



**Sent via e-mail to comments@actuary.org**

ASOP No. 4 Revision  
Actuarial Standards Board (ASB)  
1850 M Street, Suite 300  
Washington, DC 20036

*May 31, 2012*

**Subject:** Proposed Revision of Actuarial Standard of Practice (ASOP) No. 4

To members of the Actuarial Standard Board:

I would like to thank the Actuarial Standards Board for the opportunity to provide comments on the proposed revision of ASOP No. 4. These comments are different from [my comments on the proposed revision of ASOP No. 27](#) I submitted recently, even though there is a considerable overlap between them.<sup>1</sup> For the reader's convenience, three sections from the ASOP No. 27 comments that are related to these comments are presented in the Appendix.

My comments are limited to the most consequential and contentious issues in the proposed standard – actuarial present values, "liabilities," and obligations (section 3.7). We all agree that these concepts have "created challenges for actuarial communications for decades and continue to do so today." These issues have been the subject of intense discussions in recent years.

The ASB has encountered considerable difficulties in the development of sensible standards in this area. Undoubtedly, it has not been easy for the ASB and its pension committee to accommodate the "myriad viewpoints" on this subject that currently exist in the actuarial community. I agree with the Committee that developing a common vocabulary is necessary, and the Committee's efforts in this development should be greatly appreciated.

Here is the problem as I see it. For a long time, the standards for "measuring pension obligations" have focused on the issues of long-term funding and utilized deterministic measurements of policy portfolio returns for the calculations of present values. These standards have not accommodated other concerns of the stakeholders of pension plans that may require different approaches. One example of such a concern is the issue of short-term solvency.

The objective of the ASB is to revise the standards to accommodate these concerns. A good ASOP should contain sensible guidance that is based on clearly defined concepts, recognize that different objectives may require different measurements, and avoid impeding the natural evolution of actuarial practices. I believe the proposed standard falls short of creating an effective ASOP the actuarial community needs. The core of the proposed standard – section 3.7 – is inadequate and should be largely re-written.

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<sup>1</sup> See [http://www.actuarialstandardsboard.org/comments/2012/asop27\\_comments/comment\\_15.pdf](http://www.actuarialstandardsboard.org/comments/2012/asop27_comments/comment_15.pdf).



In these comments, I discuss the problems with the proposed standard first and outline the directions of a better approach next.

### **Problems with the Proposed Standard**

The proposed standard has at least three major problems. The first major problem is *the language of the proposed standard is overly casual*. Some concepts are nonchalantly relegated to the "everyone-knows-it" category. The proposed standard extensively relies on what the great late Peter Bernstein once sarcastically called "the rules of thumb and folklore."

The term "obligation" is a good example of this problem. A simple search of this term reveals that the document contains "obligations," "pension obligations" and "pension obligation" (plural and singular), "benefit obligations," "group benefit obligations," "projected benefit obligations," "plan obligations," and "financial obligations." All of them supposedly belong to the "everyone-knows-it" category. What this document does not have is a definition of the term "obligation."

This problem is exacerbated by the fact that the terms "obligations," "pension obligations," and "benefit obligations" appear to represent a *stream* of benefit payments, but the term "projected benefit obligations" is a *present value* of such a stream "under US GAAP for single employer plans" (as mentioned in the document). As a result, the document utilizes similar terms to denote fundamentally different objects.

The proponents of this language may argue that everyone knows that the term "obligation" represents a stream of payments in this context. But that is exactly the point – the term is in the "everyone-knows-it" category that requires no definitions. Besides, if the term "obligation" were explicitly defined as a stream of payments, how would this definition be reconciled with the pension accounting "obligations" (ABO, PBO, etc.), which are present values? The time has come for this ASOP to explicitly distinguish payments and their present values. My specific proposals are presented in the next section.

The second major problem is the separation of "present values based on plan assets" and "present values not based on plan assets." This separation is as conceptually deficient as it is counterproductive. Present values are always based on investment returns generated by some portfolios of assets. These portfolios include the existing policy portfolio and other hypothetical portfolios that may be relevant to the plan. See section "Discounting vs. Investing" in the Appendix for more details about this notion.

As an example, let us consider the issue of short-term solvency. The question is whether the plan's market value of assets would be sufficient to fund the accrued benefits if the plan were to terminate today. Let us assume that there exists a portfolio of tradable assets that has a high probability of making the plan's benefit payments (e.g. a high quality matching bond portfolio or a group annuity contract with a highly rated insurance company); let us call this asset "the matching asset." Then, to answer the solvency question, the price of the matching asset is compared to the plan's market value of assets.



