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UNIT 2 : NETWORK THEOREMS

* Sources of Electrical energy.

Electrical source is a basic network element that supplies energy to the network.

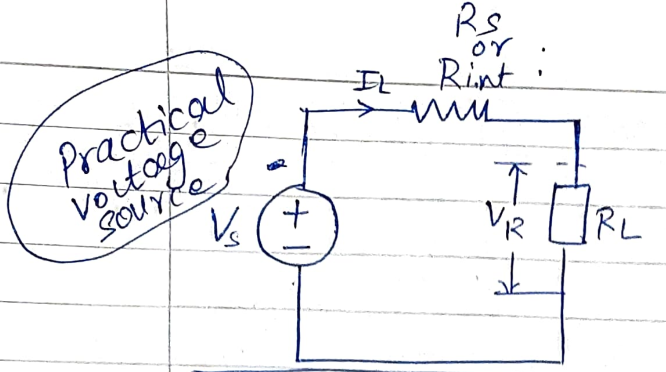
Electrical sources.

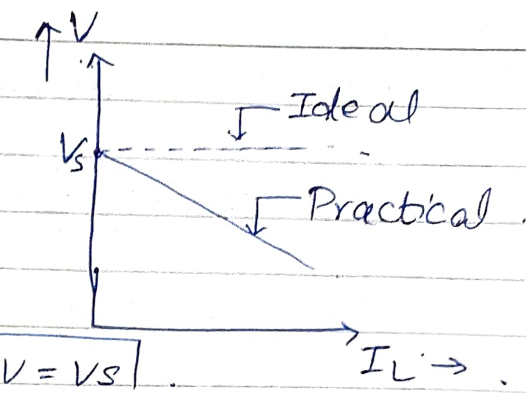
- Independent sources
  - Independent Voltage source
  - Independent current source
- Dependent sources
  - Dependent Voltage source
    - VDVS
    - CDVS
  - Dependent current source
    - CDCS
    - VDVS

① Independent voltage source.

Energy source that gives constant voltage across its terminals irrespective of the current drawn through its terminals is called "ideal voltage source".

→ But practically, it has a small internal resistance in series with it due to which voltage across its terminals decreases slightly as current increases.



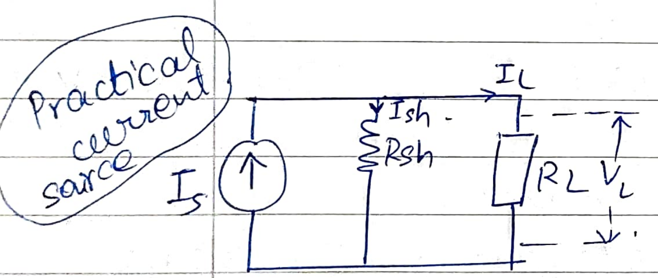


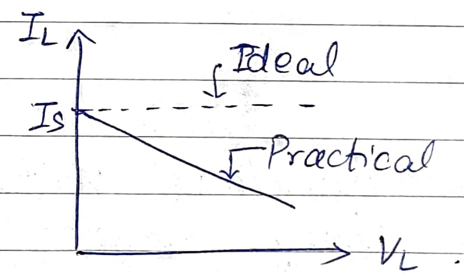
$\boxed{\text{If } R_L = 0 \Rightarrow I_L = 0 \Rightarrow V = V_s}$

② Independent current source

Source which gives constant current at its terminals irrespective of the voltage appearing across its terminals is called "ideal current source".

→ But practically, it has high internal resis. connected in parallel with it, due to which current slightly decreases as the voltage at its terminals increases.





$\boxed{I_L + I_{sh} = I_s \quad \text{As } I_{sh} \uparrow,\ I_L \downarrow}$

③ Dependent sources

Sources whose value depends on voltage or current, present somewhere else in the same circuit are called "dependent sources".

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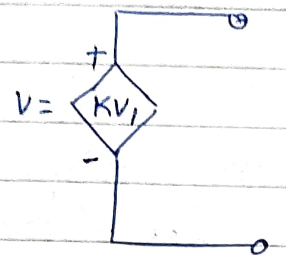
→ Such sources are indicated by a diamond and are further classified as below.

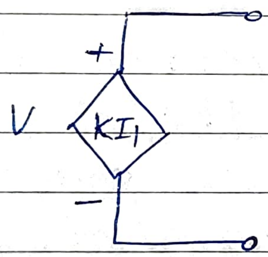
(i) Voltage dependent voltage source (VDVS)

It produces a voltage as a function of voltages elsewhere in given circuit.



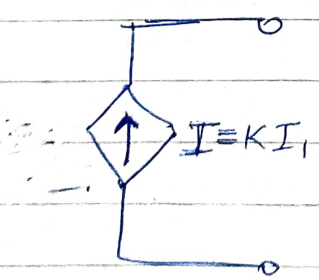
(ii) Current dependent voltage source (CDVS)

It produces voltage as a function of current elsewhere in the given circuit.



(iii) Current dependent current source (CDCS)

It produces a current as a function of current elsewhere in given circuit.



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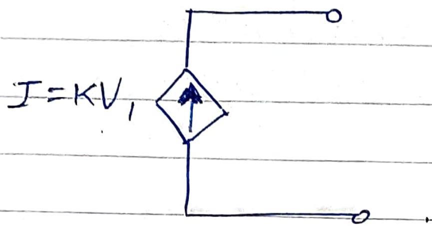
(iv) Voltage dependent current source (VDCS)

It produces a current as a function of voltage elsewhere in given circuit.

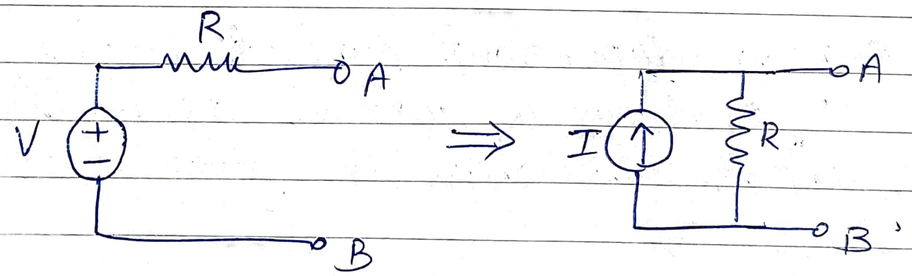


→ K is constant & V_1 & I_1 are voltage & current respectively, present elsewhere in given circuit.

★ Source Transformation

(i) Voltage to current source

If voltage source is connected in series with a resistor, the circuit can be converted into current source by connecting a resistor in parallel.



$\boxed{I = \frac{V}{R}}$

= (If voltage source is not having resistor in series, the transformation is not possible)

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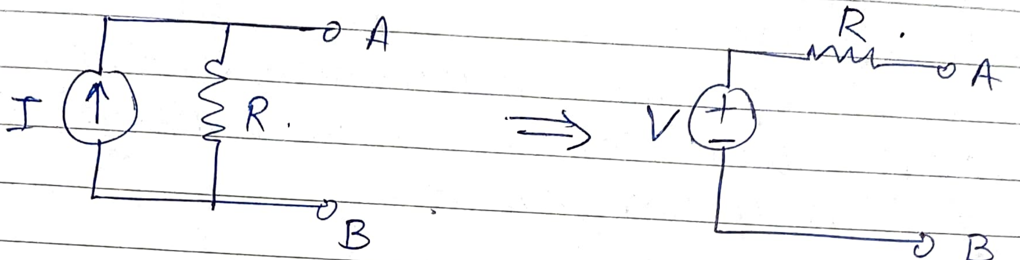
(ii) Current source to voltage source.

If a current source is connected in parallel with a resistor, it can be converted into voltage source, by connecting a resistor in series.



$\boxed{V = IR}$

★ Basic Network terminologies.

1.) Active network

Network that consists of an energy source (voltage or current) is called active network.

2.) Passive network

Network that consists of ~~and~~ passive elements like capacitor, resistor or inductor and doesn't contain ~~act~~ energy source is a passive network.

3.) Unilateral network

If the response of network depends on the direction of current, it is a unilateral network.

4.) **Bilateral network**

If the response (or characteristics) of network does not depend on direction of current through various element, it is called a bilateral network.

5.) **Linear network**

If network parameters does not change with temp., time, voltage, then it is a linear network.

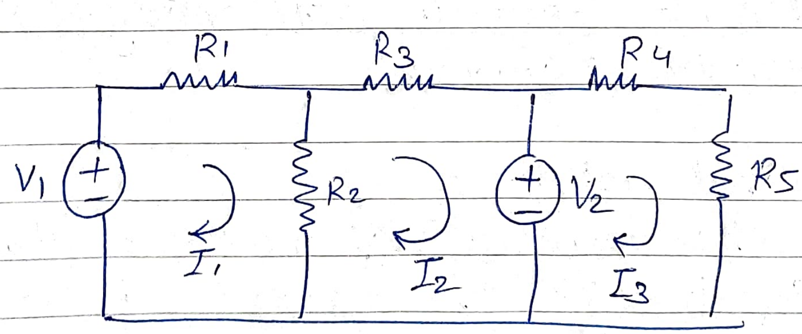
6.) **Non-linear network**

If network parameters change with time, temp., it is a non-linear network.

★ **Mesh Analysis**

KVL is applied to the circuit to get a set of loop or mesh equations.

Let us consider a circuit as below.



Mesh 1:

$$V_1 - I_1 R_1 - (I_1 - I_2) R_2 = 0$$

$$V_1 - I_1R_1 - I_1R_2 + I_2R_2 = 0$$

$$V_1 - I_1(R_1+R_2) + I_2R_2 = 0$$

$$V_1 = I_1(R_1+R_2) - I_2R_2 \rightarrow (1)$$

Mesh 2:

$$-V_2 - R_2(I_2 - I_1) - I_2R_3 = 0$$

$$-V_2 - I_2R_2 + I_1R_2 - I_2R_3 = 0$$

$$V_2 = I_1R_2 - I_2R_2 - I_2R_3$$

$$V_2 = I_1R_2 - I_2(R_2+R_3) \rightarrow (2)$$

Mesh 3:-

$$V_3 - I_3R_4 - I_3R_5 = 0$$

$$V_3 - I_3(R_4+R_5) = 0$$

$$V_3 = I_3(R_4+R_5) \rightarrow (3)$$

For mesh circuits, mesh equations can be directly written as

$$R_{11}I_1 \pm R_{12}I_2 \pm R_{13}I_3 = \pm V_1$$

$$\pm R_{21}I_1 + R_{22}I_2 \pm R_{23}I_3 = \pm V_2$$

$$\pm R_{31}I_1 \pm R_{32}I_2 + R_{33}I_3 = \pm V_2$$

$$\begin{bmatrix} \pm R_{11} & \pm R_{12} & \pm R_{13} \\ \pm R_{21} & \pm R_{22} & \pm R_{23} \\ \pm R_{31} & \pm R_{32} & \pm R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \pm V_1 \\ \pm V_2 \\ \pm V_3 \end{bmatrix}$$

Let $\Delta R = \begin{bmatrix} \pm R_{11} & \pm R_{12} & \pm R_{13} \\ \pm R_{21} & \pm R_{22} & \pm R_{23} \\ \pm R_{31} & \pm R_{32} & \pm R_{33} \end{bmatrix}$

By cramer's rule

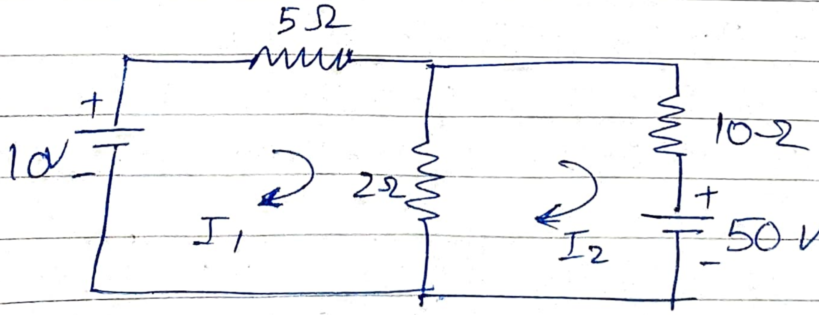
$$I_1 = \frac{\Delta R_1}{\Delta R} = \frac{\begin{bmatrix} V_1 & \pm R_{12} & \pm R_{13} \\ V_2 & \pm R_{22} & \pm R_{23} \\ V_3 & \pm R_{32} & \pm R_{33} \end{bmatrix}}{\Delta R}$$

$$I_2 = \frac{\Delta R_2}{\Delta R} = \frac{\begin{bmatrix} \pm R_{11} & V_1 & \pm R_{13} \\ \pm R_{21} & V_2 & \pm R_{23} \\ \pm R_{31} & V_3 & \pm R_{33} \end{bmatrix}}{\Delta R}$$

$$I_3 = \frac{\Delta R_3}{\Delta R} = \frac{\begin{bmatrix} \pm R_{11} & \pm R_{12} & R_{13} \\ \pm R_{21} & \pm R_{22} & R_{23} \\ \pm R_{31} & \pm R_{32} & R_{33} \end{bmatrix}}{\Delta R}$$

Numericals

1. For the given circuit, write mesh current equations and determine current



Applying KVL to loop 1.

$$10 - 5I_1 - 2(I_1 - I_2) = 0$$
$$10 - 5I_1 - 2I_1 + 2I_2 = 0$$
$$10 - 7I_1 + 2I_2 = 0$$
$$10 = 7I_1 - 2I_2 \rightarrow (1)$$

Applying KVL to loop 2.

$$-50 - 2(I_2 - I_1) - 10I_2 = 0$$
$$-50 - 2I_2 + 2I_1 - 10I_2 = 0$$
$$-50 - 12I_2 + 2I_1 = 0$$

$$-50 = -2I_1 + 12I_2 \rightarrow (2)$$

ie:

$$\begin{bmatrix} 10 \\ -50 \end{bmatrix} = \begin{bmatrix} 7 & -2 \\ -2 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta R = 84 - 4 = 80\,\Omega$$

$$\Delta R_1 = \begin{vmatrix} 10 & -2 \\ -50 & 12 \end{vmatrix} = 120 - 100 = 20\,\Omega$$

$$\Delta R_2 = \begin{vmatrix} 7 & 10 \\ -2 & -50 \end{vmatrix} = -350 + 20 = -330\,\Omega$$

By cramer's rule,

$$I_1 = \frac{\Delta R_1}{\Delta R} = \frac{20}{80} = 0.25\,A$$

$$I_2 = \frac{\Delta R_2}{\Delta R} = \frac{-330}{80} = -4.125\,A$$

Ans

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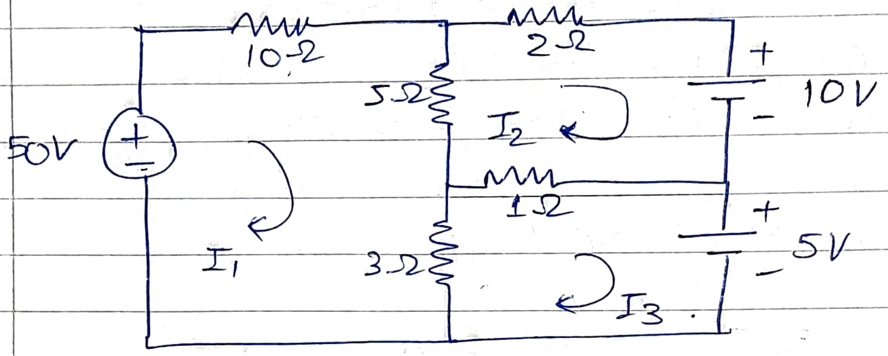
(2.) Find mesh currents I_1, I_2 & I_3 for the below network.



