}$ ✓

$m_{AM} = \frac{1}{4}$ ✓

$\boxed{\begin{array}{l} y=-1 \\ x=-3 \\ m=\frac{1}{4} \\ b=? \end{array}}$

$y = mx + b$

✓ $(-1) = \left(\frac{1}{4}\right)(-3) + b$ ✓

$(4)\ -1 = \frac{(4)\ -3}{4} + b\ (4)$

$-4 = -3 + 4b$ ✓

$\frac{-1}{4} = \frac{4b}{4}$

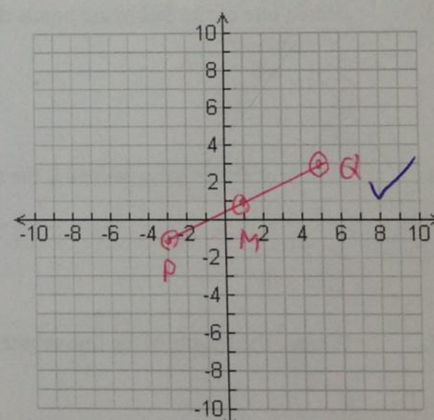
$\frac{-1}{4} = b$ ✓

$\therefore y = \frac{1}{4}x - \frac{1}{4}$ ✓ ← median from A

Out of 10

8. $P(-3, -1)$ is one endpoint of PQ. $M(1, 1)$ is the midpoint of PQ. Determine the coordinates of endpoint Q. Explain your solution.

$Q(x, y)$



$(1, 1) = \left(\frac{-3+x}{2}, \frac{-1+y}{2}\right)$ ✓

$\frac{1}{1} = \frac{-3+x}{2}$ ✓ | $\frac{1}{1} = \frac{-1+y}{2}$ ✓

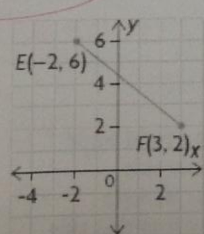
$2 = -3 + x$ | $2 = -1 + y$

$5 = x$ | $3 = y$

$\therefore Q(5, 3)$ ✓

Out of 5

9. Determine the length of the line segment



$d_{EF} = \sqrt{(3-(-2))^2 + (2-6)^2}$ ✓

$d_{EF} = \sqrt{41}$ ✓

$d_{EF} = 6.4$ units

out of 2

10. The lengths of the sides in a quadrilateral are $PQ = 4.5$ units, $QR = 4.5$ units, $RS = 4.5$ units, and $SP = 4.5$ units. What types of quadrilateral could $PQRS$ be? What other information is needed to determine the exact type of quadrilateral?

- PQRS could be a square or rhombus ✓
- need slopes of each side
  eg/ square perp. slopes ✓
  rhombus para. slopes

out of 2

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11. Use the given information to write an equation for a circle with centre (0, 0). $\boxed{x^2+y^2=r^2}$

