The Fallacy of "To vs. Through" Glide Path Classification

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ABSTRACT

This paper discusses the "to" vs. "through" classification of glide paths. The paper makes the case that this classification is deeply flawed, serves no useful purpose, and should be abolished altogether.

There are more things
In heaven and earth, Horatio,
Than are dreamt of in your philosophy.

W. Shakespeare, Hamlet

Target date funds (TDF) have been increasingly gaining popularity since their designation as a Qualified Default Investment Alternative (QDIA) in 2006. Their glide paths – one of the key attributes of TDFs – have been at the center of attention of plan participants, sponsors, regulators, and others.

Obviously, it would be highly desirable to clarify and classify TDF glide paths. One of the first attempts to "pigeonhole" glide paths into broad categories was to classify them as either "*to* retirement" or "*through* retirement." This classification remains in use to this day.

This paper discusses the origins and the essence of the "to vs. through" classification. Our conclusion is *the "to vs. through" classification is deeply flawed, serves no useful purpose, and should be abolished altogether*.

"To vs. Through": Definitions

Initially, the definition of "to" vs. "through" glide paths was based on the investor's intentions regarding the withdrawal of assets. Investors who intend to withdraw and spend most of the money around the retirement date would use "to" glide paths that would not extend too far after the retirement date. Investors who intend to withdraw money gradually after the retirement date would use "through" glide paths that would extend through retirement.

It quickly became clear, however, that this classification was not terribly helpful. For the majority of investors, it makes little sense to withdraw and spend most of their retirement assets around the retirement date. Some have advanced the argument that "to" glide paths are designed for those who would use all of their retirement assets to purchase a life annuity at retirement. Yet, full annuitization has been a highly unpopular asset allocation decision. Thus, those who would utilize "to" glide paths do not represent a major segment of the marketplace.

In light of this realization, the original "to vs. through" classification has been redefined. The latest "to vs. through" classification is defined as follows. "To" glide paths are defined as the ones that stop gliding at retirement (i.e. "stationary" in retirement). Accordingly, "to" glide paths actually extend through retirement, but under the condition that all glide path portfolios after retirement are the same. All other glide paths are defined as "through" glide paths.[1]

This classification identifies a specific shape of glide paths in retirement (the stationary one) and suggests that contrasting this shape against all other shapes is helpful. We have serious reservations regarding the utility of this suggestion. However, the "to vs. through" debate has been so prominent in the industry that the merits of stationary glide paths in retirement deserve special attention. This issue is discussed in the next section.

The Merits of Stationary Glide Paths in Retirement

This section discusses the common lines of arguments in favor of "to" glide paths. The objective of this section is to focus on the "big picture" issues (e.g. the applicability and credibility of arguments) rather than technicalities.

- *Paul Samuelson demonstrated the optimality of stationary glide paths in 1969.*

This is the most credible argument in favor of "to" glide paths. Here is how it is presented in Daverman-O'Hara [2014]:

> "*In 1969, Nobel laureates Robert Merton and Paul Samuelson each independently demonstrated in their groundbreaking papers that in the absence of labor income the optimal strategic asset allocation is constant, with the amount of risk reflecting individual risk aversion. This fact remains as true now as it was then, which is why the idea of a "through" glidepath is so puzzling.*"

This reference to Paul Samuelson's work in this area is substantively incomplete. What is missing here is Samuelson's dissatisfaction with the framework utilized in Samuelson [1969]. Samuelson had called the optimality of stationary glide paths "*a defiance of folk wisdom and casual introspection*" and long endeavored to design a framework that rationalized evolving glide paths. More than once, Samuelson used the phrase "at last" in the titles of his papers when he thought that he succeeded in this endeavor.[2]

Moreover, Samuelson [1991] contains the following remarkable statement that emphasizes the importance of the rationalization of evolving glide paths:

> "*My three-decade search for confirmation of enhanced risk tolerance among long-horizon investors has thus, in a satisfying sense, achieved success. It is hard to believe that somewhere in the literature there is not already explicit anticipation of so important a result.*"

Furthermore, Samuelson called the optimality of stationary glide paths demonstrated in Samuelson [1969] "*a failure*" in an interview in 2005:

> "*I felt that the first pass at the problem was a failure. I had not proved that you should do something different over time.*"

In this interview, Samuelson also stated that "some very considerable progress has been made," but did not declare the problem solved.[3] For more details about Samuelson's endeavors to rationalize evolving glide paths, see Mindlin [2013].

Let us put these endeavors in their proper context. To demonstrate the optimality of a glide path, the first step should be to specify assumptions/principles and build a framework based on these assumptions/principles. The next step would be to demonstrate that the glide path is a solution to a properly enunciated glide path optimization problem in this framework. In this case, we say the framework rationalizes the glide path.

Paul Samuelson's objective was to rationalize evolving glide paths. For "the first pass at the problem," the framework utilized in Samuelson [1969] failed to do so – the optimal glide paths were stationary. Therefore, according to Samuelson, the framework was "a failure," its mathematical rigor notwithstanding. Different assumptions/principles were needed.

In reality, there is a multitude of frameworks that may rationalize a multitude of glide paths. Paul Samuelson and many others clearly understood this reality and endeavored to design different models. The fact that a particular framework rationalizes a particular glide path is inconclusive by itself. The crux of the matter is the quality of the framework and its ability to credibly reflect capital markets and human behavior.

Where may the framework in Samuelson [1969] be theoretically suspect? Here are a couple of potential areas of vulnerability. First, this framework is based on the objective of maximizing expected utility. While this objective is popular among academics, this popularity is far from universal. For instance, Roy [1952] contains the following memorable statement:

> "*In calling in a utility function to our aid, an appearance of generality is achieved at the cost of a loss of practical significance and applicability in our results. A man who seeks advice about his actions will not be grateful for the suggestion that he maximise expected utility.*"

Other economists have disputed the use of expected utilities as well. Rabin-Thaler [2001], for example, employs particularly strong language:

> "*… it is time for economists to recognize that expected utility is an ex-hypothesis, so that we can concentrate our energies on the important task of developing better descriptive models of choice under uncertainty.*"

There is a multitude of sensible investment objectives other than maximizing expected utility. For example, Mindlin [2016 B] demonstrates that a straightforward extension of the mindset of Modern Portfolio Theory (MPT) to multi-period problems leads to evolving optimal glide paths. Specifically, the objective of maximizing the mean of the terminal value given the volatility of the terminal value for a simple two-period model generates evolving glide paths. It should be noted that this objective assumes that the investor will always dutifully follow the glide path designed today. We call such glide paths *"will-do" glide paths*.

Second, this framework does not incorporate the fact that the investor is at liberty to select any portfolio at any future point. The original glide path assumption should be rationally expected to hold true only if any remaining segment of the original glide path (a sub-glide path) is optimal on its own (see Mindlin [2015 A]). Thus, in order to have a realistic glide path assumption, *any sub-glide path of an optimal glide path should be optimal on its own*. We call the glide paths that follow this principle *"expected-to-do" glide paths* because they are based on rational expectations of the investor's future portfolio selections. These considerations play no role in the Samuelson framework. As demonstrated in Mindlin [2015 C], "expected-to-do" glide paths are generally evolving.

Overall, the support that Samuelson [1969] provides to "to" glide paths is rather superficial. Those who refer to Samuelson [1969] without referring to subsequent works in this area essentially ignore an important period in the history of finance and diminish the credibility of their arguments.[4]

- *Why continue to de-risk?*

Some proponents of "to" glide paths have posed a direct question: Why continue to de-risk?[5] It is somewhat surprising to see this question in serious publications decades after a number of economists (Paul Samuelson in particular) aspired to find answers to this and similar questions.

