Commitment Driven Investing: The Essentials

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Commitment Driven Investing (CDI) is a theoretical framework designed to generate optimal asset allocation, contribution, and payout strategies for institutional and individual investors with financial commitment to fund. Examples of such investors include, but are not limited to, defined benefit (DB) plans, defined contribution (DC) plans, 529 plans, sovereign funds, foundations and endowments.

Basic Definitions

The term "*financial commitment*" is defined as a series of future asset values at different points in time. Financial commitments can be out-flows (the asset values to be funded) and in-flows (contributions, or savings). Financial commitments can also be certain or uncertain in terms of their timing and magnitude.

Financial commitments play a special role in investing. Investors do not invest in a vacuum – they invest and take a multitude of risks primarily to fund their financial commitments. A typical *funding problem* involves an investor that has made commitments to contribute and invest assets in order to make pre-determined payments and/or accumulate pre-determined asset values. The investor's challenges include the selection of optimal policy portfolio as well as the determination of optimal payout and contribution strategies.

To fund an asset value at a future point in time means to ensure that the accumulated value of today's assets, future contributions, and investment returns is equal to or greater than this asset value at this future point. *To fund a commitment* means to fund every asset value in the commitment. Funding a commitment is a multi-period process. This framework is called *Commitment Driven Investing* to highlight the role of financial commitments in investing.

Generally, it is a challenge to guarantee at the present that the investor's financial commitment will be funded. To do so, the investor must purchase "the matching asset" for the investor's commitment – an asset that will certainly deliver the money when it is due. Ordinarily, the matching asset – a portfolio of tradable securities that match the commitment in terms of timing and magnitude – rarely exists. Moreover, even when the matching asset does exist, the investor may have compelling reasons not to invest in it (the asset may be too expensive, for instance). As a result, the majority of investors endeavor to fund their financial commitments by virtue of investing in risky non-matching assets.

These non-matching assets may have the capability to deliver the money when it is due, but provide no guarantee to do so. Non-matching assets are always risky in this context as the resources the investor is willing to contribute to fund the commitment may turn out to be insufficient.

Historically, institutional and individual investors have managed their investment programs primarily via the risk/return analysis of their portfolios. Portfolio optimization methodologies have been largely confined to the "asset-only" space. In these methodologies, the investor's financial commitments have played a very minor role at best.

There are two major components of the conventional portfolio optimization problem: return and risk. The picture is essentially *two-dimensional* (see **Exhibit 1**).

Exhibit 1

# The Risk-Return Line

Risk ⟷ Return
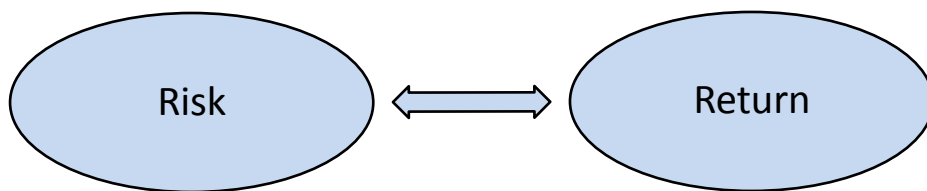
The presence of financial commitments changes this picture dramatically. Commitment out-flows and commitment in-flows are at the heart of the investor's funding problem. Portfolio risk and return obviously remain the focus of the investor's attention, but become the integral parts of a much more comprehensive funding objective.

The primary risk is defined as the shortfall event: the existing assets, the commitment in-flows, and investment returns are insufficient to fund the commitment-out-flows. To manage the primary risk, we need risk measurements. These measurements include, but are not limited to, shortfall probability, size, and volatility.

Thus, the key components of CDI are commitment out-flows, commitment in-flows, and risk. These components form *the funding triangle*, see **Exhibit 2**. The picture is essentially *three-dimensional*. The primary objective of CDI is to optimize the components of the funding triangle. The investment objectives and relationships between the components of the funding triangle are at the heart of CDI.

