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COMMUNICATION		Level

Name:

ANSWERS

UNIT 5: QUADRATIC MODELS & QUADRATIC EQUATIONS

KNOWLEDGE

1. For the following equation in standard form $y = -2x^2 + 4x + 6$

a) Convert to vertex form by completing the square.

$$c = \left(\frac{b}{2}\right)^2$$

$$c = \left(\frac{-2}{2}\right)^2$$

$$c = (-1)^2$$

$$c = 1$$

$$y = (-2x^2 + 4x) + 6$$

$$y = -2(x^2 - 2x) + 6$$

$$y = -2(x^2 - 2x + 1 - 1) + 6$$

$$y = -2(x^2 - 2x + 1) + 6 + 2$$

$$y = -2(x - 1)^2 + 8$$

out of 5

b) Using the vertex form from part a), describe the transformations applied to $y = x^2$ and then sketch a graph of the function.

$$y = -2(x - 1)^2 + 8$$

$a = -2$ → vertical reflection about the x-axis ✓

→ vertical stretch by a factor of 2 ✓

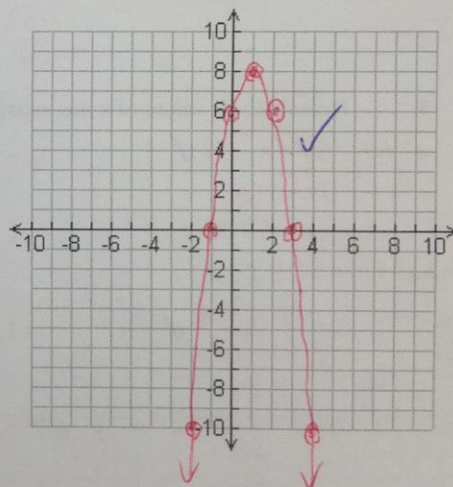
$h = 1$ → horizontal shift right 1 unit ✓

$k = 8$ → vertical shift up 8 units ✓

vertex = (1, 8)

Step pattern
1(-2), 3(-2), 5(-2)---etc ✓
-2, -6, -10---etc

out of 10



2. Use the given information to determine the equation of each quadratic relation in vertex form.

vertex at (-3, 2), passes through (-1, 4)

$$a = 0.5$$

$$h = -3$$

$$k = 2$$

$$y = 0.5(x + 3)^2 + 2$$

$$y = a(x - h)^2 + k$$

$$(4) = a[(-1) - (-3)]^2 + (2)$$

$$4 = a(-1 + 3)^2 + 2$$

$$4 = a(2)^2 + 2$$

$$4 - 2 = 4a$$

$$\frac{2}{4} = \frac{4a}{4}$$

$$0.5 = a$$

out of 5

3. Without graphing, determine the number of x-intercepts that each relation has.

a) $y = 2x^2 + 8x + 14$
 $a=2$ $b=8$ $c=14$ ✓

$$b^2 - 4ac$$

$$(8)^2 - 4(2)(14) ✓$$

$$-48 < 0 ✓$$

∴ no x-ints ✓

Out of 11

b) $y = 2x(x - 5) + 7$

$$y = 2x^2 - 10x + 7 ✓$$

$$a=2$$
 $b=-10$ $c=7$ ✓

$$b^2 - 4ac ✓$$

$$(-10)^2 - 4(2)(7) ✓$$

$$44 > 0 ✓$$

∴ 2 x-ints ✓

4. Solve each of the following equations by factoring.

a) $6x^2 - 11x - 10 = 0$

$$P = -60$$

$$(2x-5)(3x+2) = 0 ✓$$

$$S = -11$$

$$-15 \times 4 = -60 ✓$$

$$-15 + 4 = -11 ✓$$

$$\frac{-15}{6}, \frac{4}{6} ✓$$

$$\frac{-5}{2}, \frac{2}{3} ✓$$

$$x = \frac{5}{2} ✓$$

$$x = -\frac{2}{3} ✓$$

Out of 9

b) $2x^2 - 9x = 5$

$$(2x^2 - 9x - 5) = 0 ✓$$

$$P = -10$$

$$S = -9$$

$$(x-5)(2x+1) = 0 ✓$$

$$-10 \times 1 = -10 ✓$$

$$-10 + 1 = -9 ✓$$

$$x = 5 ✓$$

$$x = -\frac{1}{2} ✓$$

$$\frac{-10}{2}, \frac{1}{2} ✓$$

$$-\frac{5}{1} ✓$$

5. Determine all values of x that satisfy the equation $x^2 + 4 = 3x(x - 5)$. Round your answers to two decimal places.

$$x^2 + 4 = 3x(x - 5) ✓$$

$$x^2 + 4 = 3x^2 - 15x$$

$$x^2 - 3x^2 + 15x + 4 = 0$$

$$-2x^2 + 15x + 4 = 0 ✓$$

$$a = -2$$
 $b = 15$ $c = 4$ ✓

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-15) \pm \sqrt{(15)^2 - 4(-2)(4)}}{2(-2)} ✓$$

$$x = \frac{-15 \pm \sqrt{257}}{-4}$$

$$x = \frac{-15 \pm 16}{-4}$$

$$x_1 = \frac{-15 + 16}{-4}$$

$$x_2 = \frac{-15 - 16}{-4}$$

$$x_1 = -0.25 ✓$$

$$x_2 = 7.75 ✓$$

Out of 6

6. Determine the value of c needed to create a perfect-square trinomial.

a) $(x^2 + 19x) + c$

$$c = \left(\frac{b}{2}\right)^2$$

$$c = \left(\frac{19}{2}\right)^2 ✓$$

$$c = (9.5)^2$$

$$c = 90.25 ✓$$

Out of 6

b) $(2x^2 + 12x) + c$

$$2(x^2 + 6x) ✓$$

$$c = \left(\frac{b}{2}\right)^2$$

$$c = \left(\frac{6}{2}\right)^2 ✓$$

$$c = (3)^2$$

$$c = 9 ✓$$

$$\therefore 2 \times 9 = 18$$

APPLICATION

1. A rectangular field is going to be completely enclosed by 100 m of fencing. Create a quadratic relation that shows how the area of the field will depend on its width. Then determine the dimensions of the field that will result in an area of 575 m². Round your answers to the nearest hundredth of a metre.

