



**Sent via e-mail to comments@actuary.org**

ASOP No. 27 Revision  
Actuarial Standards Board (ASB)  
1850 M Street, Suite 300  
Washington, DC 20036

*May 29, 2012*

**Subject:** Proposed Revision of Actuarial Standard of Practice (ASOP) No. 27

To members of the Actuarial Standard Board:

I would like to thank the Actuarial Standards Board for the opportunity to provide comments on the proposed revision of ASOP No. 27. My comments are limited to the selection of discount rates and related issues.

The selection of discount rates has been the subject of intense discussions in recent years. In particular, there are serious disagreements on the issue of "portfolio return" vs. "market-consistent" assumptions. Obviously, the ASB is facing considerable difficulties trying to accommodate the views of the "traditionalist" and "financial economics" camps. This issue is crucial to the actuarial profession, and the ASB's efforts to find common ground between these camps should be greatly appreciated. So far, this common ground has eluded the ASB in particular and the actuarial community in general.

One of the most important qualities of a good standard of practice is clarity. Undoubtedly, it has not been easy for the ASB to achieve the desired clarity at the time when the actuarial community appears to be split and numerous debates have produced little more than abundant confusion. This exposure draft contains clear reflections of this confusion.

My comments have the proverbial "good news" and "bad news." The "bad news" is this section is conceptually inadequate and should be mostly re-written. The "good news" is fairly modest efforts are required to produce a solution that should satisfy both the "traditionalists" and "financial economists." I believe we can find the desired middle ground between these camps relatively easily.

Overall, I am concerned that certain segments of this exposure draft are outdated and may become an obstacle for the evolution of actuarial practices.

### **Discounting vs. Investing**

The ASB must make up its collective mind on the validity of the following principle:

*In finance, discount rates are measurements of portfolio returns.*



Dear members of the ASB, there is no way around dealing with this principle. You must either accept this principle or present valid arguments to the contrary. If accepted, this principle would serve as a solid foundation for transparent actuarial standards that would be exceedingly helpful to the actuarial community. As discussed below, this principle would serve as the desired common ground between the "traditionalists" and "financial economists."

The basis for this principle is the definition of portfolio return and a simple transformation of this definition.<sup>1</sup> Yet, I can easily imagine a couple of reasons to disagree with this principle, so let me deal with these reasons now.

First, as we all know, certain regulations require the use of "averaged" yield curves (e.g. 24-month average segment rates), which appears to contradict the "discount-rates-are-returns" principle. However, this and other "averaging" procedures have no basis in finance – they come from the area of *public policy*. These procedures belong to the set of incentives developed by policymakers to support retirement plans.

Second, the "discount-rates-are-returns" principle appears to contradict the "financial economics" perspective that the actuary may utilize a particular quality bond yield curve as discount rates. Actually, there is no contradiction here. The key points are:

- yield curves are rates of return;
- more than one portfolio may be relevant to a pension plan.

The "financial economics" perspective does suggest portfolio return based discount rates, even though this portfolio may not be the plan's actual portfolio. For example, let us assume that it is determined that the consideration of a matching Treasury bond portfolio is beneficial to the plan. Then the Treasury yield curve should be used as discount rates because the yield curve represents "portfolio returns" for this portfolio. Similar arguments are applicable to other matching portfolios (e.g. group annuity contracts).

Third, the "discount-rates-are-returns" principle appears to contradict the statement from the previous draft of this ASOP: "Discount Rate and Investment Return Link Broken." Technically, there is no contradiction here. As clarified in that draft, "... the discount rate is not necessarily the same as an investment return assumption for assets held in a pension trust." In other words, the discount rate is not necessarily related to the existing policy portfolio and may be related to some other portfolio, e.g. a matching bond portfolio. This is a perfectly valid point. But the wording of this point – "Discount Rate and Investment Return Link Broken" – is awfully poor to put it charitably. Making this statement the title of a section makes it even worse. This statement is a reflection of the confusion still present in the draft.

The "discount-rates-are-returns" principle supports the traditional practices of using various measurements of the existing policy portfolio return as discount rates. This principle also supports the desire of the proponents of the "financial economics" perspective to use various

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<sup>1</sup> See, for example, D. Mindlin, *The Good, the Bad, and the Ugly of Pension Accounting*, *CDI Advisors Research*, CDI Advisors LLC, 2009, p. 4-5, <http://www.cdiadvisors.com/papers/CDITheCaseforStochasticPV.pdf>.



yield curves as discount rates. Once again, yield curves are rates of return. In finance, there is no discounting without investing.