Applying KVL,

For loop 1,

$$50 - 10I_1 - 5(I_1 - I_2) - 3(I_1 - I_3) = 0$$
$$50 - 10I_1 - 5I_1 + 5I_2 - 3I_1 + 3I_3 = 0$$

$$50 = 18I_1 - 5I_2 - 3I_3 \rightarrow (1)$$

For loop 2,

$$-2I_2 - 10 - 1(I_2 - I_3) - 5(I_2 - I_1) = 0$$
$$-2I_2 - 10 - I_2 + I_3 - 5I_2 + 5I_1 = 0$$

$$10 = 5I_1 - 8I_2 + I_3 \rightarrow (2)$$

For loop 3,

$$-5 - 3(I_3 - I_1) - 1(I_3 - I_2) = 0$$
$$-5 - 3I_3 + 3I_1 - I_3 + I_2 = 0$$
$$5 = 3I_1 + I_2 - 4I_3 \rightarrow (3)$$

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By (1), (2) & (3).

$$\begin{bmatrix} 50 \\ 10 \\ 5 \end{bmatrix} = \begin{bmatrix} 18 & -5 & -3 \\ 5 & -8 & 1 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\Delta R = \begin{vmatrix} 18 & -5 & -3 \\ 5 & -8 & 1 \\ 3 & 1 & -4 \end{vmatrix}$$

$$= 18(32-1) + 5(-20-3) - 3(5+24)$$

$$\Delta R = 356\ \Omega.$$

$$\Delta R_1 = \begin{vmatrix} 50 & -5 & -3 \\ 10 & -8 & 1 \\ 5 & 1 & -4 \end{vmatrix}$$

$$= 50(32-1) + 5(-40-5) - 3(10+40)$$

$$\Delta R_1 = 1175\ \Omega$$

$$\Delta R_2 = \begin{vmatrix} 18 & 50 & -3 \\ 5 & 10 & 1 \\ 3 & 5 & -4 \end{vmatrix}$$

$$= 18(-40-5) - 50(-20-3) - 3(25-30)$$

$$\Delta R_2 = 355\ \Omega$$

$$\Delta R_3 = \begin{vmatrix} 18 & -5 & 50 \\ 5 & -8 & 10 \\ 3 & 1 & 5 \end{vmatrix}$$

$$= 18(-40-10) + 5(25-30) + 50(5+24)$$

$$\Delta R_3 = 525\ \Omega$$

Using Cramer's rule.

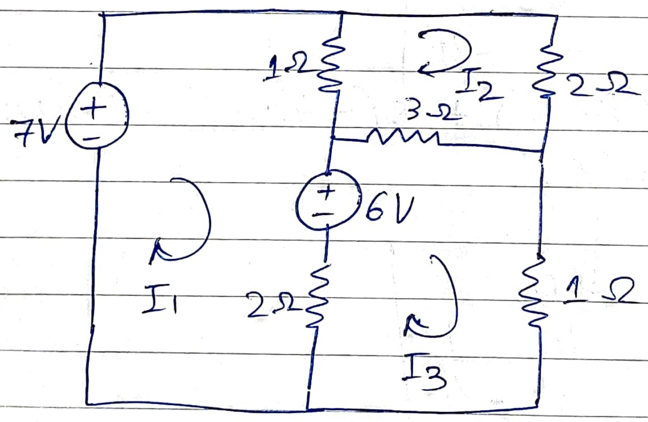
$$I_1 = \frac{\Delta R_1}{\Delta R} = \frac{1175}{356} = 3.3\,A$$

$$I_2 = \frac{\Delta R_2}{\Delta R} = \frac{355}{356} = 0.99\,A$$

$$I_3 = \frac{\Delta R_3}{\Delta R} = \frac{525}{356} = 1.47\,A$$

Ans

(3.) For a given network, find mesh current I_1, I_2 & I_3.



Applying KVL to loop 1.

$$7 - 1(I_1 - I_2) - 6 - 2(I_1 - I_3) = 0$$

$$7 - I_1 + I_2 - 6 - 2I_1 + 2I_3 = 0$$

$$1 = 3I_1 - I_2 - 2I_3 \rightarrow (1)$$

Applying KVL to loop 2.

$$-2I_2 - 3(I_2 - I_3) - 1(I_2 - I_1) = 0$$

$$-2I_2 - 3I_2 + 3I_3 - I_2 + I_1 = 0$$

$$-I_1 + 6I_2 - 3I_3 = 0 \rightarrow (2)$$

For loop 3,

$$-I_3 - 2(I_3 - I_1) + 6 - 3(I_3 - I_2) = 0$$

$$-I_3 - 2I_3 + 2I_1 + 6 - 3I_3 + 3I_2 = 0$$

$$2I_1 + 3I_2 - 6I_3 + 6 = 0$$

$$-2I_1 - 3I_2 + 6I_3 = 6 \rightarrow (3)$$

By (1), (2) & (3),

$$\begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\Delta R = \begin{vmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{vmatrix} = 3(+36 - 9) + 1(-6 - 6) - 2(+3+12) = 39\Omega.$$

$$\Delta R_1 = \begin{vmatrix} 1 & -1 & -2 \\ 0 & 6 & -3 \\ 6 & -3 & 6 \end{vmatrix} = 117\Omega$$

$$\Delta R_2 = \begin{vmatrix} 3 & 1 & -2 \\ -1 & 0 & -3 \\ -2 & 6 & 6 \end{vmatrix} = 78\Omega.$$

$$\Delta R_3 = \begin{vmatrix} 3 & -1 & 1 \\ -1 & 6 & 0 \\ -2 & -3 & 6 \end{vmatrix} = 117\Omega.$$

By Cramer's rule,

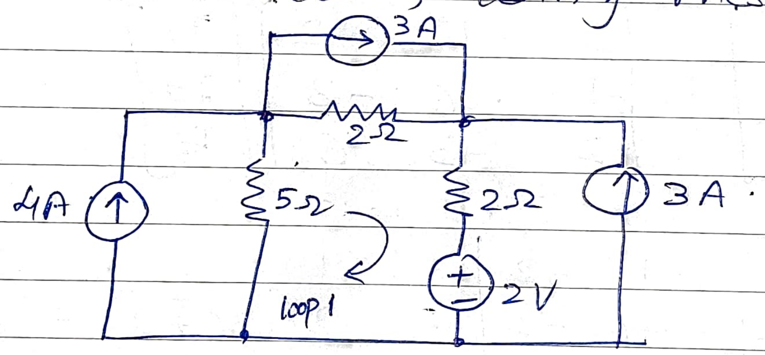
$$I_1 = \frac{\Delta R_1}{\Delta R} = \frac{117}{39} = 3\,A$$

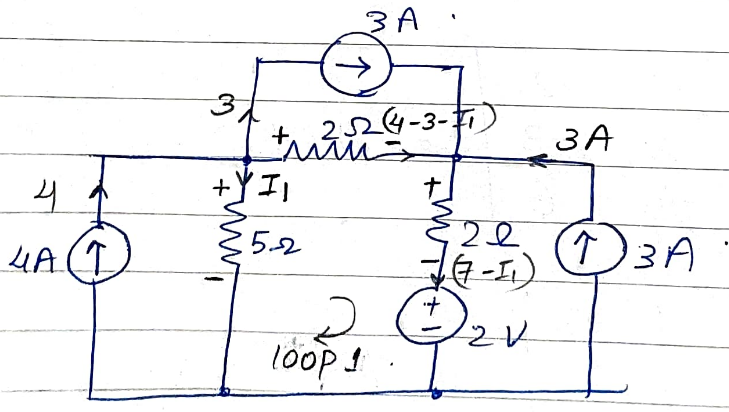
$$I_2 = \frac{\Delta R_2}{\Delta R} = \frac{78}{39} = 2\,A$$

$$I_3 = \frac{\Delta R_3}{\Delta R} = \frac{117}{39} = 3\,A$$

Ans.

(4.) Find current through 5Ω resistor in the below circuit, using mesh analysis





Applying KVL to loop 1,

$$5I_1 - 2(1-I_1) - 2(7-I_1) - 2 = 0$$

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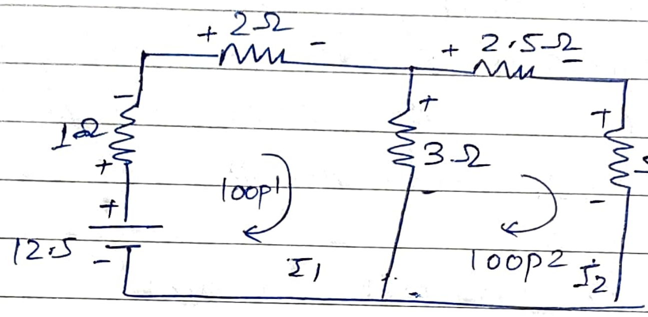
$$5I_1 - 2 + 2I_1 - 14 + 2I_1 - 2 = 0.$$

$$9I_1 = 18$$

$$\therefore I_1 = 2\ A \text{ . Ans}$$

(5.) Using mesh analysis, find the current through $1\,\Omega$ resistance.



Applying KVL to loop 1,

$$12.5 - I_1 - 2I_1 - 3(I_1 - I_2) = 0.$$

$$12.5 - 6I_1 + 3I_2 = 0.$$

$$12.5 = 6I_1 - 3I_2 \rightarrow (1.)$$

Applying KVL to loop 2.

$$-3(I_2 - I_1) - 2.5I_2 - 5I_2 = 0.$$

$$-3I_2 + 3I_1 - 2.5I_2 - 5I_2 = 0.$$

$$-10.5I_2 + 3I_1 = 0 \rightarrow (2.)$$

$$I_2 = \frac{3}{10.5} I_1 \rightarrow (3.)$$

Substituting above in (1.)

$$12.5 = 6I_1 - 3\left(\frac{3}{10.5}\right) I_1$$

$12.5 = 5.14 I_1$

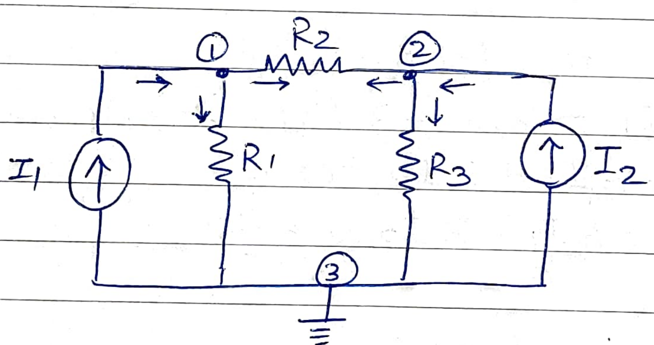
$\therefore I_1 = 2.43$ A. Ans.

* Node Analysis

→ This method is based on the application of KCL on different nodes of circuit.

→ Instead of assuming mesh currents, we assume node voltages w.r.t some reference node. and then write KCL equations for all nodes.

Let's consider a circuit below



→ 1 & 2 are principle nodes

→ 3 is the reference node (assume to zero potential)

The method:

Node 1:

Applying KCL at node 1,

$$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$

$$= \frac{V_1}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2}$$

$$I_1 = \left[\frac{1}{R_1} + \frac{1}{R_2}\right] V_1 - \frac{V_2}{R_2} \rightarrow (1)$$

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Node 2:

$$I_2 = \frac{V_2}{R_3} + \frac{V_2 - V_1}{R_2}.$$

$$= \frac{V_2}{R_3} + \frac{V_2}{R_2} - \frac{V_1}{R_2}.$$

$$I_2 = \left[\frac{1}{R_2} + \frac{1}{R_3}\right] V_2 - \frac{V_1}{R_2}.$$

In matrix form.

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & \frac{-1}{R_2} \\ \frac{-1}{R_2} & \frac{1}{R_1} + \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}.$$

In general,

$$\pm G_{11} V_1 \pm G_{12} V_2 \pm G_{13} V_3 = \pm I_1$$
$$\pm G_{21} V_1 \pm G_{22} V_2 \pm G_{23} V_3 = \pm I_2$$
$$\pm G_{31} V_1 \pm G_{32} V_2 \pm G_{33} V_3 = \pm I_3.$$

or

$$\begin{bmatrix} \pm G_{11} & \pm G_{12} & \pm G_{13} \\ \pm G_{21} & \pm G_{22} & \pm G_{23} \\ \pm G_{31} & \pm G_{32} & \pm G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\Delta G = \begin{bmatrix} \pm G_{11} & \pm G_{12} & \pm G_{13} \\ \pm G_{21} & \pm G_{22} & \pm G_{23} \\ \pm G_{31} & \pm G_{32} & \pm G_{33} \end{bmatrix}$$

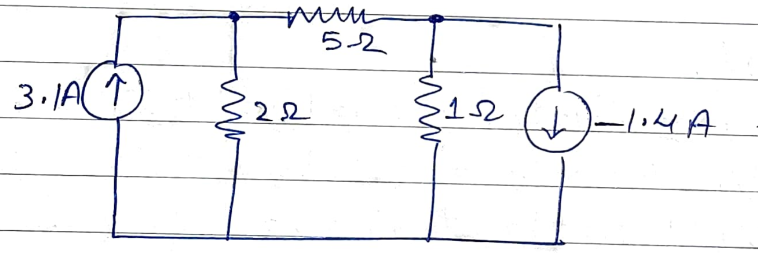
$$V_1 = \frac{\Delta G_1}{\Delta G} = \frac{\begin{bmatrix} I_1 & \pm G_{12} & \pm G_{13} \\ I_2 & \pm G_{22} & \pm G_{23} \\ I_3 & \pm G_{32} & \pm G_{33} \end{bmatrix}}{\Delta G}$$

$$V_2 = \frac{\Delta G_2}{\Delta G} = \frac{\begin{bmatrix} \pm G_{11} & I_1 & \pm G_{13} \\ \pm G_{21} & I_2 & \pm G_{23} \\ \pm G_{31} & I_3 & \pm G_{33} \end{bmatrix}}{\Delta G}.$$

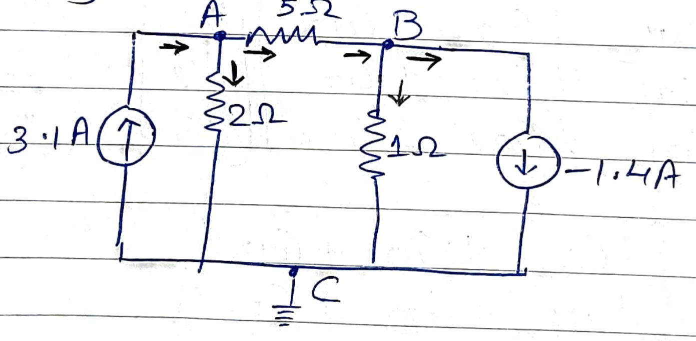
$$V_3 = \frac{\Delta G_3}{\Delta G} = \frac{\begin{bmatrix} \pm G_{11} & \pm G_{12} & I_1 \\ \pm G_{21} & \pm G_{22} & I_2 \\ \pm G_{31} & \pm G_{32} & I_3 \end{bmatrix}}{\Delta G}.$$

Numericals

5. Using nodal analysis, find voltage across $5\,\Omega$ resistor for the below network.



Redrawing.