Exhibit 2

The Funding Triangle
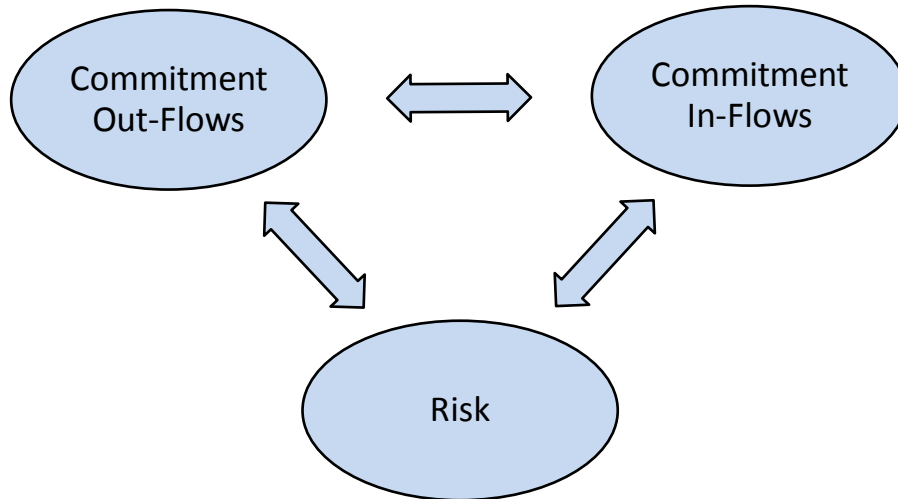
One of the most important parts of funding problems – optimal policy portfolio selection – may appear obscured in the funding triangle, but only seemingly so. Optimal policy portfolio selection is one of the key challenges the investor faces. Optimal policy portfolio selection plays a major role in the management of all components of the triangle. Optimal policy portfolio selection is one of the most important means of the optimization of the funding triangle. Optimal policy portfolio selection is in fact one of the main aspects of CDI.

To formally introduce investment portfolios, let us assume the investor has identified *M* assets classes for the investor's funding problem. A *portfolio* is defined as an *M*-dimensional vector *X* such that the portion of total assets invested in asset class *k* is equal to the kth component X_k of *X*. For simplicity, it is assumed that $X_k \geq 0$ for all $k \leq M$ and $\sum_{k=1}^{M} X_k = 1$. A policy portfolio (a.k.a. *glide path*) is a series of portfolios $\{X_t\}$, $1 \leq t \leq T$, where *T* is the number of time periods. In essence, a policy portfolio is a $M \times T$ matrix in which the sum of elements in each column is equal to 1 (a matrix with this property is called stochastic matrix).

Given policy portfolio $\{X_t\}$, here is how the investor's portfolios are expected to evolve over time. The investor's actual portfolio is X_1 at the beginning of the first period. At the beginning of the tth period, the investor is expected to reallocate (rebalance) to portfolio X_t. The process is expected to continue for all $2 \leq t \leq T$.[1]

Investment Objectives

Different investors may have different funding problems. Different funding problems may require different investment objectives. The funding triangle structure allows a concise formula for defining investment objectives. Specifically, investment objectives are specified according to the following principle:

"*given two vertices in the funding triangle, optimize the third.*"

Specifically, the investment objectives are defined as follows:

Objective *A*: *To maximize commitment out-flows* given commitment in-flows and risk.
Objective *B*: *To minimize commitment in-flows* given commitment out-flows and risk.
Objective *C*: *To minimize risk* given commitment in-flows and commitment out-flows.

It is important to note that objectives *A*, *B*, and *C* represent just general goals that require the selection of specific measurements of the commitments and risk for their implementation. In particular, the objective "to minimize risk," while common, is an abbreviated form of "to minimize the selected measurement of risk."

Here is how these objectives work for various investors.

Defined Contribution (DC) Plans

DC plans may involve several commitments.