It is essential to understand that we consider a hypothetical investment in the matching asset along with a hypothetical plan termination in this exercise. The price of the matching asset is a "present value based on plan assets," only these assets are hypothetical rather than actual. As long as a hypothetical insolvency issue is relevant to the plan, a hypothetical investment in the matching asset is relevant to the plan as well. While it is technically correct that "the discount rate does not reflect the expected return on plan assets," the discount rate used for the solvency calculations does "reflect" the expected return of the matching asset.

The deficiency of the premise "plan-assets-vs.-everything-else" leads to a perplexing definition of "market-consistent present values":

*"Market-consistent present values of pension obligations are types of present values not based on plan assets. A market-consistent present value is one that is consistent with the price at which expected plan benefit payments would trade ... "*

Remarkably, "market-consistent present values" are defined as not what they are, but as *what they are not!* The clarification that a "market-consistent present value" is one that is consistent with some market doesn't really help. A proper definition of a novel concept should specify what it is rather than what it isn't or what it is consistent with. This definition would be highly undesirable in an actuarial textbook. In an actuarial standard, this definition would be next to unacceptable.

To illustrate the fallacy of this definition, let us assume that the plan's policy portfolio is the conventional 60/40 (stocks/bonds). Then the present value calculated using this portfolio's geometric return *is not* "market-consistent" by definition. However, the present value calculated using the all-equity portfolio's geometric return *is* "market-consistent" also by definition (because this present value is "not based on plan assets"). Capital market assumptions for the all-equity portfolio can be perfectly consistent with the equity market observations. Make sense?

Here's another example of a perplexing statement that is based on the same premise:

*"Present values of pension obligations that are not based on plan assets do not vary with the allocation of the assets used to fund the obligation ..."*

In other words, "present values of pension obligations" that have nothing to do with plan assets "do not vary" with those assets. How many actuaries will be glad we got this thing straight?

The good news is the concept of "market-consistent present values" is unnecessary for this ASOP. This observation leads to the third major problem in this proposed standard. Actuarial standards are not the right place to introduce novel actuarial and/or economic concepts (e.g. "market-consistent present values" or "forward looking expected geometric returns" in the proposed standard No. 27) that would eventually "percolate" into actuarial textbooks, examination process and practices. The Committee may want to re-think the whole process.

Overall, I believe this proposed standard's approach to actuarial present values is unsound and based on a flawed premise. The next section outlines a better approach.



## Policy Portfolios and Present Values

Let us start this outline with a short exercise in semantics. I would like to reiterate my long-standing aversion to the term "liability." Due to its many meanings, this term creates substantial problems and confusion every time it is used. I understand and appreciate that

*"The Pension Committee considered removing the word "liability" from ASOP No. 4, but concluded that the word is so deeply embedded in pension vernacular that attempting to remove it would be impracticable."*

I also understand that it's probably too late to ask for reconsideration. Nonetheless, I would use the term "*required assets*" instead of "liability." I would not rule out the use of the adjective "actuarial" with the term, although the merits of this addition are open to discussion. I believe that relatively modest efforts would be required to complete an orderly transition. Or so I hope.

While we are on the subject of semantics, this ASOP needs a new term that indicates a stream of payments. The presence of the term "obligation" in the ASOP is a problem. The term "obligation" has been utilized in pension accounting and its meaning has been well-established for a long time. If clarity is a desirable quality of an ASOP, then the term "obligation" should stand for a pension accounting present value and nothing else.

I propose to use the term "*commitment*" to indicate a stream of benefit payments. At the same time, these comments are not the right place for a wide-ranging debate regarding the pros and cons of this term in particular and the "removal" of the term "liability" in general. I would like the Committee to make a note of the term "commitment" for future discussions.

Now let us get back to the problem of developing a better ASOP. The proposed standard takes a step in the right direction – it highlights the significance of *policy portfolios*. However, the proposed standard's classification of policy portfolios is overly simplistic. The proposed standard recognizes just two types of portfolios – the plan's existing policy portfolio and everything else.

This classification is vastly insufficient. A comprehensive concept of a policy portfolio, which is well-known in finance, would work much better for the purposes of this ASOP. In particular, the ASOP should distinguish conventional portfolios of risky assets and "buy-and-hold" portfolios that make regular predetermined payments.