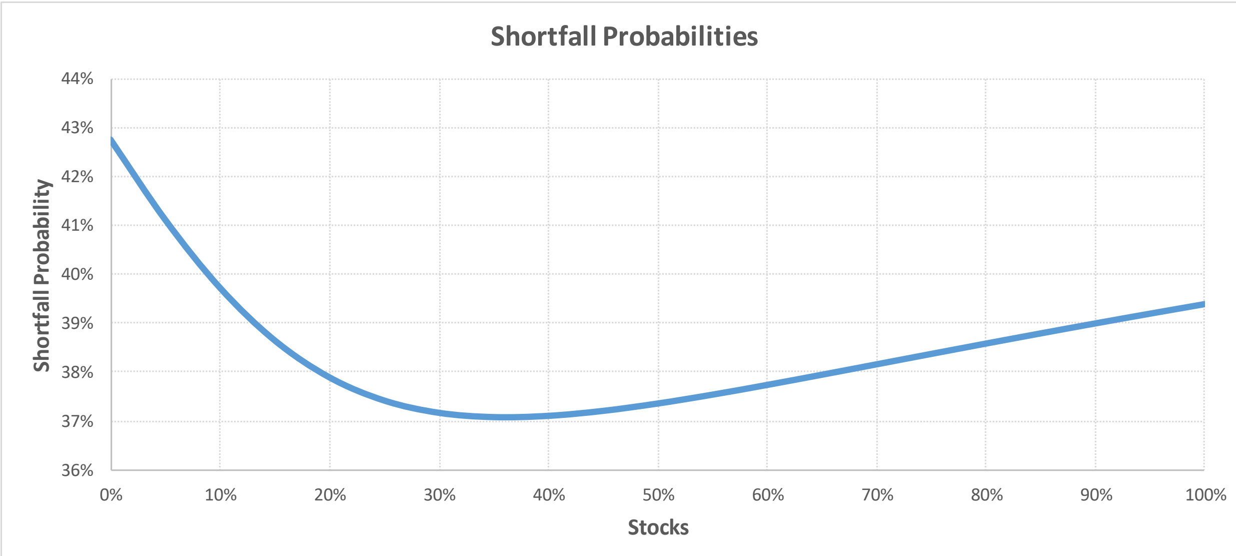
The proponents of "to" glide paths, however, seem unimpressed by the arguments presented by these economists. Their view has merits – indeed, the arguments in favor of evolving glide paths presented before Mindlin [2009] had been rather unconvincing. So, the question "Why continue to de-risk?" sounds like a direct challenge to those who may have doubts about "to" glide paths. In this case, this is a fair question that requires a straightforward answer.

Here is a short answer to this question: risk is a multi-headed monster. Here is a more detailed answer: investors face numerous risks; the relationships between these risks are complex; what looks like "de-risking" in the "asset-only" space may in fact be the evolution of a different type of risk. In particular, *constant risk* taken in the "asset-commitment" space may imply "de-risking" in the "asset-only" space, as demonstrated in Mindlin [2009] and Mindlin [2015 C].

To illustrate this point, let us consider a simplified numerical example. Think of an investor that has $97 now and has made a commitment to accumulate $100 in a year. Let us assume that there are two asset classes available to the investor: a stock index fund (geometric expected return 7%, standard deviation 15%) and a bond index fund (geometric expected return 4%, standard deviation 5%); the correlation between these funds is 0.2. *Let us also assume that the investor defines risk as the shortfall event – the failure of the commitment* (to accumulate $100 in a year). *The probability of shortfall is designated as the primary risk measurement.*

Exhibit 1 presents the shortfall probabilities for various portfolios.