Commitments out-flows:

- to accumulate a particular asset value at retirement;
- to fund a particular percentage of the last pre-retirement compensation (a.k.a. replacement rate) payable in retirement;
- to purchase an annuity that makes pre-determined payments in retirement;

Commitments in-flows:

- to contribute a particular (fixed or increasing) percentage of compensation.
- a DC plan sponsor may have a commitment to make matching contributions.

In this context, Objective *A* becomes "to maximize post-retirement spending given pre-retirement contributions and risk." Objective *A* is appropriate for a plan participant who wishes to maximize post-retirement spending given contributions and risk.

Objective *B* becomes "to minimize pre-retirement contributions given post-retirement spending and risk." Objective *B* is appropriate for a plan participant who has determined the desired level of post-retirement income and wishes to minimize the required contributions to achieve this income at a given level of risk.

The investment objectives for 529 Plans are conceptually similar to the ones of DC Plans.

Defined Benefit (DB) Plans

DB plan may have the following commitments.

Commitments out-flows:
- to fund the plan's benefits;
- to achieve a pre-determined funded status over a pre-determined time period.

Commitments in-flows:
- to make contributions according to a particular set of rules.

In this context, Objective *B* becomes "to minimize the cost of funding benefits given risk." Objective *C* becomes "to maximize the safety of benefits given the level of contributions." Objectives *B* and *C* may serve the best interests of plan participants and the sponsor.

Sovereign Funds, Foundations, Endowments

A sovereign fund, foundation, or endowment may have the commitment to achieve a particular value of the fund over a particular time period. For example, the fund may want its after-spending inflation-adjusted value to exceed a particular multiple S (e.g. 90%, 100%, 110%) of today's value of the fund in N years.

Objective *A* becomes "to maximize multiple S given contributions and risk." Objective *C* becomes "to minimize risk given contributions and multiple S."

Given several investment objectives for a funding problem, it is important to understand whether different objectives may lead to the same set of optimal portfolios (efficient frontier). These matters are considered in later sections.

Required Assets

To achieve the investment objectives outlined in the previous section, the investor have several decisions to make. These decisions may include, but are not limited to, the selections of optimal policy portfolio, sustainable commitment out-flows, and required commitment in-flows. All these decisions must be made at the present.

To do so, the investor needs appropriate measurements of financial commitments. Since the commitments are multi-period objects, the time value of money principle generally guides us to the construction of present values. The specific type of the commitments' present values must be consistent with investment objectives. It should be emphasized that CDI imposes the following "order of operations": *to specify investment objectives first and select the appropriate present values next.*

The key analytical tool in CDI is the concept of *stochastic present value* (SPV). This concept is instrumental in providing an effective analytical framework for the investment objectives.

Let $F_1, \ldots, F_N$ be commitment out-flows (end-of-period) and $R_1, \ldots, R_N$ be a series of portfolio returns generated by the plan's policy portfolio in corresponding years. Then the SPV of series $F_1, \ldots, F_N$ is defined as

$$RA = \sum_{k=1}^{N} \frac{F_k}{\left(1+R_1\right)\ldots\left(1+R_k\right)}$$

Random variable *RA* is called the *required assets* associated with the commitment out-flows and the policy portfolio. *RA* represents the asset value required to fund the commitment. The concept of *RA* is the key technical tool for the cost-risk analysis of financial commitments.

Let us assume that the investor's asset value at the present is A_0 and the primary objective is to fund $F_1, \ldots, F_N$. The shortfall event happens if and only if *RA* is greater than A_0, i.e. the asset value required to fund the commitment is greater than the existing asset value. Thus, we have an easy and intuitively clear expression of the shortfall event in terms of *RA* :

$$\text{The Shortfall Event} = \left\{RA > A_0\right\}$$

Now let $G_1, \ldots, G_N$ be commitment in-flows (end-of-period). For convenience, commitment out-flows are assumed to be positive and commitment in-flows are assumed to be negative. The corresponding *RA* is defined as

$$RA_N = \sum_{k=1}^{N} \frac{F_k - G_k}{(1+R_1)\ldots(1+R_k)}$$

Since the shortfall event may happen in any period, we also need the "intermediate" *RA* for all $n \leq N$:

$$RA_n = \sum_{k=1}^{n} \frac{F_k - G_k}{(1+R_1)\ldots(1+R_k)}$$

Now the shortfall event is defined as follows:

$$\text{The Shortfall Event} = \bigcup_{n=1}^{N} \{RA_n > A_0\}$$

In most practical applications, however, we have

$$\{RA_1 > A_0\} \subseteq \{RA_2 > A_0\} \subseteq \ldots \subseteq \{RA_N > A_0\}$$