Broadly defined policy portfolios should be at the heart of this ASOP serving as a crucial link between pension commitments and their present values. The ASOP should recognize the fundamental relationship between policy portfolios and present values of pension commitments. This recognition is the key step in the development of an ASOP that accommodates the traditional actuarial and other measurements of pension commitments, including the ones inspired by the "corporate pension finance" perspective.



I'm well-aware that the relationship between policy portfolios and present values is controversial. Some proponents of "corporate pension finance" may claim that the value of a cash flow does not depend on the way the cash flow is funded. This and related issues have been the subject of raging debates over the last several years. These comments are not the right place for these debates, here's just a brief description of my position here.<sup>2</sup>

The goal of "corporate pension finance" is to *price* a stream of payments; risk is defined as a non-payment event. In contrast, the goal of the stakeholders of pension plans is to *fund* pension benefit payments. Risk is defined as the inability of a particular funding methodology to succeed. These objectives and definitions of risk are fundamentally different.

To illustrate the difference in these definitions of risk, think of a payment of \$100 a year from now that will be certainly paid. In the "corporate pension finance" mindset, this payment is riskless. However, if the investor chooses to fund this payment via investing \$90 in the all-equity portfolio, there may or may not be \$100 accumulated in a year. So, the \$100 payment is riskless, yet the investor's decision to utilize the all-equity portfolio creates substantial risk in this funding problem.

To look at the same issue from a different angle, the pricing objective leads to a portfolio that is virtually certain to make all payments. In contrast, the funding objective may lead to portfolios that have various probabilities – a 100% probability, a 90% probability, a 80% probability, etc., – of making all payments. Thus, the funding objective is a *generalization* of the pricing objective.

Actuaries are important participants in the process of *funding* pension benefits. Present values in actuarial reports are measurements of the ability of the existing assets and the contributions determined by the funding method to fund the benefits. Again, funding is a generalization of pricing – plan sponsors are at liberty to utilize risky or risk-free (matching) assets for the purposes of funding.

In this context, the relationship between policy portfolios and present values of pension commitments is essential. A portfolio impacts the riskiness of pension benefits and the cost of funding of these benefits. A "risky" portfolio may result in the low expected but high volatility cost of funding. A "safe" portfolio may result in the high expected but low volatility cost of funding.

In this context, a present value is defined as the asset value required at the present to fund the benefits. Different portfolios may require different asset values to be invested at the present. Hence, these present values reflect the fact that the cost of funding depends on the portfolio.

However, there must be a huge warning sign here. The present values I've discussed so far are not the conventional discount rate based present values that possess the much-criticized feature "higher-risk-lower-liability." I'm talking about stochastic present values that incorporate the full range of portfolio returns and the volatility of benefit payments. As I discuss in [The Case for](#)

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<sup>2</sup> For a review of these debates, see D. Mindlin, The "Financial Economics" Debate, *CDI Advisors Research*, CDI Advisors LLC, 2009, <http://www.cdiadvisors.com/papers/CDIFinancialEconomicsDebate.pdf>.



[Stochastic Present Values](#), stochastic present values belong to the mainstream of actuarial science in particular and finance in general.<sup>3</sup> See also section "The Assumption of a Discount Rate" in the Appendix.

I'd like to make clear that I'm *not* proposing a new type of present values for traditional actuarial and accounting reports. Better "liabilities"/"obligations" for actuarial and accounting reports is a different subject. These comments are about making a better ASOP No. 4, not actuarial and accounting reports. This ASOP should adopt a comprehensive approach to policy portfolios and corresponding stochastic present values because this approach creates an effective framework that accommodates the traditional actuarial and other measurements of pension commitments.

Section 3.7 in the proposed standard would become shorter and much more transparent if it incorporated a comprehensive approach to policy portfolios. The ASOP would distinguish conventional "rebalanceable" portfolios of risky assets and "buy-and-hold" portfolios that make regular predetermined payments (e.g. matching bond portfolios or annuities).

Actuarial present values are based on policy portfolios that are relevant to the plan. The judgment regarding the relevant portfolios should be based on the input from the stakeholders of the plan that include, but are not limited to, the shareholders/taxpayers the plan sponsor represents, plan participants, and regulators.

The actuary should select one or more deterministic measurements of portfolio returns to serve as discount rates. If a relevant policy portfolio contains traditional portfolios of risky assets, then the actuary should specify the objective that leads to the selection of a discount rate for a portfolio of risky assets.

