On the Validity of Common Portfolio Return Assumptions

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The Society of Actuaries has commissioned an independent Blue Ribbon panel to issue "recommendations for strengthening public plan funding." The report issued by the commission contains a number of practical recommendations that should improve public plan funding and management. The Actuarial Standard Board is likely to consider these recommendations in the development of actuarial practice standards.

One of the panel's major findings is the recommendation to focus on "median expected future investment conditions" and "median expected outcomes." In particular, the panel recommends to use "the median expected return" as the discount rate. These recommendations significantly affect the calculations of actuarial present values, contributions, and funded status.

Yet, while the logic of the report is reasonable, its language is occasionally imprecise and open to (mis)interpretations. The wording of some statements may imply certain relationships between the concepts utilized in the report that are actually not true. Given the significance of these concepts, this paper highlights these important issues.

Median Return vs. Geometric Return

The report's key statement regarding the selection of the assumed rate of return is the following:

> "*The Panel believes the assumed rate of return should be set at the median expected return, which should be based on the geometric mean return.*"

Taken at face value, the part of this statement that claims a relationship between the median and the geometric returns is problematic. Generally, the median and the geometric returns are not the same. Normal distributions would represent one example of this observation.

But let us give this statement the benefit of the doubt and view it in the context of the current practices in the pension industry. Most pension plans use forward-looking capital market assumptions (CMA) that specify the expected return, volatility, and correlations between the major asset classes under consideration. These CMA are used to calculate the expected return and volatility of portfolio returns. Furthermore, there are robust estimates of the geometric expected return based on the expected return and volatility of return.[1]

The calculations of median returns, however, require additional assumptions that deal with the shape of return distribution. One of the most prevalent assumptions of this kind is the assumption of *lognormal portfolio returns*. Under this assumption, the median and geometric returns are the same.[2]

The assumption of lognormal portfolio returns, however, has a glaring mathematical problem. While the assumption that *asset class* returns are lognormal creates no mathematical problems, the distributions of *portfolio* returns – linear combinations of asset class returns – are not necessarily lognormal. Generally, a linear combination of lognormals is not lognormal.

Technically speaking, the assumption of lognormal portfolio returns represents a lognormal *approximation* of linear combinations of lognormals. This approximation is based on matching the first two moments of the underlying distribution. The key question is, how good is this approximation?

To answer this question, let us assume that all individual asset classes have lognormal returns and examine the impact of the lognormal *portfolio* return assumption on the key measurements of portfolio returns – the arithmetic expected return, the geometric expected return, and the median return. The choice of these measurements was driven primarily by their role in the selection of discount rates.

To estimate median returns, this paper utilizes the following approach. Given a portfolio, conventional CMA, and the assumption of lognormal *asset class* returns, we calculate the first *three* moments of the portfolio return. Then we design a known distribution that matches these three moments (this methodology is called CDI3 in this paper).[3] The median return for this distribution is compared to the median return for the lognormal distribution that matches the first *two* moments of the portfolio return.

Let us consider three asset classes (A1, A2, and A3). The conventional CMA for these asset classes are presented in the Appendix. We consider six portfolios – from aggressive to conservative. *Exhibit 1* presents the results for these portfolios.

Exhibit 1

	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6
A1	60%	55%	50%	45%	40%	35%
A2	10%	20%	30%	40%	50%	60%
A3	30%	25%	20%	15%	10%	5%
Total	*100%*	*100%*	*100%*	*100%*	*100%*	*100%*
Arithmetic Return	8.80%	8.20%	7.60%	7.00%	6.40%	5.80%
Geometric Return	7.06%	6.84%	6.58%	6.26%	5.90%	5.48%
Volatility	19.70%	17.34%	15.01%	12.74%	10.54%	5.50%
Lognormal Median						
	7.06%	6.84%	6.58%	6.26%	5.90%	5.48%
CDI3 Median						
	6.65%	6.36%	6.07%	5.79%	5.51%	5.22%
The Difference = Lognormal Median – CDI3 Median						
	0.42%	**0.49%**	**0.51%**	**0.48%**	**0.39%**	**0.25%**

Exhibit 1 demonstrates that the lognormal assumption for all portfolios can significantly overestimate the median values. *Exhibit 1* also shows that, taken at face value, the statement regarding the connection between median and geometric returns is still problematic. Pension practitioners that wish to use the median portfolio return as the discount rate should avoid computational shortcuts and utilize more comprehensive approaches.