a.) radius of 11 units

$x^2+y^2=(11)^2$ ✓

$x^2+y^2=121$ ✓

b.) x-intercepts $(-9, 0)$ and $(9, 0)$ $r=9$

$x^2+y^2=(9)^2$ ✓

$x^2+y^2=81$ ✓

c.) diameter of 12 units

$r=\frac{12}{2}=6$ ✓

$x^2+y^2=(6)^2$ ✓

$x^2+y^2=36$ ✓

Out of 7

12. Calculate the distance between point $A(6, 5)$ and the line $y = 2x + 3$. ✓

$m=\frac{2}{1}$ $b=3$

$m=-\frac{1}{2}$ is perp. to $y=2x+3$

$x=6$
$y=5$
$m=-\frac{1}{2}$
$b=?$ ✓

$y=mx+b$

$(5)=\left(-\frac{1}{2}\right)(6)+b$ ✓

$5=-3+b$

$8=b$ ✓

$\therefore y=-\frac{1}{2}x+8$ ✓

① $y=2x+3$ ✓

② $y=-\frac{1}{2}x+8$ ✓

sub ① into ②

$(2)2x+3(2)=-\frac{1}{2}x(2)+8(2)$ ✓

$4x+6=-x+16$

$\frac{5x}{5}=\frac{10}{5}$

$x=2$ ✓

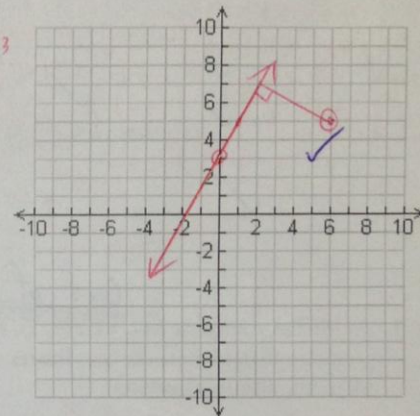
sub $x=2$ into ①

① $y=2(2)+3$ ✓

$y=7$ ✓

$\therefore (x,y)=(2,7)$ ✓

Out of 14



distance from (6,5) to (2,7)

$d=\sqrt{(2-6)^2+(7-5)^2}$ ✓

$d=\sqrt{20}$

$d=4.5$ units ✓

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APPLICATION

1. $\triangle QRS$ has vertices at $Q(2, 6)$, $R(-3, 1)$, and $S(6, 2)$. Determine the perimeter of the triangle.

$d_{RS}=\sqrt{(6-(-3))^2+(2-1)^2}$ ✓

$d_{RS}=\sqrt{82}$

$d_{RS}=9.1$ units ✓

$d_{SQ}=\sqrt{(2-6)^2+(6-2)^2}$ ✓

$d_{SQ}=\sqrt{32}$

$d_{SQ}=5.7$ units ✓

$d_{QR}=\sqrt{(-3-2)^2+(1-6)^2}$ ✓

$d_{QR}=\sqrt{50}$

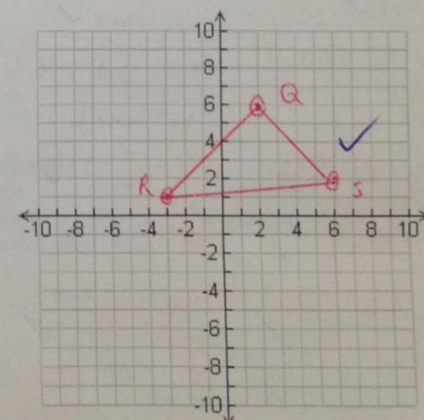
$d_{QR}=7.1$ units ✓

Perimeter

$=9.1+5.7+7.1$ ✓

$=21.9$ units ✓

Out of 9



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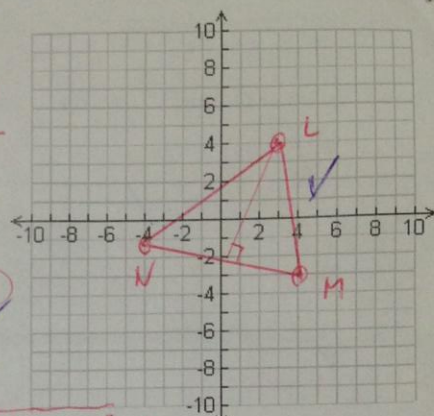
2. $\triangle LMN$ has vertices at $L(3, 4)$, $M(4, -3)$, and $N(-4, -1)$. Use analytic geometry to determine the area of the triangle.