The acceptance of the "discount-rates-are-returns" principle would not turn raging debates into bouquet throwing contests overnight. However, the subject of these debates would be entirely different. It would no longer be the foundations of actuarial science and "financial economics," but much more transparent area of investment portfolios.

There would be no need for this ASOP to introduce "market-consistent" assumptions. Instead, we would have various portfolios, their market prices and relevance to the plan. The results of this approach would be "market-consistent" and "marked-to-market" by definition. I believe the acceptance of the "discount-rates-are-returns" principle would make those discussions much more productive and serve as a good starting point for the development of better actuarial standards.

### **The Assumption of a Discount Rate**

Once again, clarity is one of the most important qualities of a good standard of practice. In particular, hidden assumptions are highly undesirable. In this exposure draft, at least two important assumptions are hidden.

As we all know, present value calculations utilize discount rates for the purposes of actuarial and accounting reporting. These present values are assumed to be deterministic, hence discounting procedures should use deterministic discount rates. Yet, there is no sound principle of actuarial science that necessitates deterministic present values and discount rates.

As was discussed in the previous section, discount rates are measurements of portfolio returns. If portfolio returns are certain, then these returns should be used as discount rates. But if portfolio returns are uncertain, as they are for most pension plans, then the utilization of discount rates is a choice, not a necessity.

There are plenty of good reasons for this choice – e.g. computational convenience, regulations, accounting and pricing considerations. Furthermore, the concept of a discount rate is one of the cornerstones of the classic actuarial science. Still, the presence of a discount rate is an assumption by itself, and this is even before we start thinking about the appropriate value for the discount rate. This assumption is hidden in this exposure draft.

The next issue is the order of operations – discount rates vs. present values. The exposure draft implies that discount rates are specified first and present values are calculated next using these discount rates. This order of operations has no basis in actuarial science or finance. In some areas, we specify rates of return first and calculate the implied present values next. In some other areas, we determine asset prices (present values) first and calculate the implied rates of return next. The order of operations "discount-rates-first-present-values-next" is, once again, a choice, not a necessity.



By now, some readers are probably wondering, "How can you calculate present values without discount rates?" For example, the distribution of a present value can be estimated via the use of the entire range of investment returns and volatilities of benefit payments. The actuary would select a measurement of the resulting stochastic present value (e.g. the mean or the median) and, subsequently, determine the implied discount rate. As we see in this example, the order of operations is "present-value-first-discount-rate-next."

As I discuss in [The Case for Stochastic Present Values](#), stochastic present values belong to the mainstream of finance.<sup>2</sup> Several classic actuarial textbooks – Winklevoss, Trowbridge, Bowers, Kellison – either mention or directly deal with stochastic present values. The classic Black-Scholes formula represents another example of stochastic present value valuation. For a detailed treatment of the basic properties of stochastic present values, see the third edition of S. Kellison's "The Theory of Interest," chapter 12, or the second edition of the same book, chapter 10.<sup>3</sup>

Moreover, several actuaries questioned the wisdom of deterministic estimates of present values when ASOP No. 27 was discussed in 2008. One of these actuaries – Fiona Liston – happens to be a member of this committee.<sup>4</sup> My personal favorite, however, was the following paragraph from [the comments provided by Douglas German of Buck Consultants](#):<sup>5</sup>

*"Finally, we believe that pension and OPEB actuarial practice in the United States has suffered and continues to suffer from a single-minded focus on identifying the mean value of the benefit commitment, with virtually no attention to the more important issues of the shape of the distribution under differing economic and demographic conditions, its associated variance and confidence levels, or how existing assets should be taken into account. While the profession is far from being able to develop any actuarial standards in this regard, anything the ASB can do to help address this deficiency should be done."*

The sentiment here is rather strong – "has suffered and continues to suffer" – and I concur wholeheartedly. These comments remain relevant to this day, even though substantial progress has been made in our understanding of stochastic present values since 2008.

To recap, I believe the standard should disclose that the presence of deterministic present values and discount rates is an assumption and give a concise justification of this assumption. The standard should give stochastic present values the right to exist.

I would like to make clear that I am proposing neither "abolition" of deterministic present values and discount rates, nor any kind of "reinventing" of current actuarial practices. The natural evolution of these practices is driven by the marketplace of ideas, which will eventually sort everything out in its due course. All I am asking the ASB to do is not to stand in the way.