Applying KCL at node A,

$$3.1 = \frac{V_A}{2} + \frac{V_A - V_B}{5}$$

$\therefore\ 0.7V_A - 0.2V_B = 3.1 \rightarrow (1)$

Applying KCL at node B

$$\frac{V_A - V_B}{5} - \frac{V_B}{1} = -1.4$$

$\therefore\ -0.2V_A + 1.2V_B = 1.4$

$$\begin{bmatrix} 0.7 & -0.2 \\ -0.2 & 1.2 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 3.1 \\ 1.4 \end{bmatrix}$$

$$\Delta G = \begin{vmatrix} 0.7 & -0.2 \\ -0.2 & 1.2 \end{vmatrix} = 0.8$$

$$\Delta G_1 = \begin{vmatrix} 3.1 & -0.2 \\ 1.4 & 1.2 \end{vmatrix} = 4$$

$$\Delta G_2 = \begin{vmatrix} 0.7 & 3.1 \\ -0.2 & 1.4 \end{vmatrix} = 1.6$$

Using Cramer's rule,

$$V_A = \frac{\Delta G_1}{\Delta G} = \frac{4}{0.8} = 5V.$$

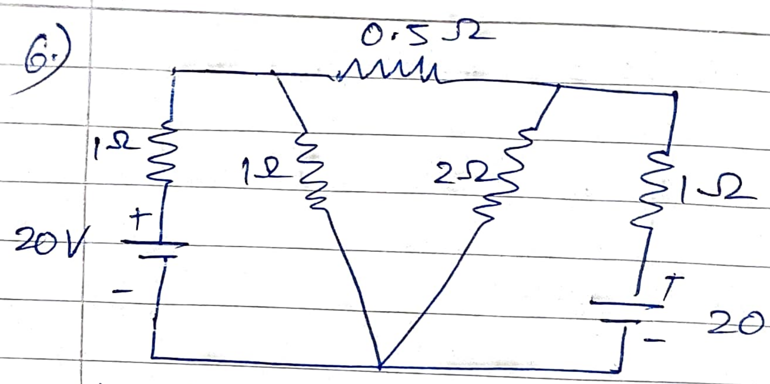
$$V_B = \frac{\Delta G_2}{\Delta G} = \frac{1.6}{0.8} = 2V.$$

Voltage across 5Ω resistor

$V = V_A - V_B$

$V = 3V$ Ans

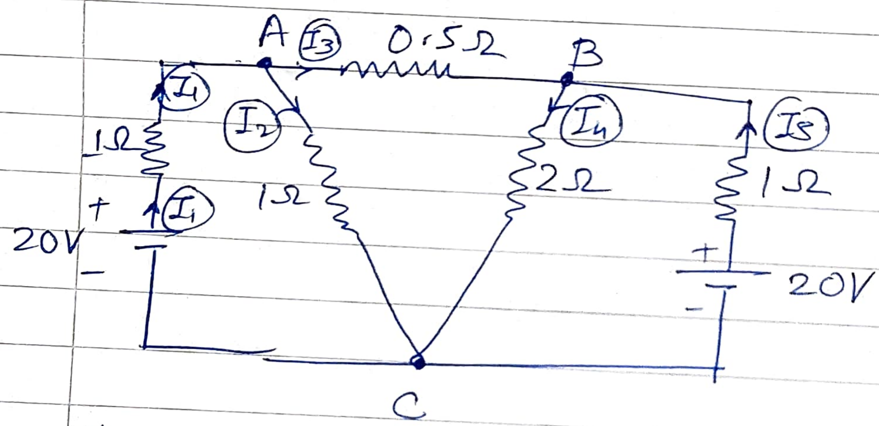
6.)



Find the values of currents in the given circuit.

Step 1:

→ Redrawing the circuit with nodes and currents . . .



Node C is the reference node

Step 2: Applying KCL at node A .

$$I_1 = I_2 + I_3$$

$$\therefore \frac{20 - VA}{1} = \frac{VA - 0}{1} + \frac{VA - VB}{0.5}$$

$$\therefore 20 - VA = VA + 2VA - 2VB$$

$$\therefore 4VA - 2VB = 20$$

$$\therefore 2VA - VB = 10 \longrightarrow (1)$$

Applying KCL at node B

$$I_3 + I_5 = I_4$$

$$\frac{VB - VA}{0.5} + \frac{20 - VB}{1} = \frac{VB - 0}{2}$$

$\_/\_/\_$

$$2(V_A - V_B) + (20 - V_B) = \frac{V_B}{2}$$

$$\therefore 4(V_A - V_B) + 2(20 - V_B) = V_B$$

$$\therefore 4V_A - 4V_B + 40 - 2V_B - V_B = 0$$

$$\therefore 4V_A - 7V_B = -40 \rightarrow (2)$$

$$2V_A - 3.5V_B = -20$$

By (1) & (2)

$$2V_A - V_B = 10$$

$$\begin{array}{r} 2V_A - 3.5V_B = -20 \\ - \quad + \quad\quad - \\ \hline 2.5V_B = 30 \end{array}$$

$$\therefore V_B = 12V$$

$$2V_A = V_B + 10 \Rightarrow V_A = 11V$$

$$I_1 = \frac{20 - V_A}{1} = \frac{20 - 11}{1} = 9A$$

$$I_2 = \frac{V_A - 0}{1} = \frac{11 - 0}{1} = 11A$$

$$I_3 = \frac{V_A - V_B}{0.5} = 2(V_A - V_B) = 2(-1) = -2A$$

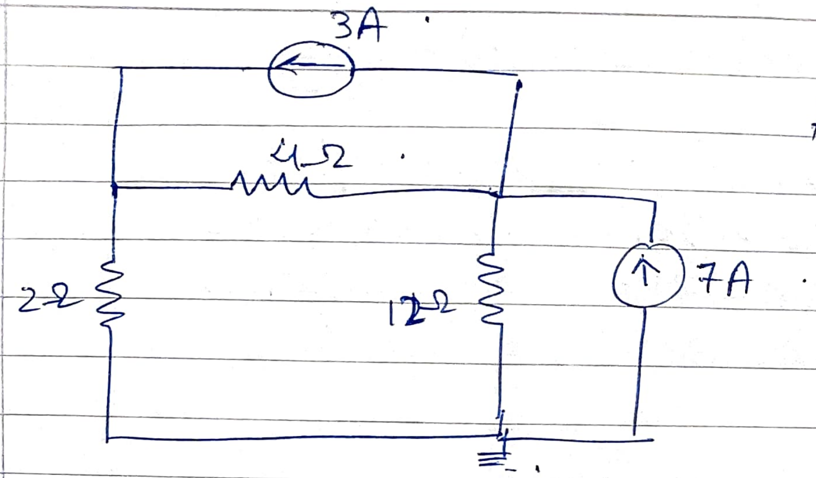
(Negative here means actual current is flowing from B to A)

$$I_4 = \frac{V_B - 0}{2} = \frac{12 - 0}{2} = 6A$$

$$I_5 = \frac{20 - V_B}{1} = \frac{20 - 12}{1} = 8A$$

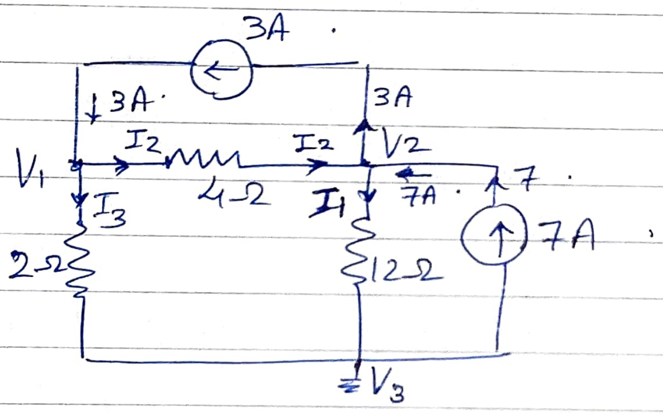
$$\boxed{I_1 = 9A,\ I_2 = 11A,\ I_3 = -2A,\ I_4 = 6A,\ I_5 = 8A}$$ Ans.

(7.)



Find the values of currents.

Redrawing with currents and nodes.



V_3 is reference node.

At node 1. (Applying KCL)

$$3 = I_2 + I_3$$

$$3 = \frac{V_1 - V_2}{4} + \frac{V_1 - 0}{2}$$

$$4 \times 3 = V_1 - V_2 + 2V_1$$

$$\therefore 3V_1 - V_2 = 12 \rightarrow (1)$$

At node 2 (Applying KCL)

$$7 + I_2 = 3 + I_1$$

$$\therefore 4 = I_1 - I_2$$

$$\therefore 4 = \frac{V_2 - 0}{12} - \frac{V_1 - V_2}{4}$$

$$\therefore 48 = V_2 - 3V_1 + 3V_2$$

$$\therefore 48 = 4V_2 - 3V_1$$

$$3V_1 - V_2 = 12$$
$$-3V_1 + 4V_2 = 48$$
$$3V_2 = 60$$
$$V_2 = 20\,V$$

$$3V_1 = 12 + V_2 = 32$$
$$V_1 = \frac{32}{3}V = 10.66\,V$$

$$I_1 = \frac{V_2 - 0}{12} = \frac{20}{12} = 1.66\,A$$

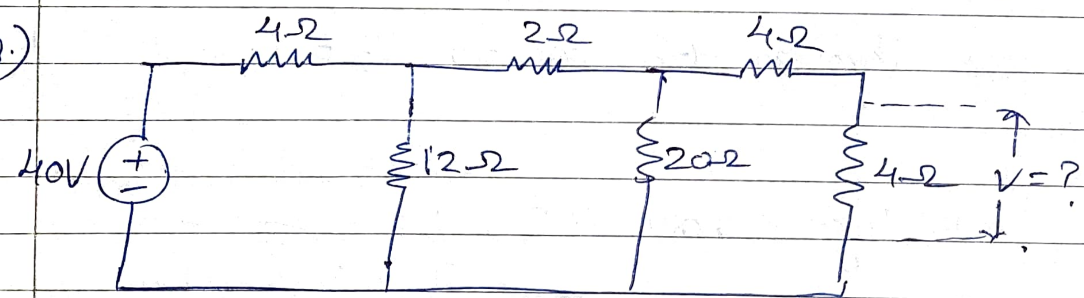
$$I_2 = \frac{V_1 - V_2}{4} = -2.33\,A$$

$$I_3 = \frac{V_1 - 0}{2} = \frac{10.66}{2} = 5.33$$

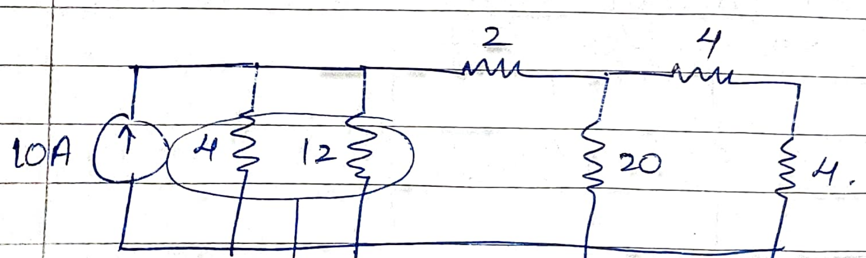
Ans

★ Source Transformation numericals

8.)

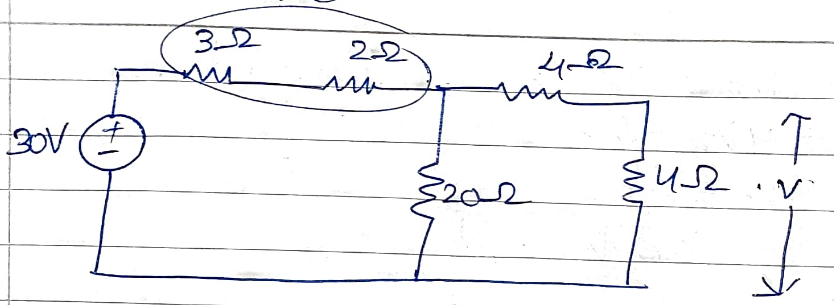


Let us convert 40V source into current source

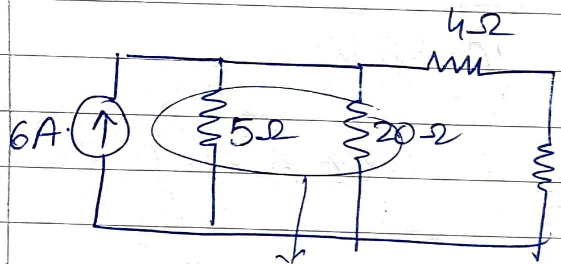


$$4 \parallel 12 = \frac{4 \times 12}{4+12} = 3\Omega$$

series.

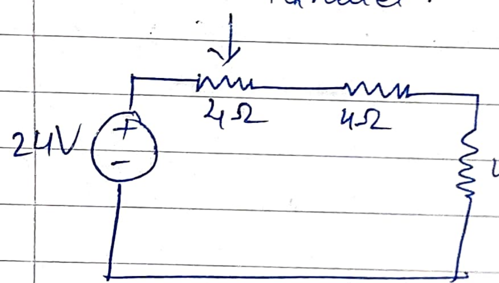


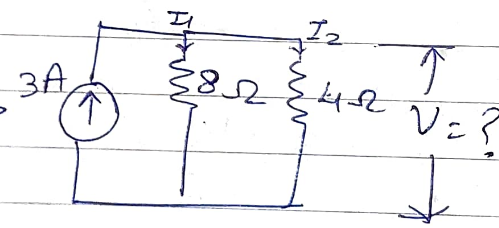
$3+2=5\,\Omega$



$5 \| 20 = \frac{5\times 20}{5+20} = 4\,\Omega$

Parallel.





~~$V = 4I_2 = 4\times\frac{8}{8+4}$~~

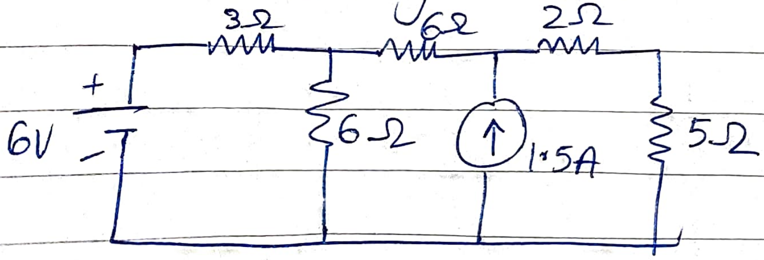
I through 4Ω resistor, $I_2 = I_T \times \frac{8}{8+4} = \frac{3\times 8}{8+4}$

$I_2 = 2A$.

Voltage across 4Ω, $V = 4I_2$

$V = 8V$ Ans

9.) Find the voltage across 5Ω resistor.