For example, the objective of "connecting" the starting and ending asset values leads to the expected geometric return. The objective of "no expected gains/losses *in the future*" leads to the expected arithmetic return. See section "Measurements of Portfolio Returns" in the Appendix for more details.

If a relevant policy portfolio is a matching "buy-and-hold" portfolio, then the specifics of this portfolio drive the discount rate selection. The actuary may determine the price of this matching portfolio first and specify the implied discount rates next, or vice versa. The assets that comprise the matching portfolio may or may not incorporate the credit-worthiness of the plan sponsor or the risk of non-payment. The resulting present values and discount rates are market-consistent by design.

Now we can define actuarial present values of a pension commitments. There are several ways to do so. The proposed standard provides a reasonable definition: an actuarial present value is "the portion of the cost of projected benefits included in the calculation, as determined by the actuarial cost method, and the types of actuarial assumptions used, as addressed more fully in ASOP Nos. 27 and 35."

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<sup>3</sup> See D. Mindlin, The Case for Stochastic Present Values, *CDI Advisors Research*, CDI Advisors LLC, 2009, <http://www.cdiadvisors.com/papers/CDITheCaseforStochasticPV.pdf>.



## Conclusion

I believe that section 3.7 in the proposed standard has significant room for improvement. As discussed in these comments, this section should be largely re-written.

Thank you for your attention to these comments. Feel free to contact me if you have any questions/comments. I would be happy to assist the ASB in the development of this standard and related issues.

Sincerely

A handwritten signature in black ink that reads "D. Mindlin".

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## APPENDIX

The following are three sections from [my comments on the proposed revision of ASOP No. 27](#). These sections are presented here for the reader's convenience.

### Discounting vs. Investing

The ASB must make up its collective mind on the validity of the following principle:

*In finance, discount rates are measurements of portfolio returns.*

Dear members of the ASB, there is no way around dealing with this principle. You must either accept this principle or present valid arguments to the contrary. If accepted, this principle would serve as a solid foundation for transparent actuarial standards that would be exceedingly helpful to the actuarial community. As discussed below, this principle would serve as the desired common ground between the "traditionalists" and "financial economists."





The basis for this principle is the definition of portfolio return and a simple transformation of this definition.<sup>4</sup> Yet, I can easily imagine a couple of reasons to disagree with this principle, so let me deal with these reasons now.

First, as we all know, certain regulations require the use of "averaged" yield curves (e.g. 24-month average segment rates), which appears to contradict the "discount-rates-are-returns" principle. However, this and other "averaging" procedures have no basis in finance – they come from the area of *public policy*. These procedures belong to the set of incentives developed by policymakers to support retirement plans.

Second, the "discount-rates-are-returns" principle appears to contradict the "financial economics" perspective that the actuary may utilize a particular quality bond yield curve as discount rates. Actually, there is no contradiction here. The key points are:

- yield curves are rates of return;
- more than one portfolio may be relevant to a pension plan.

The "financial economics" perspective does suggest portfolio return based discount rates, even though this portfolio may not be the plan's actual portfolio. For example, let us assume that it is determined that the consideration of a matching Treasury bond portfolio is beneficial to the plan. Then the Treasury yield curve should be used as discount rates because the yield curve represents "portfolio returns" for this portfolio. Similar arguments are applicable to other matching portfolios (e.g. group annuity contracts).

Third, the "discount-rates-are-returns" principle appears to contradict the statement from the previous draft of this ASOP: "Discount Rate and Investment Return Link Broken." Technically, there is no contradiction here. As clarified in that draft, "... the discount rate is not necessarily the same as an investment return assumption for assets held in a pension trust." In other words, the discount rate is not necessarily related to the existing policy portfolio and may be related to some other portfolio, e.g. a matching bond portfolio. This is a perfectly valid point. But the wording of this point – "Discount Rate and Investment Return Link Broken" – is awfully poor to put it charitably. Making this statement the title of a section makes it even worse. This statement is a reflection of the confusion still present in the draft.

The "discount-rates-are-returns" principle supports the traditional practices of using various measurements of the existing policy portfolio return as discount rates. This principle also supports the desire of the proponents of the "financial economics" perspective to use various yield curves as discount rates. Once again, yield curves are rates of return. In finance, there is no discounting without investing.