Exhibit 1



In particular, a 100/0 (stocks/bonds) portfolio has a shortfall probability of 39.4%. Reducing stock allocations from 100.0% to 36.1% would "de-risk" portfolios in both "asset-only" and "asset-commitment" spaces (so a 36/64 portfolio delivers the lowest shortfall probability). Both the standard deviation of return and the shortfall probability would decrease.

However, further reductions of stock allocations from 36.1% to 4.5% would "de-risk" portfolios in the "asset-only" space but not in the "asset-commitment" space. The standard deviation of return would continue to decrease, but the shortfall probability would increase (see *Exhibit 1*). Further reductions of stock allocations from 4.5% to 0% would "re-risk" portfolios in both "asset-only" and "asset-commitment" spaces. Both the standard deviation of return and the shortfall probability would increase.[6]

Even in this simplified example, the relationships between different risks are far from straightforward. In more realistic examples, these relationships are even more complex (for example, see Mindlin [2015 C]). Thus, proper definitions of risk are indispensable.

The question "Why continue to de-risk?" is largely rhetorical – it is a challenge to the sceptics. This section as well as the epigraph to this paper represent our response to this challenge.[7]

- *"Sequential risk" arguments*

Some authors have attempted to rationalize stationary glide paths using the concept of "*sequential risk*."[8] Mindlin [2016 A] contains a detailed discussion of this concept, so we present just a brief summary of this discussion here.

"Sequential risk" exists in funding problems that involve risky assets and financial commitments (cash flows). In essence, "sequential risk" is an observation that there are sequences of returns that have the same annualized returns but generate different outcomes in terms of accumulated asset values. These different outcomes are caused by lower returns that occur when the asset values are higher and higher returns that occur when the asset values are lower.

Initially, the "sequential risk" observation was used to prove that the "asset-only" space is insufficient for glide path analysis. In this context, "sequential risk" was a useful concept. These days, however, few glide path designers operate in the "asset-only" space. The consensus is financial commitments do matter.

The "sequential risk" observation is certainly valid. Yet, the value of this observation is uncertain outside of the "asset-only" space context. The problem is the "sequential risk" observation falls short of a proper quantitative definition of risk and fails to produce well-defined arguments.

Is it possible to define "sequential risk" properly? Perhaps. A proper definition would identify a particular scenario (e.g. substantial asset losses around the retirement date) under which the desired level of post-retirement spending is unlikely to be funded. A proper definition would contain specific measurements of the time horizon, the magnitude of losses, and the ability of the remaining assets to fund the remaining financial commitments.

A proper definition, however, would create a different problem. One should take into account *all* scenarios under which the desired level of spending is unlikely to be funded. Identifying a particular undesirable scenario and ignoring others makes little sense. For example, a glide path may control the "sequential risk" well, but have a greatly diminished ability to fund the desired level of spending due to low expected returns. This glide path may still be sub-optimal.

Overall, arguments based on "sequential risk" alone are unlikely to produce conclusive results and should be taken with a healthy dose of skepticism.

- *Behavioral finance*

Behavioral finance plays an increasingly important role in the design of retirement plans. Yet, one should be mindful of its scope of applicability in the area of glide path design. Some behavioral finance arguments carry more weight than others. One particular argument is universally accepted and should outweigh many others: people prefer more money, *ceteris paribus*. To make this argument, one should define risks and their measurements, specify the risk “budget,” and demonstrate that a particular glide path delivers more money within the risk “budget.”

In the presence of such an argument, all other arguments are inconsequential. In the absence of such an argument, all other arguments are inconclusive.

- *Monte-Carlo simulations*

Some authors have attempted to rationalize stationary glide paths using Monte-Carlo simulations. It is well-known, however, that Monte-Carlo simulations have certain intrinsic limitations. Simulation results may depend on the sample size, software and hardware platforms, seeds, as well as other factors. Most glide path designers that rely on simulations do not use statistically credible sample sizes.[9]

Most importantly, simulation based analysis typically requires pre-selected glide paths *as inputs*. Even if the results of simulation analysis identified the best glide path among the pre-selected ones, the ultimate optimality of this glide path would remain in doubt. While simulation analysis may produce useful illustrations of the pros and cons of various glide paths, simulation results are unlikely to produce definitive conclusions.