$$P = 2l + 2w \quad \checkmark$$

$$A = l \times w$$

$$\frac{100}{2} = \frac{2l}{2} + \frac{2w}{2} \quad \checkmark$$

$$50 = l + w$$

$$\textcircled{1} \quad 50 - l = w \quad \checkmark$$

$$\textcircled{2} \quad 575 = l \times w \quad \checkmark$$

sub ① into ② $\checkmark$

$$575 = l(50 - l) \quad \checkmark$$

$$575 = 50l - l^2$$

$$0 = -l^2 + 50l - 575 \quad \checkmark$$

$$a = -1 \quad b = 50 \quad c = -575 \quad \checkmark$$

$$x = \frac{-(-50) \pm \sqrt{(-50)^2 - 4(-1)(-575)}}{2(-1)} \quad \checkmark$$

$$x = \frac{-50 \pm \sqrt{200}}{-2}$$

$$x = \frac{-50 \pm 14.1}{-2}$$

$$x_1 = \frac{-50 + 14.1}{-2} \quad x_2 = \frac{-50 - 14.1}{-2}$$

$$\textcircled{x_1 = 18} \quad \checkmark \quad \textcircled{x_2 = 32} \quad \checkmark$$

out of 12

$\therefore$ the dimensions of the field are 18m by 32m $\checkmark$

2. For the quadratic relation $y = 2x^2 - 4x - 6$,
i) write the equation in factored form
ii) determine the coordinates of the vertex
iii) write the equation in vertex form
iv) sketch the graph

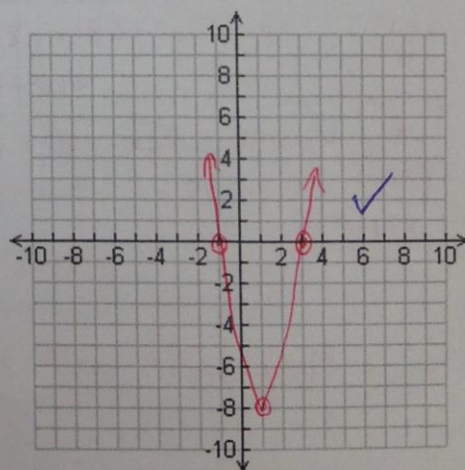
i.) $y = 2x^2 - 4x - 6$ $GCF = 2 \quad \checkmark$
 $y = 2(x^2 - 2x - 3) \quad \checkmark \quad p = -3$
 $y = 2(x-3)(x+1) \quad \checkmark \quad s = -2$
 $-3 \times 1 = -3 \quad \checkmark$
 $-3 + 1 = -2 \quad \checkmark$

ii) $0 = 2(x-3)(x+1) \quad \checkmark$
 $\downarrow \quad \downarrow$
 $x = 3 \quad x = -1 \quad \text{zeros}$
 $x \text{ of vertex } = \frac{-1 + 3}{2} = 1 \quad \checkmark$

y of vertex
 $y = 2(1)^2 - 4(1) - 6$
 $y = -8 \quad \checkmark$
 $\therefore \text{vertex} = (1, -8) \quad \checkmark$

iii) $a = 2$
 $h = 1$
 $k = -8$
 $y = 2(x-1)^2 - 8 \quad \checkmark$

out of 13



3. The volunteers at a food bank are arranging a concert to raise money. They have to pay a set fee to the musicians, plus an additional fee to the concert hall for each person attending the concert. The relation $P = -n^2 + 580n - 48\,000$ models the profit, P , in dollars, for the concert, where n is the number of tickets sold.

Calculate the number of tickets they must sell to break even. Determine the number of tickets they must sell to maximize the profit and determine the maximum profit.

$$P = 0 \leftarrow \text{break even} \checkmark$$

$$0 = -n^2 + 580n - 48\,000 \checkmark$$

$$0 = -(n^2 - 580n + 48\,000) \checkmark \quad P = 48\,000$$

$$0 = -(n - 100)(n - 480) \checkmark \quad S = -580 \checkmark$$

$$-100 \times 480 = 48\,000 \checkmark$$

$$-100 + (-480) = -580 \checkmark$$

$$n = 100 \checkmark$$

$$n = 480 \checkmark$$

$\therefore$ they must sell 100 tickets or 480 tickets to break even $\checkmark$

$$P = (-n^2 + 580n) - 48\,000$$

$$P = -(n^2 - 580n) - 48\,000 \checkmark$$

$$C = \left(\frac{b}{2}\right)^2$$

$$P = -(n^2 - 580n + 84\,100 - 84\,100) - 48\,000 \checkmark$$

$$C = \left(\frac{-580}{2}\right)^2 \checkmark$$

$$P = -(n^2 - 580n + 84\,000) - 48\,000 + 84\,000$$

$$C = (290)^2$$

$$P = -(n^2 - 580n + 84\,100) + 84\,100 - 48\,000 \checkmark$$

$$C = 84\,100$$

$$\therefore \text{vertex} = \begin{matrix} + & P \\ (290, & 36\,100) \end{matrix} \checkmark$$

$\therefore$ they must sell 290 tickets for a max profit of \$36,100 $\checkmark$

Out of 14