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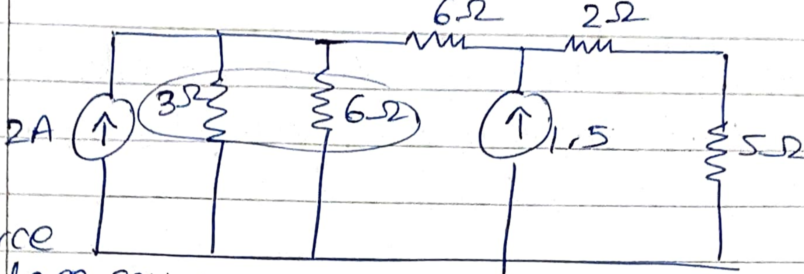
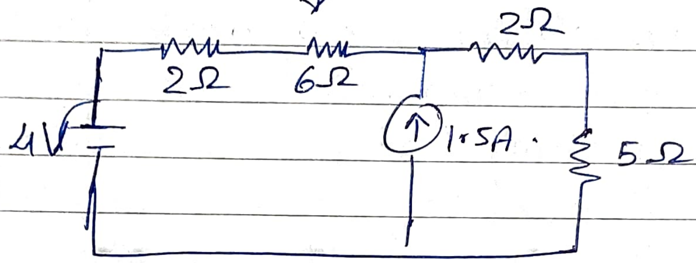
Convert 6V source into current source.



$3 \| 6 = \frac{3 \times 6}{3+6}$

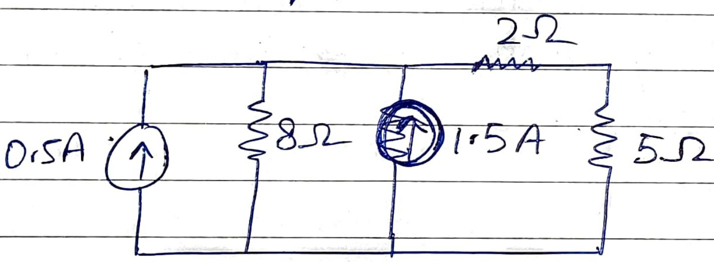
$= 2\Omega$

convert 2A source into voltage source

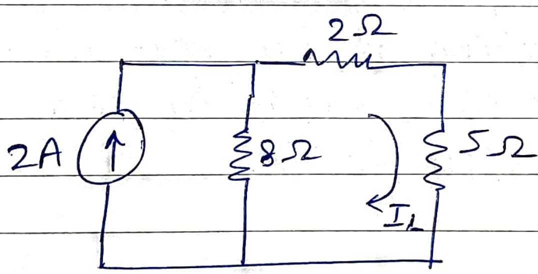


2 & 6 in series $= 8\ \Omega$.

Convert 4 V into current source



current sources in parallel, adding them



Current through 5Ω, $I_L = 2 \times \frac{8}{8+2+5}$

$I_L = 1.067$ A.

Voltage, $V_o = IR = (1.067)(5)$

$\boxed{V = 5.33\,V}$

__/__/__

* Superposition theorem

This theorem is used to solve electrical circuits that have more than one energy source.

Statement:-

In a linear electrical network having more than one independent energy source, the response in any branch (ie: voltage across any branch or current through any branch) is equal to the algebraic sum of all responses due to each individual source, acting alone with all other sources reduced to zero.

Imp. points:

→ Reducing a voltage source to zero (deactivating it) means replacing it by short circuit.
→ Reducing a current source to zero means replacing it by open circuit.

Limitations

1.) This theorem is not applicable to circuits consisting of non-linear elements like semiconductor devices.
2.) It is not applicable to circuits containing only dependent sources.
3.) It is not useful for circuits containing less than two independent sources.

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* Steps to apply superposition theorem.

Step 1: select a single source acting alone. Deactivate all other sources. (i.e. short circuit voltage sources, if any and open circuit current sources, if any).

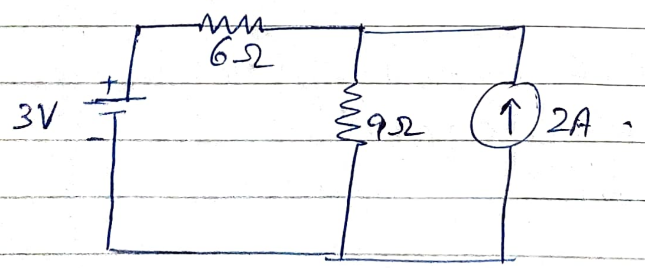
Step 2: Find current through or voltage across the required element, due to the source under consideration. Use suitable network simplification techniques.

Step 3: Repeat the above two steps for all other sources.

Step 4: Add the individual effects produced by individual sources, to obtain total current or voltage across an element.

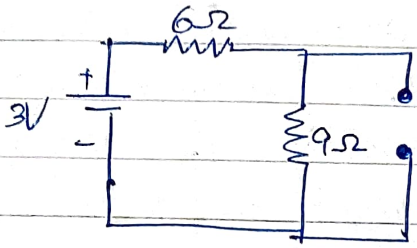
Explanation with an example.

Let us find current through 9Ω resistor in the given circuit.



Step 1: Consider 3V source alone.

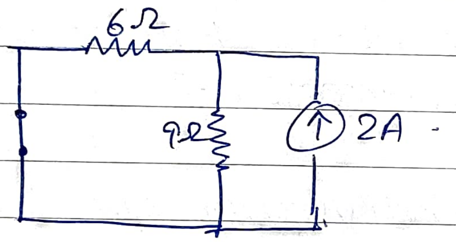
So, the current source is replaced by short circuit.



6Ω & 9Ω is series.
Total resis. = 15Ω.

$$I_9' = \frac{3}{15} = 0.2\ A.$$

Step 2: Consider 2A source alone.



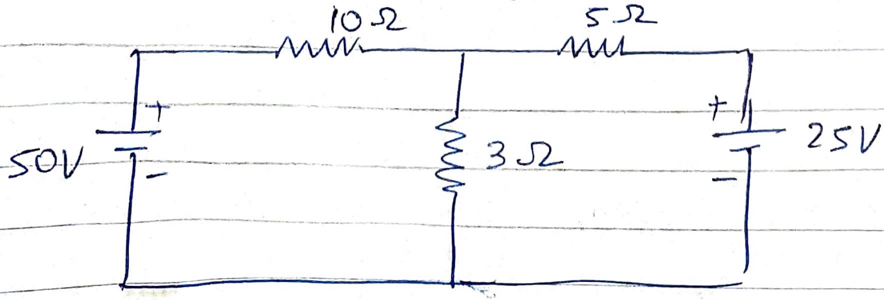
Total current is 2A.

By C.D.R,
$$I_9'' = \frac{2 \times 6}{6+9} = 0.8\ A.$$

Step 3: Calculation of I_9.
$$I_9 = I_9' + I_9''$$
$$= 0.2 + 0.8$$
$$\boxed{I_9 = 1\ A}$$

Numericals.

10.) Determine the current through each resistor in below circuit.



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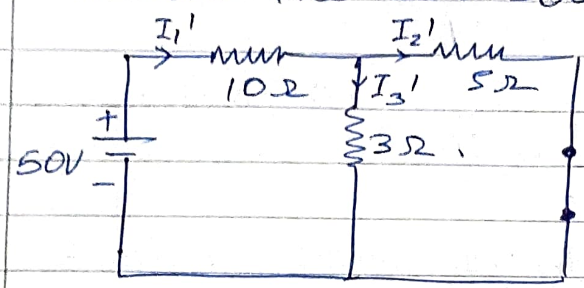
Step 1 :

Consider 50 V source alone. let's short circuit the 25 V source.



Total resistance

$= 10 + 5 \| 3$

$= 10 + \frac{5 \times 3}{5+3}$

$= 11.9 \Omega$

Current supplied by 50 V source.

$I_T = \frac{V}{R} = \frac{50}{11.9} = 4.2 A$

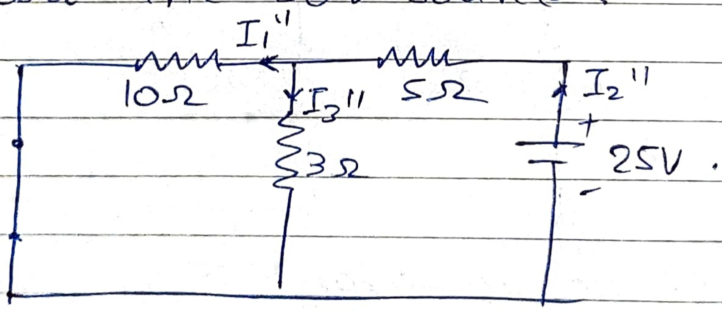
Current $I_1' = 4.2 A$

Current $I_2' = 4.2 \times \frac{3}{3+5} = 1.58 A$

Current $I_3' = 4.2 \times \frac{5}{3+5} = 2.63 A$

Step 2 :

Consider 25 V source alone. let's short circuit the 50 V source.



Total resistance

$= 5 + 10 \| 3$

$= 5 + \frac{10 \times 3}{10+3}$

$= 7.31 \Omega$

Current through 25 V source

$= \frac{25}{7.31} = 3.42 A$

___/___/___

Current $I_2'' = 3.42$ A,

Current $I_3'' = 3.42 \times \frac{10}{10+3} = 2.63$ A.

Current $I_1'' = 3.42 \times \frac{3}{3+10} = 0.78$ A.

Step 3: calculation of total currents.

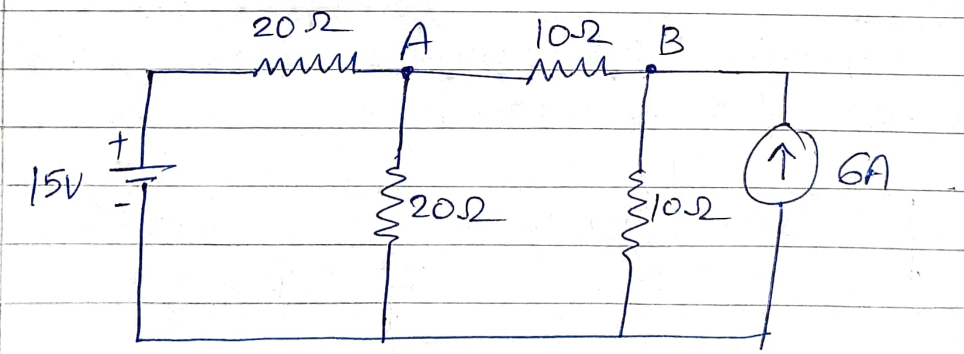
Current $I_1 = I_1' - I_1'' = 4.2 - 0.78 = 3.42$ A.

Current $I_2 = I_2'' - I_2' = 3.42 - 1.58 = 1.84$ A.

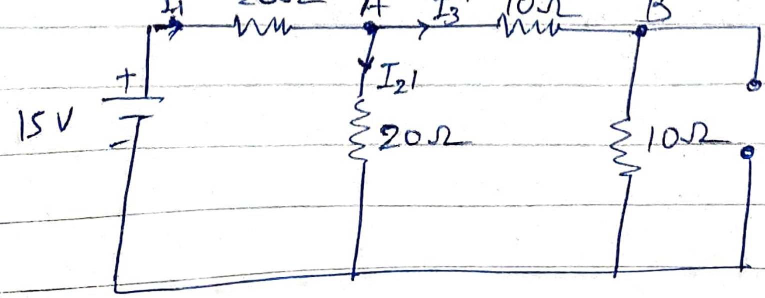
Current $I_3 = I_3' + I_3'' = 2.63 + 2.63 = 5.26$ A

Ans

(11) using superposition theorem, find the current I_{AB} in the circuit below.



Step 1: let us consider 15V source alone. and we open circuit 6A source.



Let us find total resistance

$= 20 + 20 \parallel (10+10)$

$= 20 + \frac{20 \times 20}{20+20}$

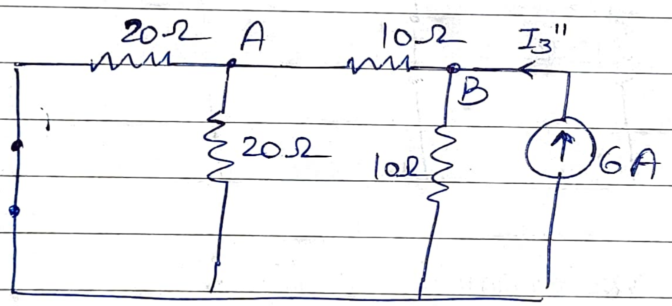
$= 30\,\Omega$.

Battery current is $I_1' = \frac{15}{30} = 0.5A$.

Current through AB $= I_3' = 0.5 \times \frac{20}{20+10+10}$

$= 0.25\,A$ (from A to B)

Step 2: Let us consider 6A source alone. Short circuit the voltage source.



Here, 20Ω & 20Ω are in parallel

$20 \parallel 20 = \frac{20 \times 20}{20+20}$

$= 10\,\Omega$.

Current I_3'' divides into two at point B.

Current through AB $= 6 \times \frac{10}{10+10+10}$

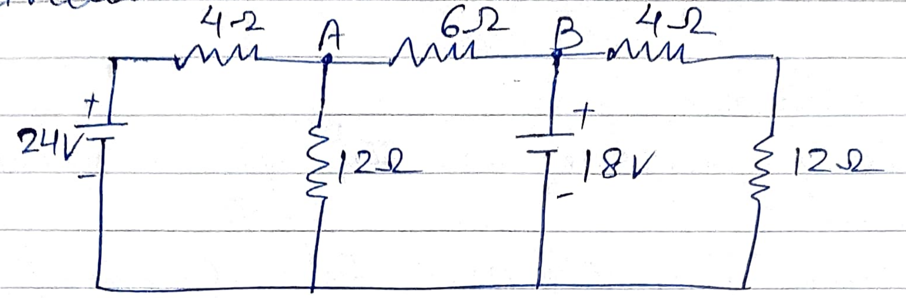
$= 2A$. (from B to A)

Step 3: Calculate the current in branch AB

$I_{AB} = I_3'' - I_3' = 2 - 0.25$

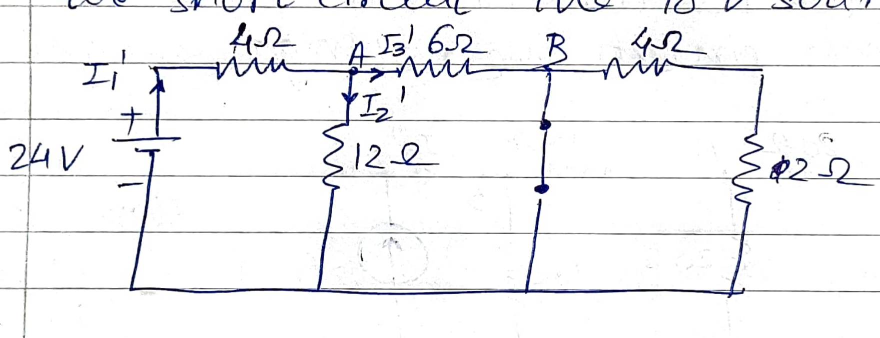
$I_{AB} = 1.75\,A$ Ans

(12.) Using superposition theorem, find the current through 6Ω resistor in the given circuit.



Step 1:

Let us consider 24V source alone and we short circuit the 18V source.



4 & 2Ω are in series & this combination is in parallel with short ckt.

So, total resistance will be.

$$4 + 6 \| 12 = 4 + \frac{6 \times 12}{6+12} = 8\Omega$$

Battery current $I_1' = \frac{24}{8} = 3A$.

Current through 6Ω resistor,

$$I_3' = 3 \times \frac{12}{12+6}$$

$I_3' = 2A$. (from A to B)

Step 2:

Let us consider 18V source alone. So we short circuit 24V source.