The acceptance of the "discount-rates-are-returns" principle would not turn raging debates into bouquet throwing contests overnight. However, the subject of these debates would be entirely

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<sup>4</sup> See, for example, D. Mindlin, *The Good, the Bad, and the Ugly of Pension Accounting*, *CDI Advisors Research*, CDI Advisors LLC, 2009, p. 4-5, <http://www.cdiadvisors.com/papers/CDITheCaseforStochasticPV.pdf>.





different. It would no longer be the foundations of actuarial science and "financial economics," but much more transparent area of investment portfolios.

There would be no need for this ASOP to introduce "market-consistent" assumptions. Instead, we would have various portfolios, their market prices and relevance to the plan. The results of this approach would be "market-consistent" and "marked-to-market" by definition. I believe the acceptance of the "discount-rates-are-returns" principle would make those discussions much more productive and serve as a good starting point for the development of better actuarial standards.

### **The Assumption of a Discount Rate**

Once again, clarity is one of the most important qualities of a good standard of practice. In particular, hidden assumptions are highly undesirable. In this exposure draft, at least two important assumptions are hidden.

As we all know, present value calculations utilize discount rates for the purposes of actuarial and accounting reporting. These present values are assumed to be deterministic, hence discounting procedures should use deterministic discount rates. Yet, there is no sound principle of actuarial science that necessitates deterministic present values and discount rates.

As was discussed in the previous section, discount rates are measurements of portfolio returns. If portfolio returns are certain, then these returns should be used as discount rates. But if portfolio returns are uncertain, as they are for most pension plans, then the utilization of discount rates is a choice, not a necessity.

There are plenty of good reasons for this choice – e.g. computational convenience, regulations, accounting and pricing considerations. Furthermore, the concept of a discount rate is one of the cornerstones of the classic actuarial science. Still, the presence of a discount rate is an assumption by itself, and this is even before we start thinking about the appropriate value for the discount rate. This assumption is hidden in this exposure draft.

The next issue is the order of operations – discount rates vs. present values. The exposure draft implies that discount rates are specified first and present values are calculated next using these discount rates. This order of operations has no basis in actuarial science or finance. In some areas, we specify rates of return first and calculate the implied present values next. In some other areas, we determine asset prices (present values) first and calculate the implied rates of return next. The order of operations "discount-rates-first-present-values-next" is, once again, a choice, not a necessity.

By now, some readers are probably wondering, "How can you calculate present values without discount rates?" For example, the distribution of a present value can be estimated via the use of the entire range of investment returns and volatilities of benefit payments. The actuary would select a measurement of the resulting stochastic present value (e.g. the mean or the median) and, subsequently, determine the implied discount rate. As we see in this example, the order of operations is "present-value-first-discount-rate-next."



As I discuss in [The Case for Stochastic Present Values](#), stochastic present values belong to the mainstream of finance.<sup>5</sup> Several classic actuarial textbooks – Winklevoss, Trowbridge, Bowers, Kellison – either mention or directly deal with stochastic present values. The classic Black-Scholes formula represents another example of stochastic present value valuation. For a detailed treatment of the basic properties of stochastic present values, see the third edition of S. Kellison's "The Theory of Interest," chapter 12, or the second edition of the same book, chapter 10.<sup>6</sup>

Moreover, several actuaries questioned the wisdom of deterministic estimates of present values when ASOP No. 27 was discussed in 2008. One of these actuaries – Fiona Liston – happens to be a member of this committee.<sup>7</sup> My personal favorite, however, was the following paragraph from [the comments provided by Douglas German of Buck Consultants](#):<sup>8</sup>

*"Finally, we believe that pension and OPEB actuarial practice in the United States has suffered and continues to suffer from a single-minded focus on identifying the mean value of the benefit commitment, with virtually no attention to the more important issues of the shape of the distribution under differing economic and demographic conditions, its associated variance and confidence levels, or how existing assets should be taken into account. While the profession is far from being able to develop any actuarial standards in this regard, anything the ASB can do to help address this deficiency should be done."*

The sentiment here is rather strong – "has suffered and continues to suffer" – and I concur wholeheartedly. These comments remain relevant to this day, even though substantial progress has been made in our understanding of stochastic present values since 2008.

To recap, I believe the standard should disclose that the presence of deterministic present values and discount rates is an assumption and give a concise justification of this assumption. The standard should give stochastic present values the right to exist.