Overall, the arguments in favor of stationary glide paths in retirement are debatable at best. Contrasting debatable glide paths to other glide paths neither classifies nor clarifies.

Our Position

Our position is based on the principles of *Commitment Driven Investing* (CDI) – an economic framework designed to optimize the process of funding financial commitments. See Mindlin [2014] for the essentials of CDI.

We believe that strategic analysis of investment and saving programs is vital for long-term investors. Investors need to select glide paths as long as investors have financial commitments to fund. Retirement investors' glide paths should extend *through* their lifetimes. Designing glide paths just *to* retirement is imprudent.

We believe that a glide path is an indispensable component of the process of optimal portfolio selection, not a mere disclosure. Today's optimal portfolio is generated via the process of either backward or forward induction. The direction of this process depends on investment objectives. Today's portfolio always comes with an optimal "through" glide path.[10]

We believe that a constructive glide path classification should be based on the measurements of investors' commitments, objectives, and risk tolerance. The following aspects of glide path design should be clearly disclosed:

- *Measurements*. Most components of funding problems are uncertain and should be modeled as random variables. Defining appropriate measurements of these random variables is of paramount importance. For example, the primary risk may be defined as the *shortfall risk* – a failure to fund a pre-determined level of post-retirement income. Then valuable measurements may include shortfall probability, shortfall size, and shortfall volatility. The selection of a suitable measurement may materially impact the resulting optimal glide path.
- *Commitments*. Investors' commitments are based on their demographic characteristics (e.g. age and retirement age), accumulated assets, and may include saving rates and the desired level of post-retirement income.
- *Objectives*. Investors' objectives may include to maximize post-retirement income given saving rates and risk tolerance, to minimize required saving rates given the desired post-retirement income and risk tolerance, and to minimize risk given the desired post-retirement income and saving rates.
- *Risk tolerance*. Risk tolerance and its evolution through the investor's lifetime is one of the key components of optimal glide path design.

The process of optimal glide path design is a multifaceted endeavor. A useful classification should be rather elaborate. Distinguishing "expected-to-do" and "will-do" glide paths should be a sensible first step.

Conclusion

The "to vs. through" classification is deeply flawed. It has gone through multiple definitions and been of little use to retirement plan participants and sponsors.

In recent years, the "to vs. through" debate has essentially abandoned its classification origins. Instead, it has evolved into a debate about the merits of stationary glide paths in retirement. While stationary glide paths may be optimal under certain specific conditions, the general arguments for the absolute superiority of stationary glide paths in retirement are disputable at best. For most investors that strive to optimize the outcomes of their investment programs, optimal glide paths should evolve throughout the investors' lifetimes.

The "to vs. through" classification is an imprudent misnomer.[11] It is time to abolish this misnomer altogether.

REFERENCES

Chiappinelli, P., Thirukkonda, R., [2015]. Who Ate Joe's Retirement Money? GMO LLC, August, 2015, link to the original.

Daverman, T., O'Hara, M., [2014]. Reexamining "To versus Through", New Research into an Old Debate, *BlackRock, Inc.*, May 2014, link to the original.

Fullmer, R., [2014]. Glide Path Classification: Sensibly Reframing "To versus Through", *T. Row Price*, April 2014, link to the original.

Ibbotson, R., Milevsky, M., Chen P., Zhu, K. [2007]. Lifetime Financial Advice: Human Capital, Asset Allocation, and Insurance, *The Research Foundation of CFA Institute*, 2007.

Knowles, K., Gardner, D., [2015]. The Date Debate Revisited, *Russell Research*, Russell Investments, May 2015, link to the original.