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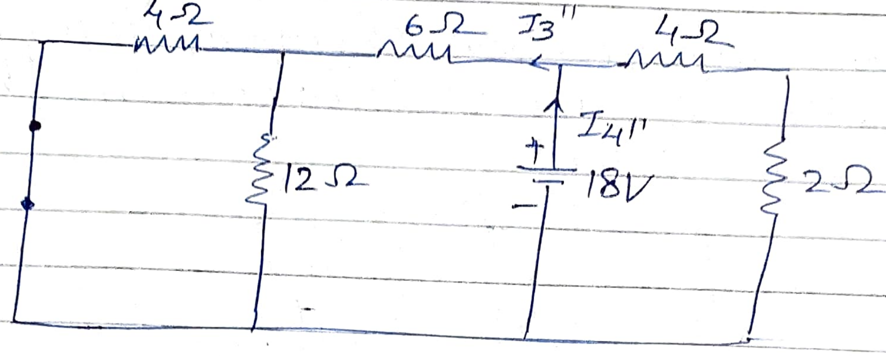


→ Let us find total resistance.
We have 4Ω & 12Ω in parallel and this is in series with 6Ω.

ie: $(4 \| 12) + 6 = \frac{4 \times 12}{4+12} + 6 = 9\Omega$

→ Also, 4Ω & 2Ω are in series, giving 6Ω.
Now, 9Ω & 6Ω are in parallel.

Total resistance $= 9 \| 6 = \frac{9 \times 6}{9+6} = 3.6\Omega$.

→ Battery current $I_4'' = \frac{18}{3.6} = 5A$.

→ Current through 6Ω resistor, I_3''

$I_3'' = 5 \times \frac{6}{6+9}$

$I_3'' = 2A$ (from B to A)

Step 3: Calculation of total current through 6Ω resistor

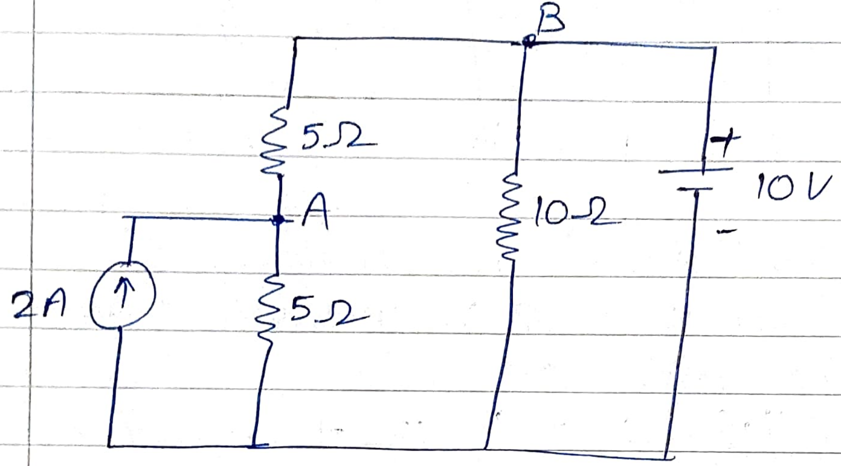
$I_6 = I_3' - I_3''$

$= 2 - 2$

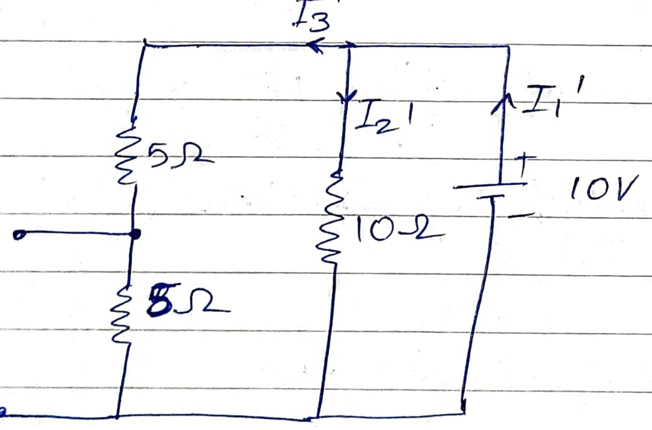
$\boxed{I_6 = 0\,A}$ (when both sources act simultaneously)

Ans.

13. Using superposition theorem, determine currents in all resistances in the network below.



Step 1: Let us consider only 10V source and we open circuit the current source.



Let us find total resistance

$= (5+5) \| 10$

$= 10 \| 10$

$= \frac{10 \times 10}{10+10} = 5\Omega$

→ Battery current $I_1' = \frac{10}{5} = 2A$

$I_2' = 2 \times \frac{10}{10+10} = 1A$

$I_3' = 2 \times \frac{10}{10+10} = 1A$.

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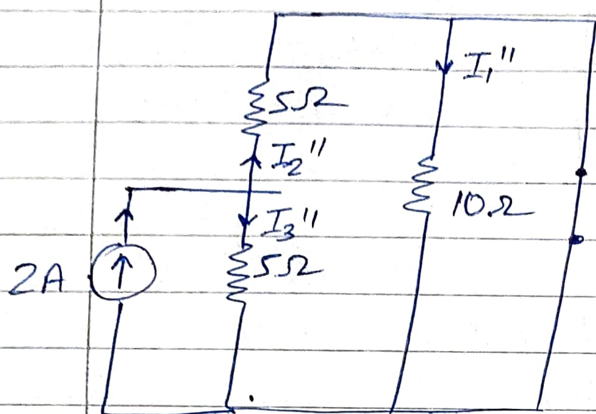
Step 2: Let us consider 2A source alone and we short circuit the voltage source



Total current is 2A.

~~I_2''~~ = 10Ω resistor is shorted. (in parallel with short circuit)

So, $I_1'' = 0$.

$$I_2'' = \frac{2 \times 5}{5+5} = 1\,A.$$

$$I_3'' = \frac{2 \times 5}{5+5} = 1\,A.$$

Step 3: Let us calculate total currents.

Current through 10Ω = $I_1' + I_1'' = 1 + 0 = \underline{1A}$

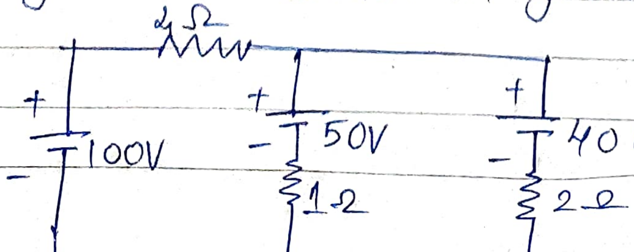
Current through 5Ω (upper) = $I_2'' - I_3' = 1 - 1 = \underline{0A}$

Current through 5Ω (lower)

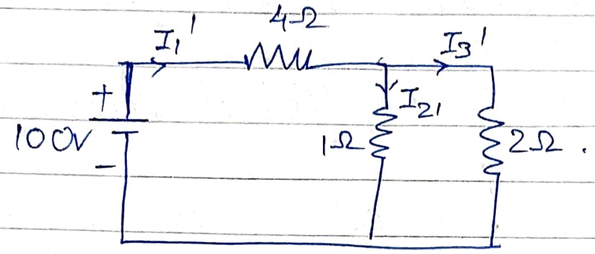
$= I_3' + I_3''$

$= 1 + 1 = \underline{2A}$.

(14.) Using Superposition theorem, find current through 1Ω resistor in given circuit.



Step 1: Let us consider 100 V source alone.



Total resistance
$= 4 + (1 \| 2)$
$= 4 + \frac{2}{3}$

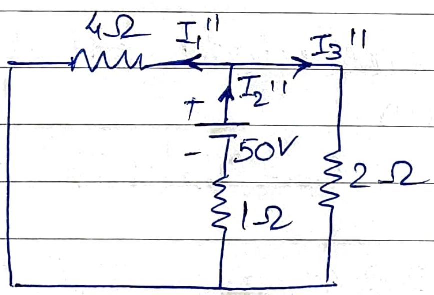
$= 4.66\,\Omega$.

Total current due to 100V source
$= \frac{100}{4.66} = 21.45\,A$.

Current through 1Ω, $I_2' = 21.45 \times \frac{2}{1+2}$

$I_2' = 14.3\,A\ (\downarrow) \rightarrow (1)$.

Step 2: Let us consider 50 V source alone.



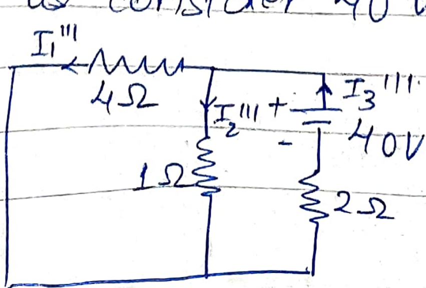
Total resistance
$= 1 + (4 \| 2)$
$= 1 + \frac{4 \times 2}{4+2}$

$= 2.33\,\Omega$.

Total current due to 50V source
$= \frac{50}{2.33} = 21.45\,A$.

Current through 1Ω resistor $= 21.45\ (\uparrow) \rightarrow (2)$

Step 3: Let us consider 40 V source only.



Total resistance $= 2 + (4 \| 1)$
$= 2 + \frac{4}{5}$
$= 2.8$.

Total current due to 40V source
$= \frac{40}{2.8} = 14.28 \text{ A}$.

Current through 1Ω, $I_2''' = 14.28 \times \frac{4}{4+1}$

$I_2''' = 11.43 \text{ A}$ (↓) → (3)

Current through 1Ω resistor due to three sources acting simultaneously

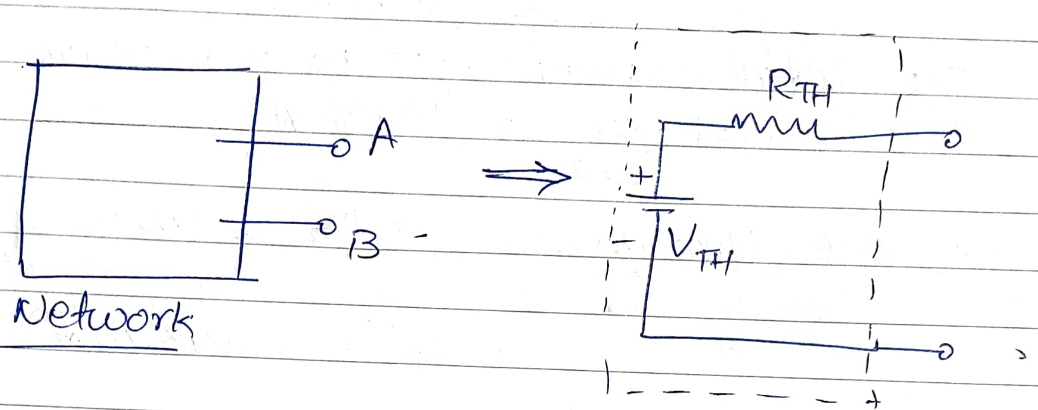
$I_2 = I_2' - I_2'' + I_2'''$
$= 14.3 - 21.45 + 11.43$
$I_2 = 4.28 \text{ A}$. Ans

★ Thevenin's theorem

Statement:
It states that any network having a number of energy sources and resistances when viewed from open, its terminals A & B can be replaced by simple equivalent network consisting of a single equivalent voltage source (V_{TH}), in series with a single equivalent resistance (R_{TH}).

→ V_{TH} = Thevenin's equivalent voltage source
= Open circuit voltage across the terminals A & B.

R_{TH} = Thevenin's equivalent resistance
= Equivalent resistance across AB terminals when all sources set to zero.



Thevenin's equivalent network,

* Applications

It is used to determine current in any given element in the network..

* Steps to apply Thevenin's theorem.

Step 1: Remove the branch resistance through which current is to be calculated (open circuit it).

Step 2: Calculate the voltage across the open terminals (V_{TH}) by using any of the network simplification techniques.

Step 3: Calculate the equivalent resistance (R_{TH}), by replacing voltage sources by short circuit and current sources

_/_/_

by open circuit.

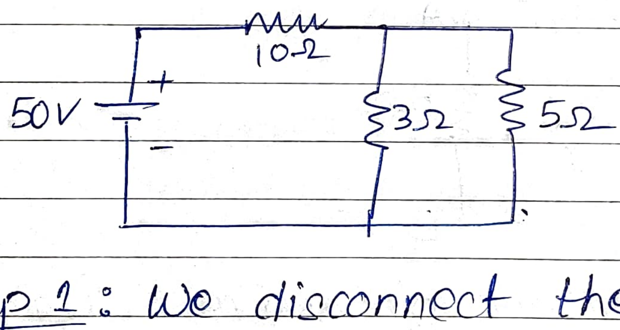
Step 4: Draw Thevenin's equivalent circuit showing V_{TH} & R_{TH} (in series with it) across the terminals of branch of interest.

Step 5: Reconnect the branch resistance (R_L). The current is determined by

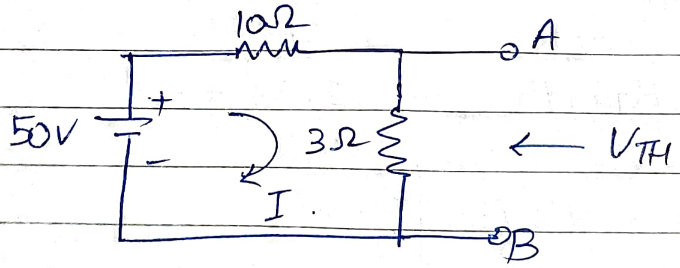
$$\boxed{I = \frac{V_{TH}}{R_{TH} + R_L}}$$

Numericals-

(15.) Using Thevenin's theorem, calculate the current in 5Ω resistor.



Step 1: We disconnect the 5Ω resistor from network.



Step 2: Let us determine V_{TH}.

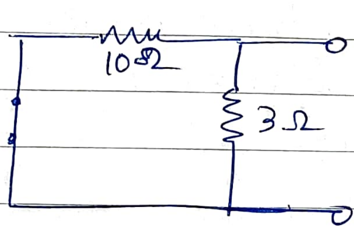
$$I = \frac{50}{10+3} = \frac{50}{13} = 3.48\ A$$

VTH is actually same as the voltage across 3 Ω resis.

$V_{TH} = I(3)$

$V_{TH} = 10.44\,V \rightarrow (1)$

Step 3: Determination of R_{TH}. For this, we short circuit the voltage source.

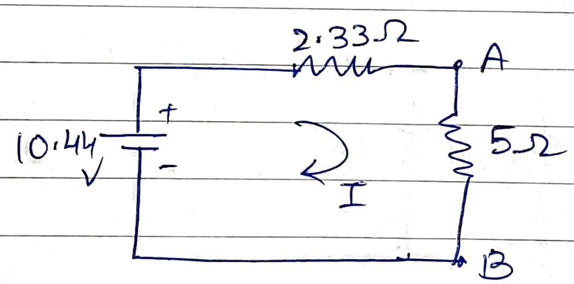


$R_{TH} = 10 \| 3$

$= \frac{10 \times 3}{10+3} = \frac{30}{13}$

$R_{TH} = 2.33\,\Omega$.

Step 4: Let us draw the thevenin equivalent circuit. and reconnect 5Ω resistor.