$m_{MN} = \frac{-1-(-3)}{-4-4}$ ✓

$m_{MN} = -\frac{1}{4}$ ✓

$m = 4$ is perp. to this ✓

$x = 3$, $y = 4$, $m = 4$, $b = ?$

$y = mx + b$

$(4) = (4)(3) + b$

$4 = 12 + b$

$-8 = b$ ✓

$\therefore y = 4x - 8$ ✓

① $y = 4x - 8$ ✓

② $y = -\frac{1}{4}x - 2$ ✓

sub ① into ②

$(4)4x - 8^{(4)} = -\frac{1}{4}x^{(4)} - 2(4)$ ✓

$16x - 32 = -1x - 8$

$\frac{17x}{17} = \frac{24}{17}$ ✓

$m = -\frac{1}{4}$, $x = 4$, $y = -3$, $b = ?$

$y = mx + b$

$(-3) = (-\frac{1}{4})(4) + b$ ✓

$-3 = -1 + b$

$-2 = b$ ✓

$\therefore y = -\frac{1}{4}x - 2$ ✓

sub $x = \frac{24}{17}$ into ①

① $y = 4(\frac{24}{17}) - 8$ ✓

$y = \frac{-40}{17}$ ✓

$\therefore (x, y) = (\frac{24}{17}, \frac{-40}{17})$ ✓

O

$d_{MN} = \sqrt{(4-(-4))^2 + (-3-(-1))^2}$

$d_{MN} = \sqrt{68} = 8.2$ base ✓

$d_{LO} = \sqrt{(\frac{24}{17} - 3)^2 + (\frac{-40}{17} - 4)^2}$

$d_{LO} = 6.6$ height ✓

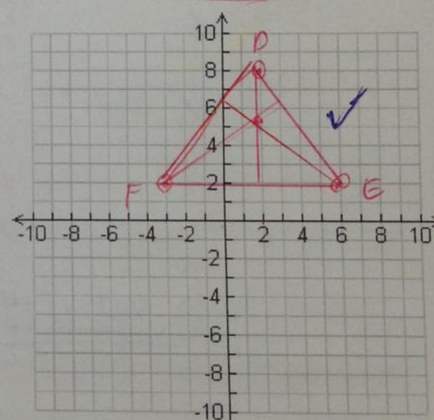
$A = \frac{bh}{2}$

$A = \frac{(6.6)(8.2)}{2}$ ✓

$A = 27 \text{ units}^2$ ✓

Out of 22

3. $\triangle DEF$ has vertices at $D(2, 8)$, $E(6, 2)$, and $F(-3, 2)$. Use analytic geometry to determine the coordinates of the orthocentre (the point where the altitudes intersect).



equation of altitude from F

$m_{DE} = \frac{2-8}{6-2}$ ✓

$m_{DE} = -\frac{3}{2}$ ✓

$m = \frac{2}{3}$ is perp. to this ✓

use $\frac{2}{3}$ & $F(-3, 2)$

$y = mx + b$

$(2) = (\frac{2}{3})(-3) + b$ ✓

$2 = -2 + b$

$4 = b$ ✓

$\therefore y = \frac{2}{3}x + 4$ ✓

equation of altitude from E

$m_{FD} = \frac{8-2}{2-(-3)}$ ✓

$m_{FD} = \frac{6}{5}$ ✓

$m = \frac{-5}{6}$ is perp. to this ✓

use $\frac{-5}{6}$ & $E(6, 2)$

$y = mx + b$

$(2) = (\frac{-5}{6})(6) + b$ ✓

$2 = -5 + b$

$7 = b$ ✓

$\therefore y = \frac{-5}{6}x + 7$ ✓

P.O.I

① $y = \frac{2}{3}x + 4$ ✓

② $y = \frac{-5}{6}x + 7$ ✓

sub ① into ②

$^{2(6)}\frac{2}{3}x + 4^{(6)} = \frac{-5}{6}^{(6)}x + 7^{(6)}$ ✓

$4x + 24 = -5x + 42$

$\frac{9x}{9} = \frac{18}{9}$

$x = 2$ ✓

sub $x = 2$ into ②

② $y = \frac{-5(2)}{6} + 7$ ✓

$y = \frac{-5}{3} + \frac{7 \times 3}{1 \times 3}$

$y = \frac{16}{3}$ ✓

$\therefore (x, y) = (2, \frac{16}{3})$ ✓

Out of 20

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