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<sup>2</sup> See D. Mindlin, *The Case for Stochastic Present Values*, *CDI Advisors Research*, CDI Advisors LLC, 2009, <http://www.cdiadvisors.com/papers/CDITheCaseforStochasticPV.pdf>.

<sup>3</sup> S. Kellison, *The Theory of Interest*, Third Edition, *McGraw-Hill Irwin*, 2009.

<sup>4</sup> See Fiona Liston's comments at [http://www.actuarialstandardsboard.org/comments/asop\\_27\\_rfc\\_comments/comment\\_13.pdf](http://www.actuarialstandardsboard.org/comments/asop_27_rfc_comments/comment_13.pdf).

<sup>5</sup> See at [http://www.actuarialstandardsboard.org/comments/asop\\_27\\_rfc\\_comments/comment\\_24.pdf](http://www.actuarialstandardsboard.org/comments/asop_27_rfc_comments/comment_24.pdf).



## Arithmetic vs. Geometric Returns: A Numerical Example and Beyond

I would like to illustrate the points presented in the previous section numerically in the example presented in section C of Appendix 3 in the exposure draft. I have certain reservations about this example, but since it is already in the draft, I would like to use it.

This example contains a portfolio that has a 50% probability to generate a return of 30% and a 50% probability of to generate a return of 0% for each of the next two years. The draft contains the calculations of two measurements of this portfolio return – so-called *forward looking expected arithmetic* (15.00%) and *geometric* (14.51%) *returns*. The example employs these returns in a forward-looking manner and appears to suggest to these values are good candidates for discount rates.

Let us try to bring this example closer to actual actuarial practices and calculate present values this portfolio generates. Let us assume that the plan's financial commitment is to make one payment of \$1,000 at the end of the second year. Then the forward looking expected arithmetic (15.00%) and geometric (14.51%) returns imply present values of \$756.14 (at 15.00%) and \$762.65 (at 14.51%).

Now let us take a look at the actual present values – the asset value required at the present to fund the commitment. This present value is a random variable that has a value of \$591.72 with a 25% probability, a value of \$769.23 with a 50% probability, and a value of \$1,000.00 with a 25% probability. All these three values are different from the present values implied by the rates suggested in the draft.

If it is desirable to select a deterministic value to serve as "the present value," I would select \$769.23. This "the present value" implies a discount rate of 14.02%, which is different from both forward looking expected arithmetic and geometric returns suggested in the draft.<sup>6</sup>

The troubles with this example do not stop here. Section D of Appendix 3 contains the following statement:

*"... a forward looking expected geometric return for an asset class can be approximated by taking the forward looking expected arithmetic return and subtracting one-half of the variance ..."*

The variance of portfolio used in the example is 2.25%. The suggested approximation estimates the forward looking expected geometric return as 13.88% (= 15.00% - 2.25%/2). Comparing this

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<sup>6</sup> This value of 14.02% is the expected geometric return for this portfolio. The value of 14.51% presented in the exposure draft is something else. Note that I specified the deterministic "present value" first and the discount rate next. At this point, a reader may ask, "Why do we need a discount rate *after* the present value has been calculated?" Good question, dear reader. We may need those discount rates for some compliance purposes. Or not. In any case, this question is outside of the scope of these comments.



value to the value of 14.51%, we see that the approximation understates the forward looking expected geometric return by 63 basis points!

This result is especially disconcerting because several estimates that connect arithmetic return, geometric return, and portfolio variance are readily available. The estimate that produces the exact result for this example has probably been known for more than half-a-century. The draft refers to the well-known textbook of Bodie-Kane-Markus. This is an excellent book that has many well-written chapters, but the section about arithmetic and geometric returns is not one of them. Alternative sources of information on this subject are presented in the next section.

A couple of statements made in the draft deserve a closer look. Section 3.8.3 (j) contains the following statement regarding accumulated asset values:

*"The use of a forward looking expected geometric return as an investment return assumption will produce an accumulated value that generally converges to the median accumulated value as the time horizon lengthens."*

This statement is correct – but only if there are no future contributions. Otherwise, the statement is problematic. A related statement regarding present values is presented in the introduction to Appendix 3:

*"The use of a forward looking expected geometric return as a discount rate will produce a present value that generally converges to the median present value as the time horizon lengthens."*

This statement is also correct – but only if the financial commitment contains just one future payment. Otherwise, the statement is problematic. Needless to say, an actuarial standard should strive to avoid problematic statements.