Therefore, the shortfall event definition is simplified to the following:

$$\text{The Shortfall Event} = \{RA_N > A_0\}$$

Thus, stochastic present value *RA* is instrumental in quantifying the primary risk of the funding problem and translating it into the standard language of probability theory.

CDI vs. Modern Portfolio Theory

Modern Portfolio Theory (MPT) is an economic theory for optimal portfolio selection that focuses on the interplay between portfolio return and risk. The ideas of MPT were pioneered by Harry Markowitz in his seminal paper Markowitz [1952]. The ideas eventually became one of the cornerstones of modern finance and asset management.

Today, countless institutional and individual investors use MPT as the basis for their asset allocation decisions. Many educational programs present MPT as a great example of a scientifically rigorous approach to investing. Overall, MPT has withstood the test of time.

Yet, MPT is not perfect. Its assumptions and effectiveness have been challenged from various perspectives. As far as this article is concerned, the following aspects of MPT are essential.

- The original MPT is a single-period framework applicable to a hypothetical investor rather than a specific investor with well-defined financial commitments. In MPT, financial commitments play no role in the portfolio optimization process.

  MPT disregards some of the most essential characteristics of the investor. For DB plans, for example, the nature of the plan (public vs. private), funded status, and funding cost play no direct role in the optimization process. For DC plans, for example, the age, retirement age, compensation, saving rates, and the existing assets play no direct role in the optimization process. The inflation and interest rate risks play no role in the optimization process as well. One may think of CDI as an attempt to incorporate financial commitments by virtue of replacing portfolio return by *RA* as the primary object of analysis and then following the spirit of MPT.

- MPT attempts to optimize the *future value* of today's $1. MPT minimizes the volatility of the future value given the mean of the future value. In other words, MPT assumes that the asset value is known at the present, and the challenge is to minimize the volatility of the future value given the expected future value.

  For an investor with financial commitments to fund, the situation is exactly the opposite. The future values – the commitments – are given. The present value – the existing asset value plus the present value of future contributions – is uncertain and much more volatile than the future values. Consequently, the roles of the future and present values are reversed in CDI compared to MPT.

  In the simplest version of CDI, given $1 in the future, CDI minimizes the volatility of the SPV of this dollar given the mean of the SPV. In other words, CDI assumes that the future asset value is known, and the challenge is to minimize the volatility of the SPV given the expected present value. In this sense, CDI is MPT "in reverse."

- MPT provides a valuable guidance for the optimization of stochastic objects. In MPT, the primary object of interest is the investment return of a portfolio of risky assets, which is uncertain by definition. If the objective is to maximize the investment return, then the solution is a set of efficient portfolios (*efficient frontier*). While the investment return is obviously important, investors have vested interests in other important stochastic objects as well, e.g. the SPV of financial commitments (*RA*).

As demonstrated in subsequent sections, the similarities between MPT and CDI are profound, although dissimilarities exist as well.[2]

The Funding Triangle Optimization

Investment objectives *A*, *B*, and *C* require measurements of the commitments and risk for their implementation. This section presents several ways to implement these objectives.

Let us start with a DB plan with a given stream of benefits (commitment out-flows) and the objective to minimize the funding cost (objective *B*). For simplicity, let us assume that the plan employs *the same portfolio X in all years*. The corresponding RA_X, which is directly related to the funding cost minimization, is defined as

$$RA_X = \sum_{k=1}^{N} \frac{B_k}{(1+R_1)\ldots(1+R_k)}$$

where $B_1,\ldots,B_N$ are the benefit payments, $R_1,\ldots,R_N$ are identically distributed returns of portfolio *X*. The challenge is to "minimize" this RA_X, even though RA_X is not a conventional function that can be minimized using traditional methods.