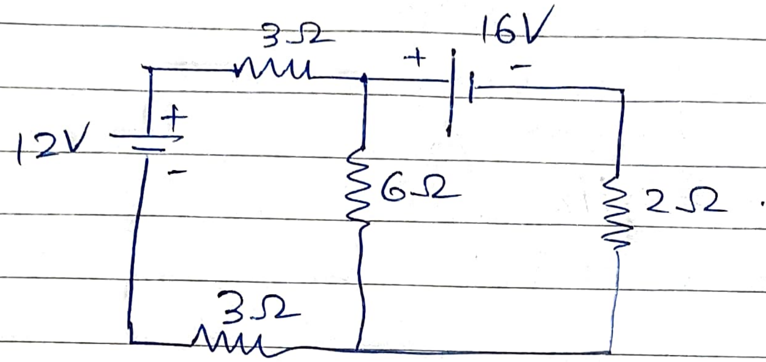


Step 5: Calculation of current through 5Ω.

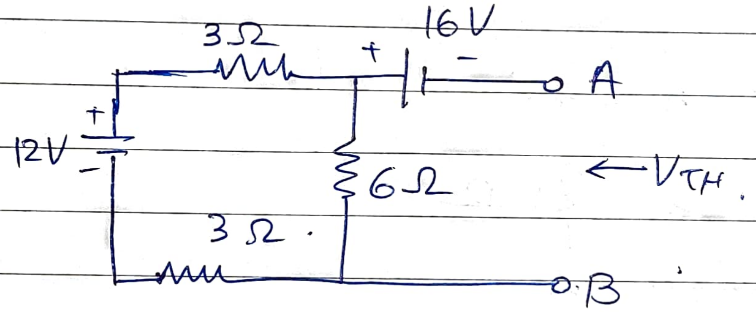
$I = \frac{V_{TH}}{R_{TH} + R_L} = \frac{10.44}{2.33 + 5}$

$\boxed{I = 1.43\,A}$ Ans.

(16.) Using Thevenin's theorem, calculate the current in 2Ω resistor.



Step 1: Let us disconnect the 2Ω resistor.



Step 2: Let us now calculate V_{TH}.

V_{TH} is same as the 16V − voltage across 6Ω resistor.

ie: $V_{TH} = 6 \times I$,

where,

$$I = \frac{V}{R} = \frac{12}{3+3+6} = 1A.$$

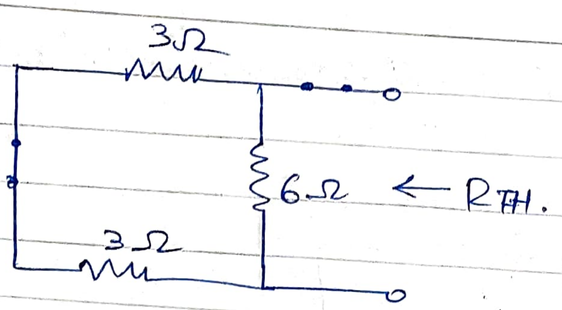
Voltage across 6Ω resistor $= (6)(1) = 6V$.

$$V_{TH} = 16 - 6 = \underline{10V} \rightarrow (1).$$

Step 3: Determination of R_{TH}.

Let us short circuit the voltage sources.

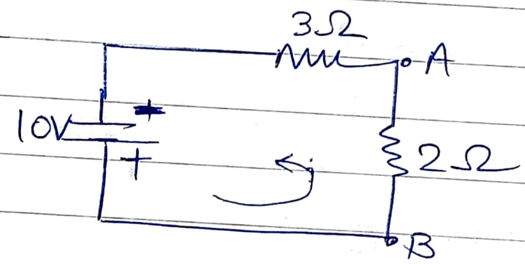
/ /



$R_{TH} = (3+3)||6$

$= \frac{6 \times 6}{6+6} = 3\Omega$.

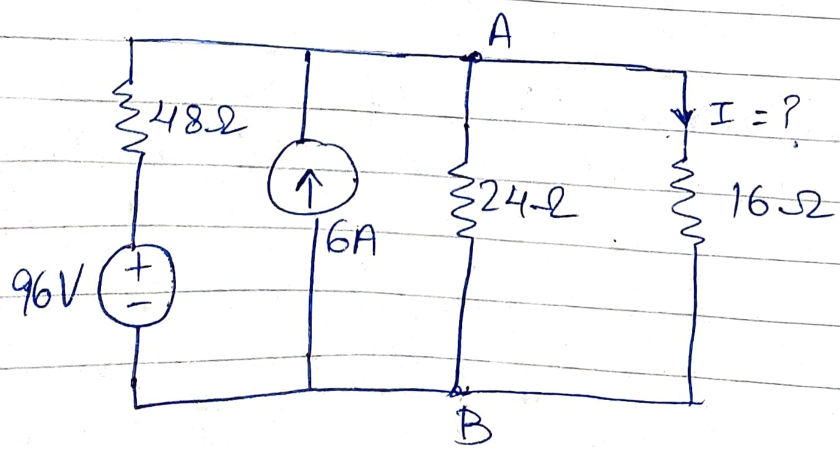
Step 4: Let's draw Thevenin equivalent circuit and reconnect 2Ω resistor



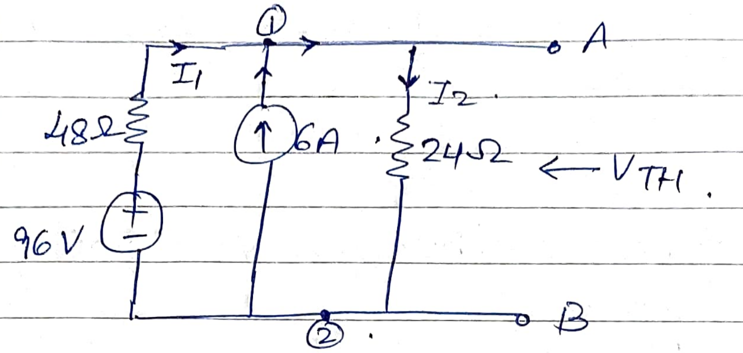
Step 5: Calculation of current.

$I = \frac{V_{TH}}{R_{TH}+R_L} = \frac{10}{3+2} = 2\ A$. Ans

(17.) Reduce portion of below circuit to left of terminals A & B to Thevenin's equivalent circuit and find current through 16Ω resistor.



Step 1: Let us disconnect the 16 Ω resistor and find V_{TH}.



Let us apply KCL at Node 1.

$$I_1 + 6 = I_2$$

$$\frac{-V_A+96}{48} + 6 = \frac{V_A - 0}{24}$$

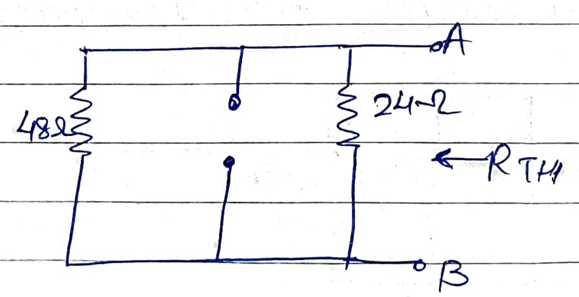
$$96 - V_A + (6\times 48) = 2V_A$$

$$3V_A = 384$$

$$V_A = 128V$$

$$V_{TH} = V_A = 128V \rightarrow (1)$$

Step 2: Let us now find R_{TH}. We open circuit the current source and short circuit the voltage source.

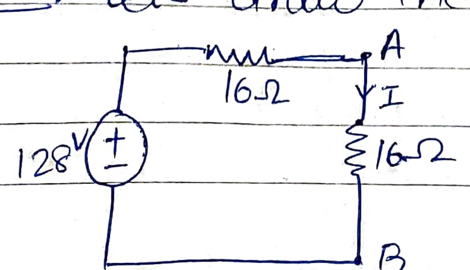


$$R_{TH} = 48 \| 24$$

$$= \frac{48\times 24}{48+24}$$

$$R_{TH} = 16\Omega$$

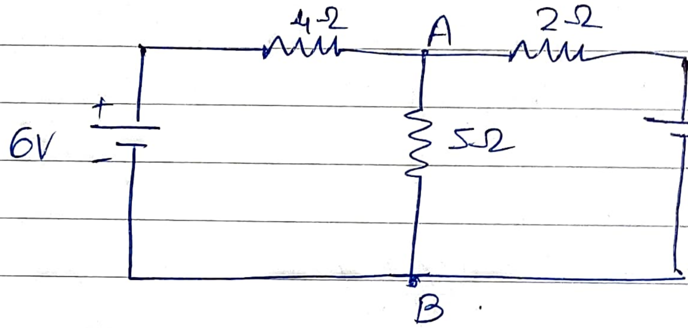
Step 3: Let's draw Thevenin's equivalent circuit



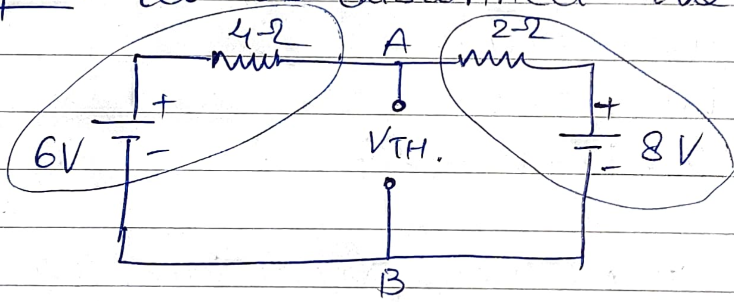
Step 4: Calculation of I.

$$I = \frac{V_{TH}}{R_{TH}+R_L} = \frac{128}{16+16} = 4A \text{ Ans.}$$

(18.) Find the current through AB using Thevenin's theorem.

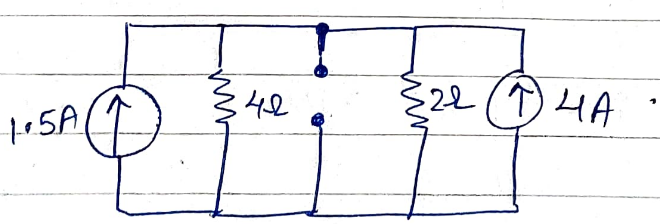


Step 1: Let us disconnect the 5Ω resistor.



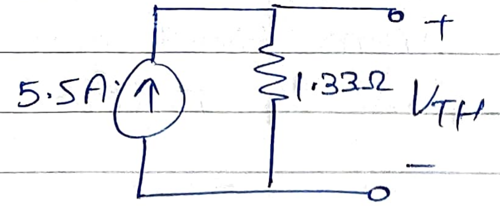
Step 2: Calculation of V_{TH}

Let us apply source transformation here.



Current sources are parallel, can be added. and 4Ω & 2Ω in parallel

$$= \frac{4 \times 2}{4+2} = 1.33\Omega$$

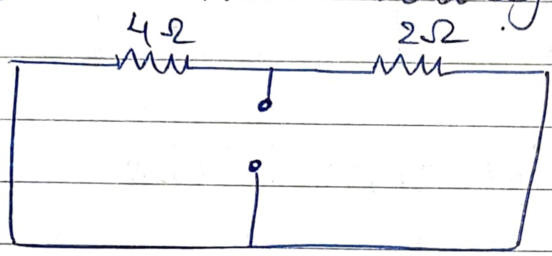


$V_{TH} = (1.33)(5.5)$

$V_{TH} = 7.33V \rightarrow (1)$

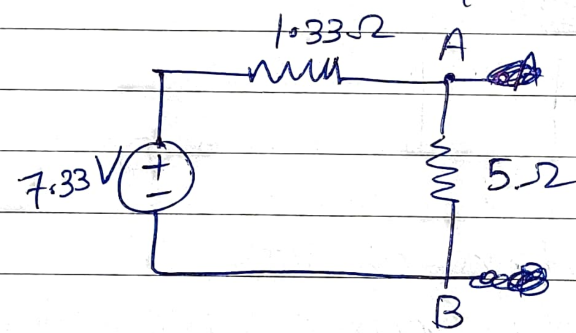
<u>Step 3:</u> Calculation of R_{TH}.

Let us short voltage sources.



$$4 \parallel 2 = \frac{4 \times 2}{4+2} = 1.33\,\Omega.$$

<u>Step 4:</u> Thevenin's equivalent circuit.



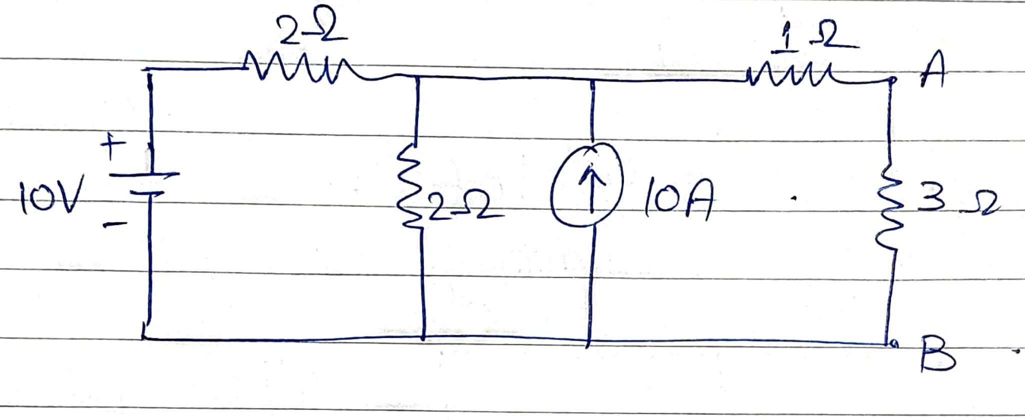
<u>Step 5:</u> Calculation of current.

$$I = \frac{V_{TH}}{R_{TH} + R_L}$$

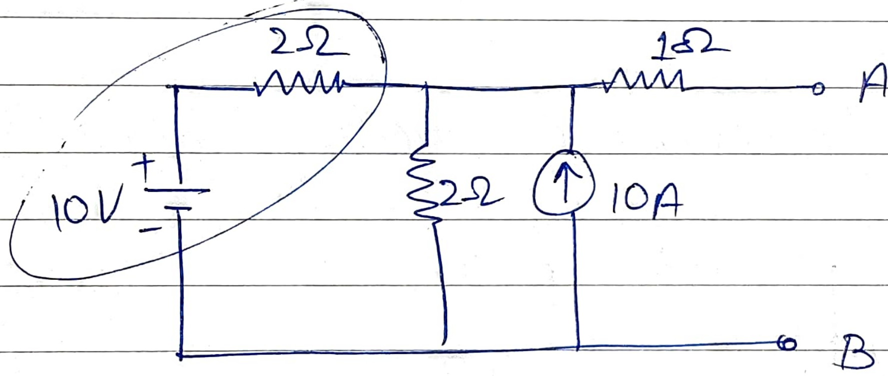
$$I = \frac{7.33}{1.33 + 5}$$

$$\boxed{I = 1.157\text{ A}}$$ Ans.

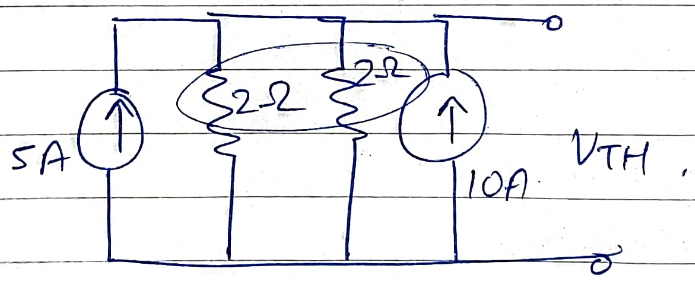
Using Thevenin's theorem, find current through branch AB, in the below circuit.