I would like to make one last observation before I conclude this section. The draft offers no formal definitions for the forward looking expected arithmetic and geometric returns. These terms appear to be brand new financial concepts. The "forward looking" part of their naming conventions appears to be redundant and misleading. The numeric example designed to illustrate these concepts contains significant shortcomings. Moreover, I would question the wisdom of introducing new concepts and presenting simplified numerical examples in an actuarial standard. There are better places for these activities.

Section 4 of the request for comments on page viii has the following two questions.

*Do you agree that the guidance on arithmetic and geometric returns in section 3.8.3(j) is appropriate?* No, I do not agree. This guidance should be largely re-written.

*Is the language about the proper incorporation of forward looking expected geometric returns into a building block exercise clear?* No, it is not clear. The terms "forward looking expected geometric returns" and "forward looking expected arithmetic returns" should not be used in the standard. The next section provides more information regarding these and similar concepts.



## Measurements of Portfolio Returns

This section assumes that deterministic discount rates are desirable. The section contains a concise review of appropriate measurements of portfolio returns for the development of discount rates.

As discussed in prior sections, the discount rate is equal to the portfolio return if the portfolio return is certain. The challenge is to select an appropriate deterministic measurement of portfolio return when the portfolio return is uncertain.

The first step in this selection is the development of capital market assumptions. These assumptions are briefly discussed in section D of Appendix 3. The actuarial standard should have a detailed section dedicated to capital market assumptions in the body of the standard (not in an appendix). Capital market assumptions (returns/risks/correlations) are discussed in many textbooks and belong to the mainstream of finance.

The standard should not indicate that "the actuary will receive capital market assumptions from an investment consultant," as it does in the draft. Actuaries work for investment consulting firms; actuaries work as investment consultants; actuaries participate in the development of capital market assumptions; the standard does not need to draw the line between actuaries and investment consultants.

Given capital market assumptions, we can calculate various measurements of portfolio returns. These measurements include, but are not limited to, arithmetic and geometric expected returns, percentiles, and moments. In order to develop the right measurement to serve as a discount rate, one needs to specify the objective for the discount rate, as discussed later in this section.

Arithmetic and geometric expected returns are the most popular candidates to serve as discount rates. The draft correctly indicates that there is a need for estimates that establish relationship between arithmetic expected returns, geometric expected returns, and variance. Having done extensive research in this area, I present the following four estimates and their justifications in "[On the Relationship between Arithmetic and Geometric Returns](#)":<sup>7</sup>

$$G \approx A - V/2 \quad (R1)$$

$$(1+G)^2 \approx (1+A)^2 - V \quad (R2)$$

$$1+G \approx (1+A) \exp\left(-\frac{1}{2}V(1+A)^{-2}\right) \quad (R3)$$

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<sup>7</sup> D. Mindlin, On the Relationship between Arithmetic and Geometric Returns, *CDI Advisors Research*, CDI Advisors LLC, 2010, <http://www.cdiadvisors.com/papers/CDIArithmeticVsGeometric.pdf>.



$$1 + G \approx (1 + A) \left( 1 + V(1 + A)^{-2} \right)^{-1/2} \quad (\text{R4})$$

where  $G$  is the geometric expected return,  $A$  is the arithmetic expected return,  $V$  is the variance.

Among other things, the paper offers some evidence to suggest that formula (R4) should be expected to produce better results than the rest. However, the paper recommends the utilization of all four estimates.

Discount rate objectives and resulting discount rates are discussed in "[Present Values, Investment Returns and Discount Rates](#)".<sup>8</sup> The paper discusses the following four objectives for a discount rate.

Objective 1: To "connect" the starting and ending asset values.

Objective 2: To have a "safety cushion" (called "adverse deviation" in the draft).

Objective 3: No expected gains/losses *in the future*.

Objective 4: No expected gains/losses *at the present*.

In particular, Objective 1 implies the geometric expected return, Objective 3 implies the arithmetic expected return.

I am optimistic that these papers would be useful for the development of better actuarial standards and educational purposes.

## Conclusion

I believe that the exposure draft has significant room for improvement. As discussed in these comments, certain sections of the draft are not appropriate and should be re-written.

Thank you for your attention to these comments. Feel free to contact me if you have any questions/comments. I would be happy to assist the ASB in the development of this standard and related issues.

Sincerely

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<sup>8</sup> D. Mindlin, Present Values, Investment Returns and Discount Rates, *CDI Advisors Research*, CDI Advisors LLC, 2010, <http://www.cdiadvisors.com/papers/CDIDiscountRate.pdf>.



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