I would like to make clear that I am proposing neither "abolition" of deterministic present values and discount rates, nor any kind of "reinventing" of current actuarial practices. The natural evolution of these practices is driven by the marketplace of ideas, which will eventually sort everything out in its due course. All I am asking the ASB to do is not to stand in the way.

## Measurements of Portfolio Returns

This section assumes that deterministic discount rates are desirable. The section contains a concise review of appropriate measurements of portfolio returns for the development of discount rates.

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<sup>5</sup> See D. Mindlin, *The Case for Stochastic Present Values*, *CDI Advisors Research*, CDI Advisors LLC, 2009, <http://www.cdiadvisors.com/papers/CDITheCaseforStochasticPV.pdf>.

<sup>6</sup> S. Kellison, *The Theory of Interest*, Third Edition, *McGraw-Hill Irwin*, 2009.

<sup>7</sup> See Fiona Liston's comments at [http://www.actuarialstandardsboard.org/comments/asop\\_27\\_rfc\\_comments/comment\\_13.pdf](http://www.actuarialstandardsboard.org/comments/asop_27_rfc_comments/comment_13.pdf).

<sup>8</sup> See at [http://www.actuarialstandardsboard.org/comments/asop\\_27\\_rfc\\_comments/comment\\_24.pdf](http://www.actuarialstandardsboard.org/comments/asop_27_rfc_comments/comment_24.pdf).



As discussed in prior sections, the discount rate is equal to the portfolio return if the portfolio return is certain. The challenge is to select an appropriate deterministic measurement of portfolio return when the portfolio return is uncertain.

The first step in this selection is the development of capital market assumptions. These assumptions are briefly discussed in section D of Appendix 3. The actuarial standard should have a detailed section dedicated to capital market assumptions in the body of the standard (not in an appendix). Capital market assumptions (returns/risks/correlations) are discussed in many textbooks and belong to the mainstream of finance.

The standard should not indicate that "the actuary will receive capital market assumptions from an investment consultant," as it does in the draft. Actuaries work for investment consulting firms; actuaries work as investment consultants; actuaries participate in the development of capital market assumptions; the standard does not need to draw the line between actuaries and investment consultants.

Given capital market assumptions, we can calculate various measurements of portfolio returns. These measurements include, but are not limited to, arithmetic and geometric expected returns, percentiles, and moments. In order to develop the right measurement to serve as a discount rate, one needs to specify the objective for the discount rate, as discussed later in this section.

Arithmetic and geometric expected returns are the most popular candidates to serve as discount rates. The draft correctly indicates that there is a need for estimates that establish relationship between arithmetic expected returns, geometric expected returns, and variance. Having done extensive research in this area, I present the following four estimates and their justifications in "[On the Relationship between Arithmetic and Geometric Returns](#)".<sup>9</sup>

$$G \approx A - V/2 \quad (R1)$$

$$(1+G)^2 \approx (1+A)^2 - V \quad (R2)$$

$$1+G \approx (1+A) \exp\left(-\frac{1}{2}V(1+A)^{-2}\right) \quad (R3)$$

$$1+G \approx (1+A)\left(1+V(1+A)^{-2}\right)^{-1/2} \quad (R4)$$

where  $G$  is the geometric expected return,  $A$  is the arithmetic expected return,  $V$  is the variance.

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<sup>9</sup> D. Mindlin, On the Relationship between Arithmetic and Geometric Returns, *CDI Advisors Research*, CDI Advisors LLC, 2010, <http://www.cdiadvisors.com/papers/CDIArithmeticVsGeometric.pdf>.



Among other things, the paper offers some evidence to suggest that formula (R4) should be expected to produce better results than the rest. However, the paper recommends the utilization of all four estimates.

Discount rate objectives and resulting discount rates are discussed in "[Present Values, Investment Returns and Discount Rates](#)."<sup>10</sup> The paper discusses the following four objectives for a discount rate.

Objective 1: To "connect" the starting and ending asset values.

Objective 2: To have a "safety cushion" (called "adverse deviation" in the draft).

Objective 3: No expected gains/losses *in the future*.

Objective 4: No expected gains/losses *at the present*.

In particular, Objective 1 implies the geometric expected return, Objective 3 implies the arithmetic expected return.

I am optimistic that these papers would be useful for the development of better actuarial standards and educational purposes.

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<sup>10</sup> D. Mindlin, Present Values, Investment Returns and Discount Rates, *CDI Advisors Research*, CDI Advisors LLC, 2010, <http://www.cdiadvisors.com/papers/CDIDiscountRate.pdf>.