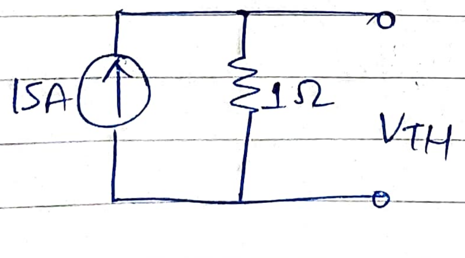
Step 1: let us disconnect the 3Ω resistor.



Step 2: Calculation of VTH.
we will apply source transformation here.



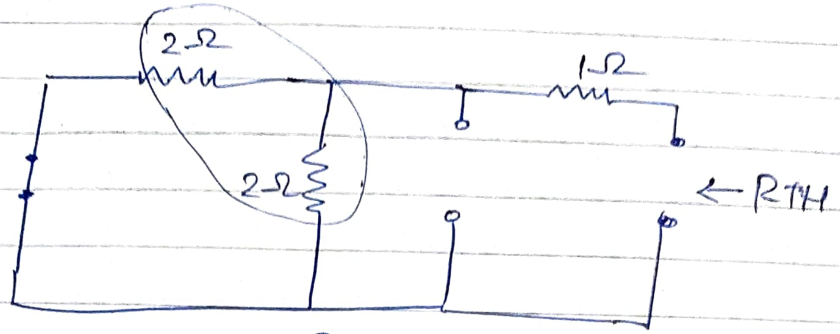
$2 \| 2 = \frac{2 \times 2}{2+2} = 1\Omega$



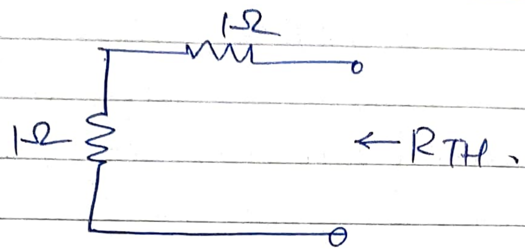
$V_{TH} = IR = \underline{15\ V} \rightarrow (1)$

_/_/_

Step 3: Calculation of R_{TH}.

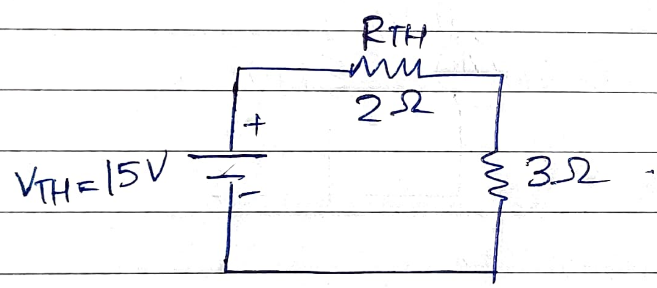


$2 \| 2 = 1\Omega$.



$R_{TH} = 1 + 1 = 2\Omega$.

Step 4: Thevenin's equivalent circuit.



Step 5: Calculation of current.

$$I = \frac{V_{TH}}{R_{TH} + R_L}$$

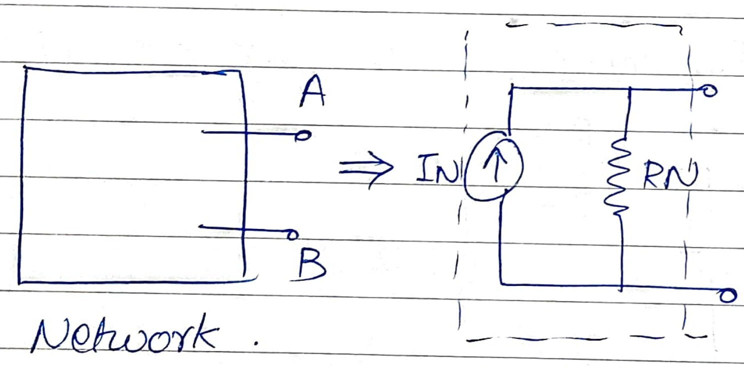
$$= \frac{15}{2+3}$$

$$I = 3\,A$$ Ans

★ Norton's theorem

Statement:

Norton's theorem states that any network having a no. of energy sources and resistances, when viewed from its open output terminals A & B, can be replaced by a simple equivalent network consisting of a single equivalent current source, in parallel with a single equivalent resistance (RN).



Norton's equivalent network.

IN (or Isc) is the Norton's equivalent current source.
= Current passing through the short circuit applied at open output terminal A & B.

RN (or Req) = Equivalent resistance across AB terminals when all sources set to zero.

Applications

The application of this theorem is to determine current in any element of a given

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Steps to Apply Norton's theorem

Step 1: Short the branch through which current is to be calculated.

Step 2: Obtain the current through this short circuited branch, using any of the network simplification technique. This is Norton's current (I_N).

Step 3: Calculate the equivalent resistance (R_{eq} or R_N) as viewed through the terminals of interest, by removing the branch resistance and deactivating all independent sources.

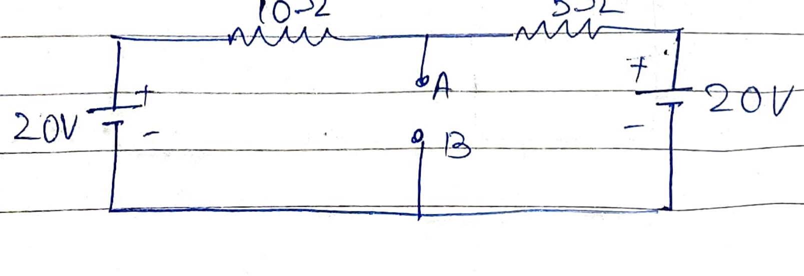
Step 4: Draw Norton's equivalent circuit showing current source I_N, with R_N parallel to it.

Step 5: Reconnect the branch resistance. Now, using current divider rule, find branch current.

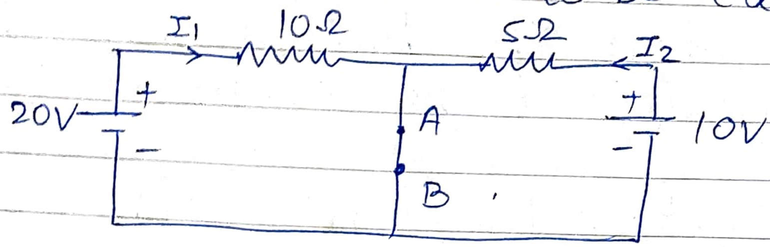
$$I = I_N \times \frac{R_{eq}\,(\text{or } R_N)}{R_{eq}\,(\text{or } R_N) + R_L}.$$

★ **Numericals:**

20.) Draw Norton's equivalent circuit at terminal AB for the circuit shown below.



Step 1 : Let us short the branch through which current is to be calculated.



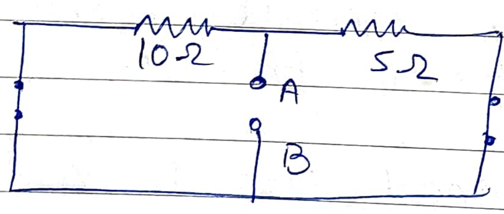
Step 2: Calculation of current through the short circuited branch.

$$I_N = I_1 + I_2$$

$$= \frac{20}{10} + \frac{10}{5}$$

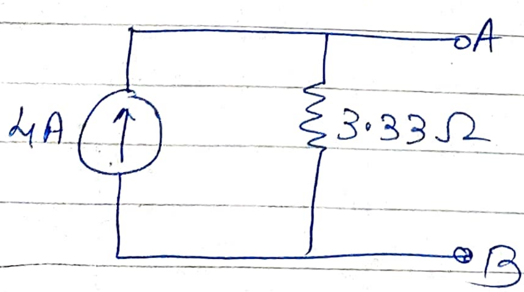
$$I_N = 4\ A \rightarrow (1)$$

Step 3: Determination of R_N. (Open the desired branch and deactivate all sources)



$$R_N = 10 \| 5 = \frac{10 \times 5}{10+5} = 3.33\ \Omega \rightarrow (2)$$

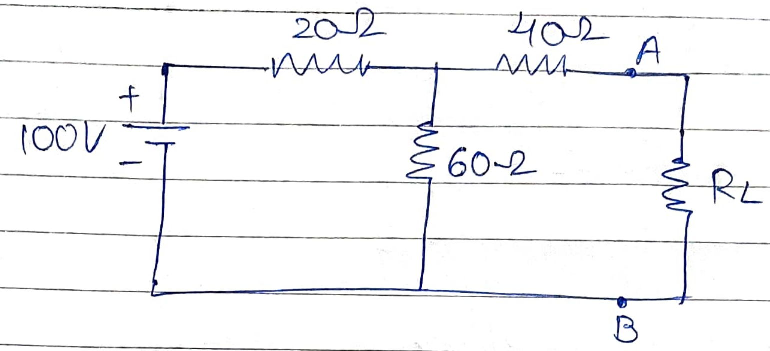
Step 4: Norton's equivalent circuit.



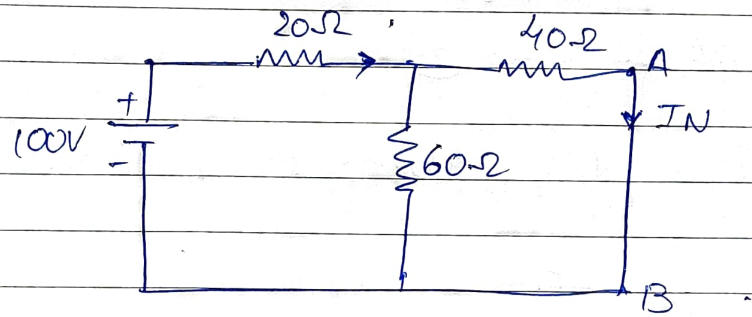
Ans.

_/_/_

(20.) Draw Norton's equivalent circuit for the below given circuit.



Step 1: Calculation of I_N (or I_{SC})



To find I_N, let us first find total current

$$I_T = \frac{100}{R_{eq}}$$

$$R_{eq} = 20 + (60 \| 40)$$

$$= 20 + \frac{60 \times 40}{60 + 40}$$

$$= \frac{100}{44}$$

$$R_{eq} = 44\Omega$$

$$I_T = 2.27A$$

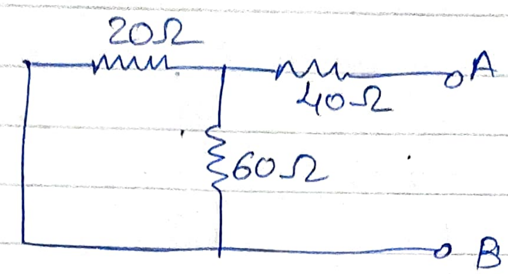
Let's find I_N by C.D.R

$$I_N = \frac{60}{40+60} \times 2.27$$

$$I_N = 1.36A \rightarrow (1)$$

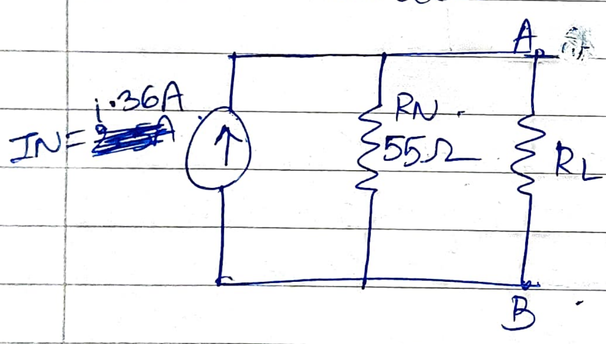
Step 2: Calculation of R_N

We will open circuit the branch AB and deactivate all sources.



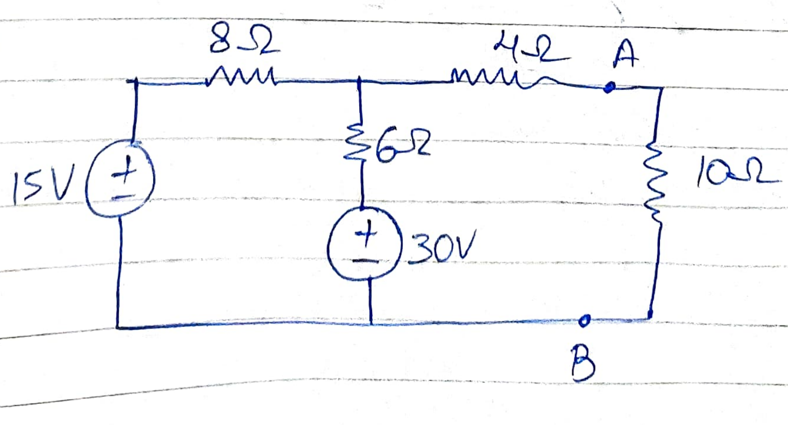
$$R_N = 40 + (20 \| 60)$$
$$= 40 + \left(\frac{20 \times 60}{20 + 60}\right)$$
$$R_N = 55\ \Omega \rightarrow (2)$$

Step 3: Let's draw Norton's equivalent circuit.



Ans.

(21.) Determine the current in 10Ω resistor using Norton's theorem, in the circuit below.



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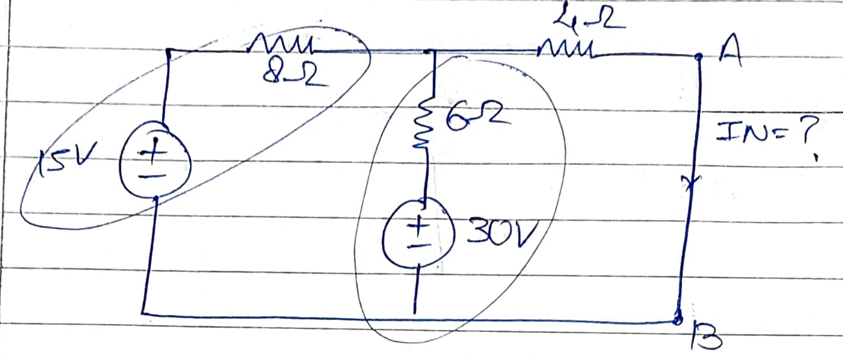
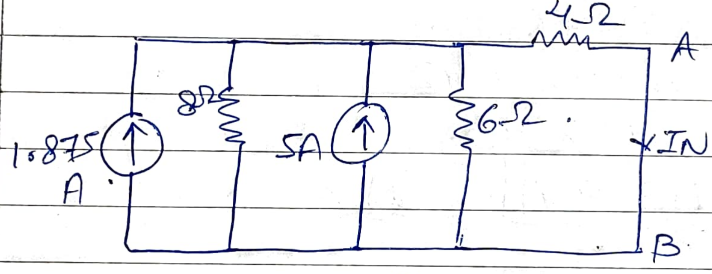
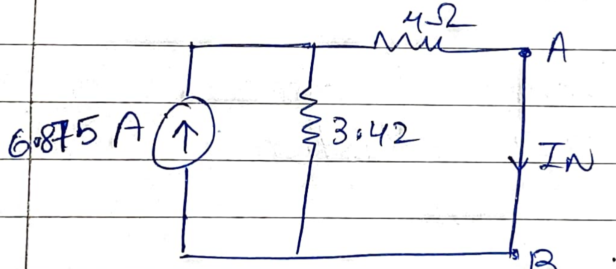
Step 1 : Determination of Isc.