Here is how MPT deals with a similar problem. For a given portfolio *X* and risk aversion parameter $t \geq 0$, "risk-adjusted expected return" *R* of portfolio return R_X is defined as follows:

$$R = E_X - t \cdot S_X$$

where E_X is the mean of R_X and S_X is the standard deviation of R_X. This equation connects measurements of return and risk. On the right side of this equation, E_X and $t \cdot S_X$ can be interpreted as the investor's "reward" and "penalty" for risk taking. The objective is dual:

- to *maximize* risk-adjusted expected return *R* for each risk aversion parameter *t* ;
- to *maximize* risk aversion parameter *t* for each risk-adjusted expected return *R*.

Both of these optimization procedures generate the classic mean-variance efficient portfolios. Let us follow a similar approach to optimize the above-defined RA_X. For a given policy portfolio *X* and risk aversion parameter $t \geq 0$, let us define "risk-adjusted expected cost" *C* as

$$C = E_X + t \cdot S_X$$

where E_X is the mean of RA_X and S_X is the standard deviation of RA_X. This equation connects measurements of funding cost and risk. We call this equation *the policy portfolio equation*. On the right side of this equation, E_X and $t \cdot S_X$ can be interpreted as the investor's "reward" and "penalty" for risk taking. The investment objective is dual:

- to *minimize* risk-adjusted expected cost C for each risk aversion parameter t;
- to *maximize* risk aversion parameter t for each risk-adjusted expected cost C.

The definitions of "risk-adjusted expected return" R and "risk-adjusted expected cost" C are similar, yet different. "Risk-adjusted expected return" R is equal to the *sum* of "reward" E_X and "penalty" $t \cdot S_X$. "Risk-adjusted expected cost" C is equal to the *difference* between "reward" E_X and "penalty" $t \cdot S_X$. This is a reflection of the directional difference between CDI and MPT – investors want *high* returns and *low* cost. Therefore, the "penalty" is subtracted from the "reward" when we deal with returns and added to the "reward" when we deal with cost.

Exhibit 3 contains side-by-side comparison of the optimization procedures in CDI and MPT.

Exhibit 3.

Commitment Driven Investing vs. Modern Portfolio Theory		
	CDI	**MPT**
Object of Analysis	Required Assets RA_X	Portfolio Return R_X
Object Preferred	Low	High
The Policy Portfolio Equation	$C = E_X + t \cdot S_X$ where E_X is the mean of RA_X S_X is the standard deviation of RA_X	$R = E_X - t \cdot S_X$ where E_X is the mean of R_X E_X is the standard deviation of R_X
Optimization Problems	Given C, Maximize t Given t, Minimize C	Given R, Maximize t Given t, Maximize R

The key difference between CDI and MPT is the object of analysis – stochastic present value *RA* in CDI vs. portfolio return in MPT. The directional difference between CDI and MPT is

reflected in the formulations of the optimization problems, which are analogous yet different (see *Optimization Problems* in **Exhibit 3**).

Another method of optimizing portfolios is offered by the <u>Roy's Safety-First</u> approach. For a given portfolio *X*, let threshold return *R* be equal to the pth percentile of portfolio return R_X :

$$p = \Pr(R_X < R)$$

Similar to MPT, this equation connects measurements of return and risk. The investment objective is dual:

- to *minimize* probability *p* for each threshold return *R*;
- to *maximize* threshold return *R* for each probability *p*.

Let us follow a similar approach to optimize the above-defined RA_X . For a given portfolio *X*, let us define threshold cost *C* as the pth percentile of portfolio return RA_X :

$$p = \Pr(RA_X < C)$$

Similar to the "mean-variance" version of CDI, this equation (also called the policy portfolio equation) connects measurements of funding cost and risk. The investment objective is dual:

- to *maximize* probability *p* for each threshold cost *C*;
- to *minimize* threshold cost *C* for each probability *p*.