Investment Conditions vs. Outcomes

Let us continue giving the abovementioned statement the benefit of the doubt. Let us assume that the "median expected return" means the long-term median return, not the portfolio median return. The following statement support this conjecture:

> "*Plans should be using rates of return that they believe can be achieved over the next 20- to 30-year period with a 50 percent probability.*"

Technically speaking, this statement is based on the observation that the long-term accumulated asset value of today's $1 is approximately lognormallly distributed. Therefore, the "long-term" median return is close to the geometric return.[4]

There are several problems with this logic. First, this observation is generally invalid for multiple payment cash flows (in addition to today's $1). Even if the accumulated asset value of every payment is lognormal, the sum of lognormals is generally not lognormal. Second, if the portfolio return distribution is not lognormal, then the short- and mid-term accumulated values may not be close to lognormal. Yet, they may be responsible for a substantial portion of the present value. In both cases, the relationship between the "long-term" median return and the geometric return is unclear.

But the biggest problem is reflected in the following statement.

> "*In practice, this means that funding should at a minimum provide for benefits if the median expected future* ***investment conditions*** *occur. By focusing on the median* ***expected outcomes****, the adequacy concept considers both return volatility and those scenarios in which* ***investment return*** *assumptions are not realized.*"[5]

Dear reader, did you notice a quick journey from investing to outcomes and back to investing? This statement implies that median returns generate median outcomes. Even if it is true for each payment, the sum of medians is not necessarily equal to the median of the sum.

To illustrate this issue, let us consider the following numerical example: ten end-of-year contributions of $1 and their accumulated value after ten years. We assume that Portfolio 3 (50% of A1, 30% of A2, 20% of A3) is utilized in all years. For simplicity, let us assume that portfolio returns are lognormal.[6]

We calculate the deterministic accumulated value ($13.54) of these contributions using the median return 6.58%. Then we calculate the first three moments of the stochastic accumulated value and design a known distribution that matches these three moments (the CDI3 methodology). The median value for this distribution is $13.65. This accumulated value implies 6.75% return, which is higher than the geometric return 6.58%. The results are summarized in *Exhibit 2*.

Exhibit 2

Median Return	Accumulated Value	CDI3 Median	Implied Return	**The Difference**
6.58%	$13.54	$13.65	6.75%	**0.17%**

Thus, median investment conditions and median outcomes are not necessarily closely connected.

Conclusion

Some popular approximations may have convenient features and, at the same time, generate considerable errors. These approximations should be properly identified and disclosed. Pension practitioners should exercise caution with these approximations and utilize more comprehensive approaches to the calculations of measurements of portfolio returns in particular and outcomes of retirement programs in general.

APPENDIX: Capital Market Assumptions

Asset Class	Arithmetic Return	Volatility	Correlations		
			A1	A2	A3
A1	8.00%	16.00%	1		
A2	4.00%	5.00%	0.2	1	
A3	12.00%	35.00%	0.9	0.1	1

REFERENCES

Mindlin, D., [2010]. On the Relationship between Arithmetic and Geometric Returns, CDI Advisors Research, CDI Advisors LLC, 2010, http://www.cdiadvisors.com/papers/CDIArithmeticVsGeometric.pdf.

Mindlin, D. [2011]. Present Values, Investment Returns and Discount Rates, *CDI Advisors Research*, CDI Advisors LLC, 2011, http://www.cdiadvisors.com/papers/CDIDiscountRate.pdf

Endnotes

[1] See Mindlin [2010] for more details.
[2] See Mindlin [2011] for more details.
[3] The design of this distribution and the moment- matching technique involve certain technicalities that are outside of the scope of this paper.
[4] See Mindlin [2011] for more details.
[5] Emphasis added.
[6] The reader may notice, that the lognormal assumption used in this section is inconsistent with the message of the previous section. In this section, we use the lognormal assumption for simplicity. The technical details required to evaluate stochastic accumulated values for non-lognormal portfolio returns are outside of the scope of this paper.