Let us use source transformation.



$6 \| 8 = \frac{6 \times 8}{6+8} = 3.42\ \Omega$



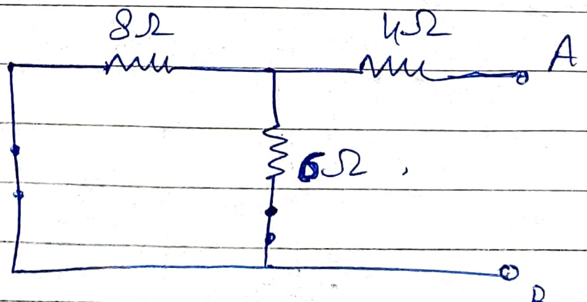
Current IN =

$$\frac{6.875 \times 3.42}{3.42+4}$$

$\therefore I_N = 3.17\ A \rightarrow (1)$

Step 2 : Determination of RN.

Let us open the branch AB and deactivate all independent sources.

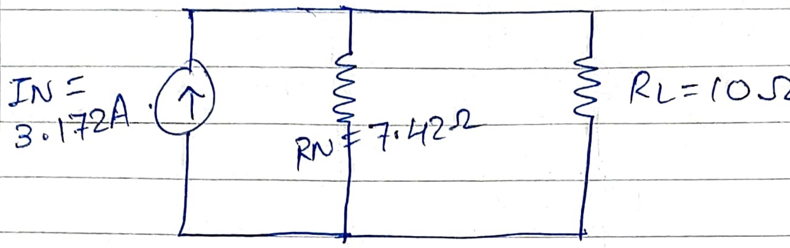


$$R_N = 4 + (8 \| 6)$$

$$= 4 + \frac{8 \times 6}{8+6}$$

$\therefore R_N = 7.42\ \Omega \rightarrow (2)$

Step 3: let's draw Norton's equivalent circuit.



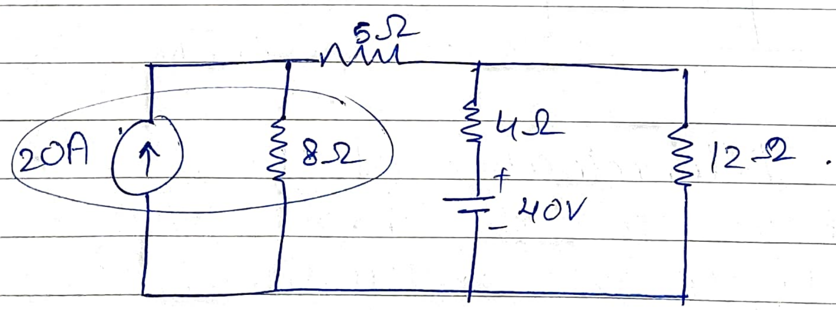
Current through 10Ω resistor

$$I = \frac{I_N \cdot R_N}{R_N + R_L} \quad \text{(By C.D.R)}$$

$$\therefore I = \frac{3.172 \times 7.42}{7.42 + 10}$$

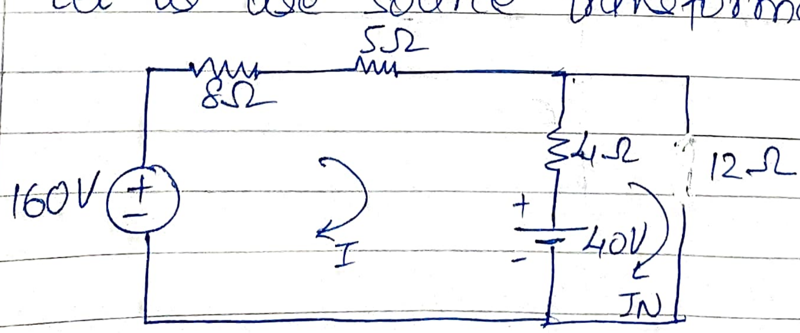
$$\therefore \boxed{I = 1.329\ A} \quad \text{Ans}$$

(22.) Draw Norton's equivalent circuit and find current through 12Ω resistor.



Step 1: Determination of Isc.

Let us use source transformation



Applying KVL to loop 1,

$$160 - 13I - 4(I - I_N) - 40 = 0$$
$$120 - 13I - 4I_1 + 4I_N = 0$$
$$17I - 4I_N = 120 \rightarrow (1)$$

Applying KVL to loop 2,

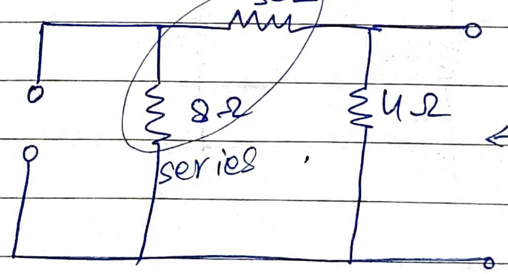
$$40 - 4(I_N - I) = 0$$
$$40 - 4I_N + 4I = 0$$
$$4I = 4I_N - 40 \rightarrow (2)$$

or

$$I = \frac{4I_N - 40}{4} \rightarrow \text{Substituting in (1)}$$
$$= I_N - 10$$

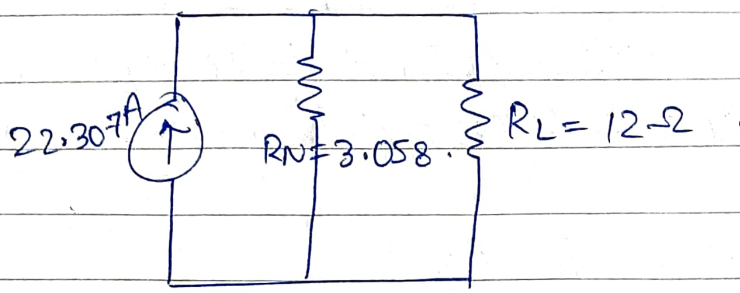
$$17(I_N - 10) - 4I_N = 120$$
$$17I_N - 170 - 4I_N = 120$$
$$13I_N = 290$$
$$I_N = 22.307\ A \rightarrow (1)$$

Step 2: Determination of RN.



$$R_N = 13 \| 4 = \frac{13 \times 4}{13 + 4} = 3.058\ \Omega \rightarrow (2)$$

Step 3: Lets draw the Norton's equivalent circuit.

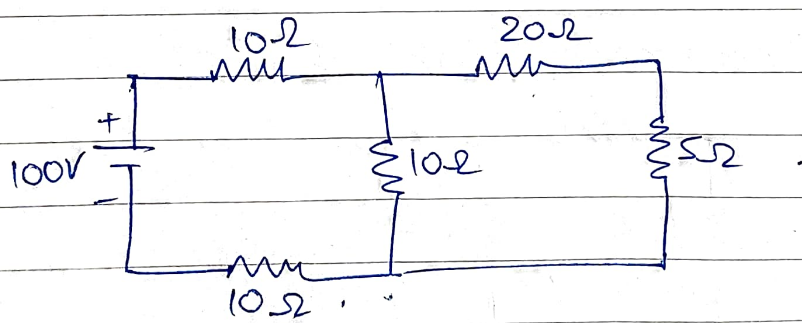


Current through 12 Ω resistor, I.

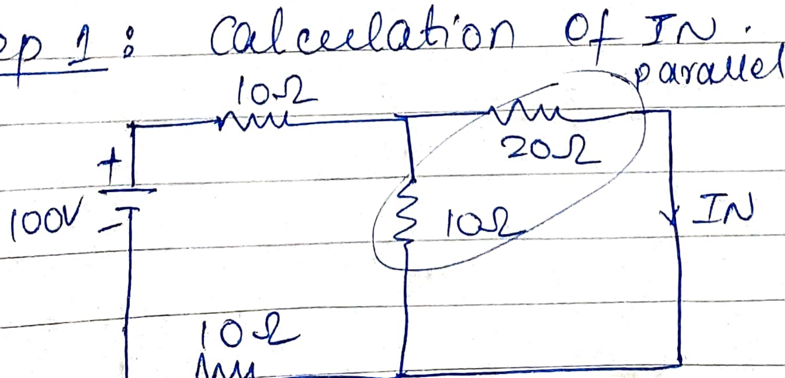
$$I = \frac{I_N \cdot R_N}{R_N + R_L} = \frac{22.307 \times 3.058}{3.058 + 12}$$

$$\boxed{I = 4.53A}$$ Ans.

(23.) Determine the value of current through 5 Ω resistance using Norton's theorem in the below circuit.



Step 1: Calculation of I_N.



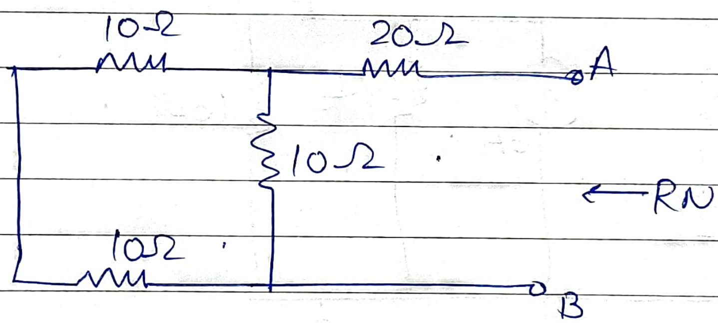
Let us first find total current.

$$R_{eq} = 10 + (10 || 20) + 10$$
$$= 10 + 6.66 + 10$$
$$R_{eq} = 26.66\ \Omega$$

$$I = \frac{100}{26.66} = 3.75\ A$$

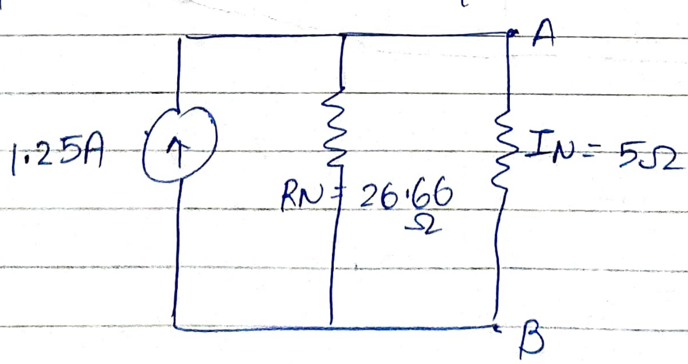
$$I_N = \frac{3.75 \times 10}{10+20} = 1.25\ A \rightarrow (1)$$

Step 2: Determination of R_N.



$$R_N = (10+10) || 10 + 20$$
$$= 20 || 10 + 20$$
$$R_N = 26.66\ \Omega \rightarrow (2)$$

Step 3: Norton's equivalent circuit.



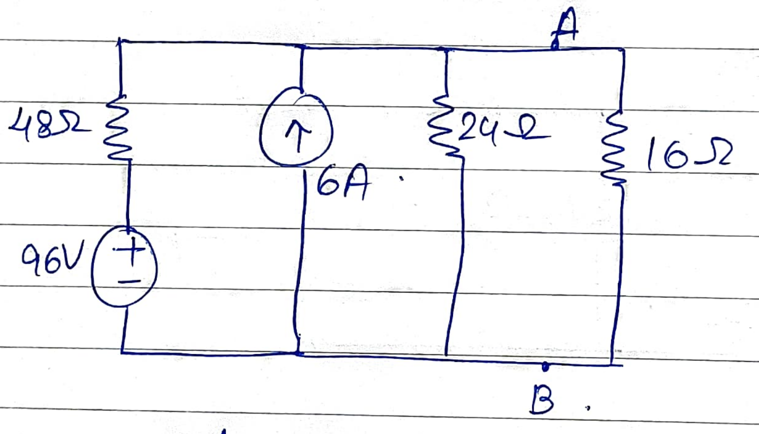
Current through 5 Ω resistor, I.

$$I = \frac{I_N \cdot R_N}{R_N + R_L}$$

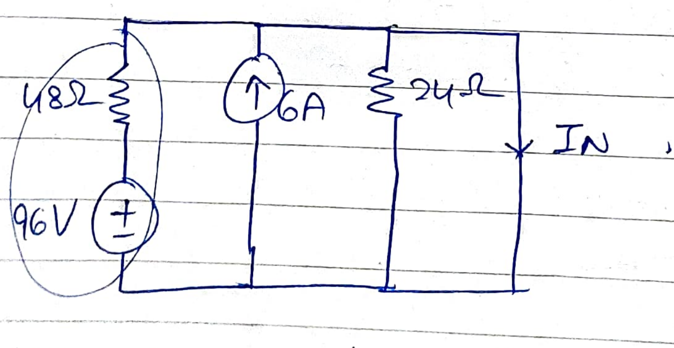
$$I = \frac{1.25 \times 26.66}{26.66 + 5}$$

$\therefore$ $\boxed{I = 1.05 A}$ Ans

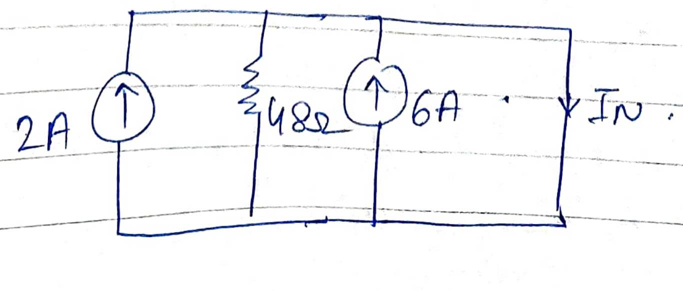
24. Consider the circuit given below. Find the current through 16 Ω resistor using Norton's theorem.



Step 1: Determination of I_N.



Using source transformation.



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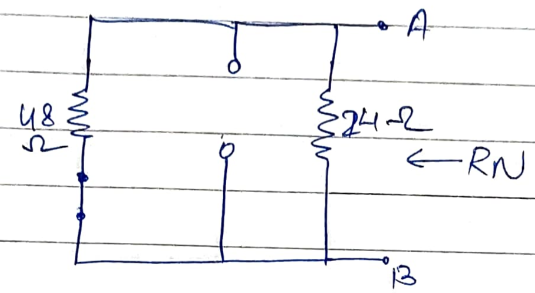
48 Ω resistor has both the terminals shorted, no current will flow through it.

So, $I_N = 6 + 2 = 8A \rightarrow (1)$

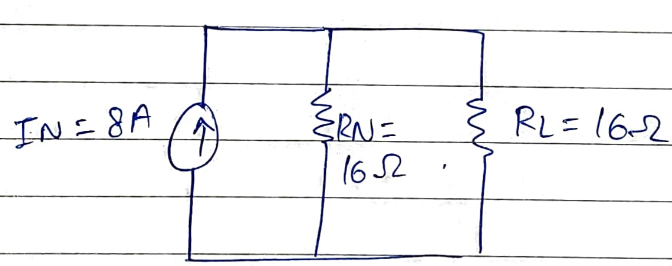
Step 2: Determination of RN.



$$R_{eq} = 48 \| 24 = \frac{48 \times 24}{48 + 24}$$

$R_{eq} = 16\,\Omega \rightarrow (2)$.

Step 3: Draw Norton's equivalent circuit.



Current through 16 Ω is I,

$$I = \frac{I_N \cdot R_N}{R_N + R_L} = \frac{8 \times 16}{16 + 16}$$

$\boxed{I = 4\,A}$. Ans.