Exhibit 4 contains side-by-side comparison of the conventional "Safety-First" approach and its counterpart in CDI. The key difference between them is the object of analysis. The directional difference between these approaches is reflected in the formulations of the optimization problems, which are analogous yet different (see *Optimization Problems* in **Exhibit 4**).

The approach presented in **Exhibit 4** represents *the "safety-first" version of CDI*. It is important to note that the CDI framework can incorporate a variety of commitment and risk measurements, policy portfolio equations, and other aspects of funding problems. In particular, there are several types of the "safety-first" version of CDI.

Exhibit 4.

<table>
<tr><td colspan="3" align="center">Commitment Driven Investing vs. Safety-First</td></tr>
<tr><td></td><td align="center">CDI</td><td align="center">Safety-First</td></tr>
<tr><td align="center"><i>Object of Analysis</i></td><td align="center">Required Assets RA_X</td><td align="center">Portfolio Return R_X</td></tr>
<tr><td align="center"><i>Object Preferred</i></td><td align="center">Low</td><td align="center">High</td></tr>
<tr><td align="center"><i>The Policy
Portfolio Equation</i></td><td align="center">$p = \Pr(RA_X < C)$</td><td align="center">$p = \Pr(R_X < R)$</td></tr>
<tr><td align="center"><i>Optimization
Problems</i></td><td align="center">Given C, Maximize p
Given p, Minimize C</td><td align="center">Given R, Minimize p
Given p, Maximize R</td></tr>
</table>

This section presents two additional examples of the "safety-first" version of CDI. The first example involves the series of payroll values $P_1,\ldots,P_N$ and assumes that the plan's contributions are equal to the fixed percentage f of payroll. The corresponding RA is defined as follows:

$$RA_X(f) = \sum_{k=1}^{N} \frac{B_k}{(1+R_1)\ldots(1+R_k)} - f \sum_{k=1}^{N} \frac{P_k}{(1+R_1)\ldots(1+R_k)}$$

where $B_1,\ldots,B_N$ are the benefit payments. The "safety-first" policy portfolio equation is defined as follows:

$$p = \Pr\left(RA_X(f) < A_0\right)$$

where A_0 is the existing asset value, p is the probability that A_0 and contribution rate f are sufficient to fund the benefits.

The investment objective is dual:

- to *maximize* probability p for each contribution rate f;
- to *minimize* contribution rate f for each probability p.

In the second example, a DC plan participant wishes to make a series of contributions $C_1,\ldots,C_N$ to the plan to fund a series of post-retirement payments equal to the last pre-retirement income times replacement rate r. The corresponding RA is defined as follows:

$$RA_X(r) = r\sum_{k=1}^{N}\frac{I_k}{(1+R_1)\ldots(1+R_k)} - \sum_{k=1}^{N}\frac{C_k}{(1+R_1)\ldots(1+R_k)}$$

where $I_1,\ldots,I_N$ is the series of the last pre-retirement income values possibly adjusted for inflation and mortality.

The "safety-first" policy portfolio equation is defined as follows:

$$p = \Pr\left(RA_X(r) < A_0\right)$$

where A_0 is the existing asset value, p is the probability that A_0 and contributions $C_1,\ldots,C_N$ are sufficient to fund the standard of living implied by replacement rate r.

The investment objective is dual:

- to *maximize* probability p for each replacement rate r;
- to *maximize* replacement rate r for each probability p.

Overall, the CDI framework can deal with a variety of funding problems and commitment/risk measurements. The right version of CDI should be determined based on the specifics of the funding problem.

Efficient Frontier Invariance Theorems

In the prior section, we presented several policy portfolio equations and corresponding dual investment objectives. For each policy portfolio equation, the following question arises naturally: Would the corresponding dual objectives generate the same set of optimal policy portfolios?