Mindlin, D., [2009]. Commitment Driven Investing and Time Diversification, *CDI Advisors Research*, CDI Advisors LLC, 2009, link to the original.

Mindlin, D., [2013]. A Tale of Three Epiphanies, *CDI Advisors Research*, CDI Advisors LLC, March, 2013, link to the original.

Mindlin, D., [2014]. Commitment Driven Investing: The Essentials, *CDI Advisors Research*, CDI Advisors LLC, March, 2014, link to the original.

Mindlin, D., [2015 A]. The Glide Path Assumption, *CDI Advisors Research*, CDI Advisors LLC, 2015, link to the original.

Mindlin, D., [2015 B]. The Glide Path "Takeoff", *CDI Advisors Research*, CDI Advisors LLC, 2015, link to the original.

Mindlin, D., [2015 C]. Why Glide Paths Evolve: "Expected-to-Do" Glide Paths, *CDI Advisors Research*, CDI Advisors LLC, 2015, link to the original.

Mindlin, D., [2016 A]. The Pitfalls of "Sequential Risk", *CDI Advisors Research*, CDI Advisors LLC, 2016, link to the original.

Mindlin, D., [2016 B]. Why Glide Paths Evolve: "Will-Do" Glide Paths, *CDI Advisors Research*, CDI Advisors LLC, 2016, link to the original.

Rabin, M., Thaler, R. [2001]. Anomalies: Risk Aversion, *Journal of Economic Perspectives*, 15, 219-32, 2001.

Samuelson, P., [1969]. Lifetime portfolio selection by dynamic stochastic programming, *The Review of Economics and Statistics*, 51, 239-246, 1969.

Samuelson, P., [1989]. A Case At Last for Age-Phased Reduction in Equity, *Proc. Natl. Acad. Sci. USA*, Vol. 86, 9048-9051, 1989.

Samuelson, P., [1991]. Long Run Risk Tolerance When Equity Return Are Mean Regressing: Pseudoparadoxes and Vindication of "Businessman's Risk", In Money, Macroeconomics, and Economic Policy: Essays in Honor of James Tobin, Edited by W. C. Brainard, W. D. Nordhaus, H. W. Watts, *MIT Press*, 1991.

Samuelson, P. [1992]. At Last, a Rational Case for Long-Horizon Risk Tolerance and for Asset-Allocation Timing? Active Asset Allocation, edited by Arnott, R.D. and Fabozzi, F.J., *Probus Publishing Company*, 1992.

ENDNOTES

[1] Fullmer [2014] contains a useful discussion of the definitions of "to vs. through" glide paths.
[2] See Samuelson [1989], Samuelson [1991], Samuelson [1992].
[3] From Paul Samuelson's interview conducted for the History of Finance project sponsored by the American Finance Association in 2005. This interview is quoted according to the transcript posted at http://www.afajof.org/details/video/2870851/Paul-Samuelson-Interview.html
[4] As a solution to the Samuelson problem, Mindlin [2009] contains an example of a simple framework that rationalizes evolving glide paths for random walk investment returns and no "human capital." This example is further simplified in Mindlin [2015 C]. Another simplified solution is presented in Mindlin [2016 B].
[5] See Daverman-O'Hara [2014], Knowles-Gardner [2015].
[6] Details of these calculations are available upon request (certain conditions apply).
[7] That is in addition to Mindlin [2009], Mindlin [2015 C], and Mindlin [2016 B].
[8] For example, see Chiappinelli-Thirukkonda [2015], Knowles-Gardner [2015].
[9] For example, Knowles-Gardner [2015] analyzes the median and the 5th percentile bequest using 2,000 simulated paths. So, the calculations of the 5th percentile rely on just 100 observations, which can hardly be considered statistically credible.
[10] Note that backward and forward inductions produce *Nash equilibrium* glide paths.
[11] Daverman-O'Hara [2014] and Fullmer [2014] concur with this statement.

Important Information