A similar question arises if we look at the problem of the funding triangle optimization from a different angle. As was discussed in a prior section, the investment objectives are specified according to the principle "given two vertices in the funding triangle, optimize the third." For example, if *X*, *Y*, and *Z* are the vertices in the funding triangle, then we may have the following investment objectives:

- given vertices *X* and *Y*, optimize *Z*;
- given vertices *X* and *Z*, optimize *Y*.

Taking *X* out of these objectives as given, the objectives become as follows:

- given *Y*, optimize *Z*;

- given *Z*, optimize *Y*.

As was just discussed, the following question arises naturally: Would these objectives generate the same set of optimal policy portfolios? This section demonstrates that, in many cases, the answer is affirmative.

CDI Mean-Variance Efficient Frontier Invariance Theorem. For given commitment out-flows $\{B_k\}$, let us define required assets RA_X as

$$RA_X = \sum_{k=1}^{N} \frac{B_k}{(1+R_1)\ldots(1+R_k)}$$

and the policy portfolio equation as

$$C = E_X + t \cdot S_X$$

where E_X and S_X are the mean and standard deviation of RA_X. Then the objectives "given *C*, maximize *t*" and "given *t*, minimize *C*" generate the same set of optimal policy portfolios.

CDI Safety-First Efficient Frontier Invariance Theorem. For given commitment out-flows $\{B_k\}$, let us define required assets RA_X as

$$RA_X = \sum_{k=1}^{N} \frac{B_k}{(1+R_1)\ldots(1+R_k)}$$

and the policy portfolio equation as

$$p = \Pr(R_X < R)$$

Then the objectives "given *R*, maximize *p*" and "given *p*, minimize *R*" generate the same set of optimal policy portfolios.

CDI Safety-First Efficient Frontier Invariance Theorem (contribution minimization). For given commitment out-flows $\{B_k\}$ and commitment in-flows equal to the fixed percentage *f* of payroll $\{P_k\}$, let us define required assets RA_X as

$$RA_X(f) = \sum_{k=1}^{N} \frac{B_k}{(1+R_1)\ldots(1+R_k)} - f \sum_{k=1}^{N} \frac{P_k}{(1+R_1)\ldots(1+R_k)}$$

and the policy portfolio equation as

$$p = \Pr\left(RA_X(f) < A_0\right)$$

where A_0 is the existing asset value. Then the objectives "given f, maximize p" and "given p, minimize f generate the same set of optimal policy portfolios.

CDI Safety-First Efficient Frontier Invariance Theorem (replacement rate maximization). For given commitment in-flows $\{C_k\}$ and commitment out-flows equal to the fixed percentage r of series $\{I_k\}$, let us define required assets RA_X as

$$RA_X(r) = r \sum_{k=1}^{N} \frac{I_k}{(1+R_1)\ldots(1+R_k)} - \sum_{k=1}^{N} \frac{C_k}{(1+R_1)\ldots(1+R_k)}$$

and the policy portfolio equation as

$$p = \Pr\left(RA_X(r) < A_0\right)$$

where A_0 is the existing asset value. Then the objectives "given r, maximize p" and "given p, maximize r" generate the same set of optimal policy portfolios.

Thus, the corresponding dual objectives, under certain general conditions, generate the same set of optimal policy portfolios. For a given vertice X in the funding triangle, the objectives "given Y, optimize Z" and "given Z, optimize Y" generate the same set of optimal policy portfolios.

CDI, Time diversification, and Nash Equilibria

So far, we have assumed for simplicity that policy portfolios are stationary, i.e. the same portfolio is employed in all years. This assumption is no longer made in this section. Thus, we assume that policy portfolios may evolve over time.

This assumption is commonly made for the participants of DC plans. It is broadly believed that a DC plan participant's portfolio should evolve from more aggressive to more conservative over

the participant's life cycle. It is also common for certain investment products to disclose the expected policy portfolio (glide path). Target date funds, for example, routinely do so.

Economists have researched the theoretical foundation of the belief "more stocks for the young, more bonds for the old" for decades. Paul Samuelson, in particular, was one of the most prominent researchers of this belief. The results of the efforts of many economists in this area, however, have been somewhat inconclusive.

This belief is related to the famously controversial time diversification problem ("does the riskiness of stocks decreases over time?"). If one believes in the time diversification properties of stocks, then the principle "more stocks for the young, more bonds for the old" holds true (since the young have more time by definition). But if one is neutral on the issue of time diversification, then the belief "more stocks for the young, more bonds for the old" is still open to discussion.

The problem is to find a sensible framework in which optimal policy portfolios evolve over time. We call this problem *the Samuelson problem* in recognition of Paul Samuelson's contribution to this area. This section presents CDI as a solution to the Samuelson problem.

Several factors make CDI a good candidate for a solution. First, CDI is a quintessential outcome-driven framework as the primary risk in CDI is defined as the shortfall event. Second, CDI is a multi-period framework. Third, CDI contains well-developed optimization objectives and analytical tools. As we see shortly, all these factors play important roles.

As a first step in the development of a solution, we need to answer the following key question. *Why is it desirable to envision the whole policy portfolio at the present*? The answer to this question is vital to this development.

CDI offers the following answer. It is desirable to envision the whole policy portfolio at the present because a realistic comprehensive forward-looking model is necessary for the development of optimal spending, contributions, and asset allocation strategies. CDI recognizes fundamental relationships between commitment out-flows, in-flows, and policy portfolios as epitomized in the funding triangle. For example, today's contribution rate should be set in anticipation of future spending and evolving portfolios.

The requirement for this model to be *realistic* has profound consequences. Let us assume that investors re-examine their policy portfolios regularly. This assumption is a reflection of prudent investment practices most investors follow.

Now let us assume that the investor has selected an optimal policy portfolio in the context of the funding objective. When the time comes to re-examine the policy portfolio at some future point, the investor is not obligated to follow the original policy portfolio. At the time, the investor should optimize the remaining "sub"-policy portfolio (the original policy portfolio without the past portfolios) in the context of the funding objective. The investor would follow the original policy portfolio only if the remaining "sub"-policy portfolio is optimal at the time. Therefore, in a realistic model, *all "sub"-policy portfolios of an optimal policy portfolio are optimal as well*. This observation implies that policy portfolios should be optimized via the process of "backward induction" – from the last portfolio to the first.

The design of optimal policy portfolios can be put in a much more expansive theoretical framework of game theory. The investor and his ageing "clones" that make future asset allocation decisions can be viewed as "players" that have objectives, actions, and preferences. Under common rationality assumptions, an optimal policy portfolio should represent a Nash Equilibrium (NE) strategy – one of the key concepts of game theory.

In general, *Nash equilibrium policy portfolios are not stationary*. Thus, the CDI framework justifies evolving policy portfolios.[3]

CDI vs. LDI

CDI is related to Liability Driven Investing (LDI) – a much publicized framework for managing pension plan liabilities that offers a range of liability driven investment strategies. CDI and LDI have the same starting point – both frameworks are based on the objective to fund pension benefits. The key difference is in the next step.

In LDI, the next step is the definition of a deterministic present value of pension benefits called "liability." The investment objectives are specified next. The "liability" plays a major role in the investment objectives – it actually "drives" pension investing, as advertised in the title. LDI imposes the following "order of operations": *to define the "liability" first and specify investment objectives next*. Furthermore, LDI promotes asset-"liability" matching as a proxy for the funding objective.

As discussed earlier, the "order of operations" in CDI is exactly the opposite: *to specify investment objectives first and select the appropriate present values next*. CDI incorporates the funding objective directly without any proxy. The assumptions in CDI are less restrictive than in LDI, therefore CDI is a generalization of LDI.[4]

CDI and Pension De-Risking

CDI offers powerful analytical tools to design optimal de-risking strategies for retirement plans. CDI distinguishes traditional "rebalanceable" portfolios and "buy-and-hold" assets that make pre-determined payments that offset (or match) the plan's commitments out-flows. CDI presents transparent measurements of the ability of a particular de-risking strategy to serve the best interests of plan participants.[5]

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Important Information