The Case for Annuities and Matching Bonds in Retirement Plans

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The problem of "de-risking" retirement plans has attracted a lot of attention in recent years. The most straightforward way to reduce (or eliminate) the riskiness of a retirement plan is to invest in assets that make regular predetermined payments that offset (or match) the plan's financial commitments. In this paper, such assets are called generically "buy-and-hold" assets. Examples of "buy-and-hold" assets include, but are not limited to, annuities for DC plan participants as well as matching bonds and group annuities for DB plans.

Many believe that investing in these and other "buy-and-hold" assets is not as prevalent as it should be. The reality is most individual and institutional investors have been reluctant to invest in matching "buy-and-hold" assets despite recommendations of numerous economists, consultants, financial advisors and asset managers. This reluctance is not exactly unjustified – there are many economic, behavioral, institutional, statutory and other obstacles to investing in "buy-and-hold" assets.

This paper does not attempt to overcome these obstacles en masse. The paper's objective is to present a relatively simple and accessible quantitative model that demonstrates the advantages of "buy-and-hold" assets. The methodology and numerical examples in this paper are specifically designed to require only basic knowledge of portfolio analysis and probability theory. All calculations for the examples discussed in this paper fit in a modest size Excel worksheet.[1]

The paper employs a simplified version of *Commitment Driven Investing* (CDI), a cost/risk management framework that utilizes multi-period cost/risk measurements and incorporates traditional portfolios and multi-period "buy-and-hold" assets. In the CDI framework, traditional ("rebalanceable") portfolios and "buy-and-hold" assets are intrinsically connected – an allocation to "buy-and-hold" assets comes with a traditional portfolio for the remaining assets.[2]

The CDI framework offers the tools to design decision factors that identify trigger points for "de-risking" retirement plans. These decision factors are measurements of the investor's ability to fund the commitments; they quantify the benefits of investing in "buy-and-hold" assets to this ability. In particular, the paper demonstrates that a partial allocation to "buy-and-hold" assets may increase the likelihood of funding the commitment.

One of the paper's main messages is "buy-and-hold" assets add another dimension to the process of optimal policy portfolio selection and may improve the investor's ability to fund the commitments.[3]

The Conventional Approach and "Buy-and-Hold" Assets

Optimal policy portfolio selection is the predominant challenge of investment management. Countless publications present various aspects of optimal policy portfolio selection. The foundation of the mainstream conventional approach to optimal portfolio selection – Modern Portfolio Theory (MPT) – is renowned and considered classic. The MPT based conventional approach serves as the basis for asset allocation decisions for a multitude of investors.

Yet, the events of the last two decades have compelled many institutional and individual investors to question the adequacy of the conventional approach to their goals. It is becoming increasingly clear the solutions the conventional approach generates may in fact be sub-optimal. Many investors recognize that the approach is not broad enough to incorporate the challenges today's investors face.

The conventional approach assumes that all assets are perfectly liquid and "rebalanceable" within its single-period time horizon. The multi-period nature of most investors' financial commitments plays no part in the single-period optimization procedure the approaches employs. Nor does the approach properly accommodate "buy-and-hold" assets.

In all fairness, MPT does well what it is intended to do. MPT is perfectly open about the fact that it is a "*portfolio*" theory rather than an "*investor*" theory. MPT is not designed to incorporate multi-period "buy-and-hold" assets that may match the investor's financial commitments. Regardless of these commitments, MPT starts with $1 at the present and performs a single-period optimization of the future value. By design, the investor's commitments play no role in the optimization.

The importance of a proper asset allocation model that incorporates all assets potentially beneficial to the investor should not be underestimated. The process of making an asset allocation decision should include clearly articulated goals and transparent quantitative measurements of the ability to achieve these goals. The conventional approach comes short of satisfying the needs of investors with multi-period financial commitments.

This paper focuses on two specific problems in the area of optimal policy portfolio selection that have been hotly debated for a long time and remain unresolved. The first problem is the desirability of annuities for DC plan participants. Many economists and advisors recommend annuities, but, for various reasons, most DC plan participants have so far rejected even partial annuitization of their post-retirement spending needs.

The second problem is the desirability of "buy-and-hold" assets that match the benefit payments for DB plans (called "matching assets" in this paper). Many DB plans wish to de-risk the process of funding their pension commitments, and investing in matching assets would be a sensible way to do so. But, for various reasons, most DB plans have refrained from investing in matching assets so far.

DB and DC plans operate in different environments, and different forces are in play in their asset allocation decisions. One thing, however, is clear. The conventional approach is not helpful in demonstrating the advantages of annuities and matching assets. There is a need for a more comprehensive framework. This paper attempts to satisfy this need.

The Funding Problem and Required Assets

This section introduces the concept of a funding problem, the investor's primary objective, and the necessary technical tools for finding optimal policy portfolios.

While there are major differences between DB and DC plans, the problems of annuitization for DC plans and matching assets for DB plans have two important common attributes. First, both types of investors have future financial commitments to fund (post-retirement spending for DC plan investors; benefit payments for DB plans). Second, annuities and matching assets are quintessential "buy-and-hold" assets – they are bought and held for the purpose of making predetermined regular payments.

Therefore, these problems have a similar structure. There is an investor with a financial commitment to fund (i.e. there are a series of future payments to be made). The investor's objective is to fund the commitment – to make sure the money is readily available every time a payment is due. This objective is called *the funding problem*.

The investor has certain assets at the present. The investor's task is to allocate a portion of the assets to "buy-and-hold" assets and the remaining assets to a traditional portfolio to be regularly rebalanced. The challenge is to make optimal allocations in terms of the investor's ability to fund the commitment.

Let us describe the funding problem in quantitative terms. The investor has made a commitment to make N end-of-year payments. The payment in year k is

$$\$P(1+I_1)\ldots(1+I_k)$$

where $\$P$ is a constant and $I_1,\ldots,I_k$ are stochastic adjustments (e.g. inflation). The investor's current asset value is $\$A$, and it would be highly desirable if these assets were sufficient to fund the commitment.

We assume that there exists the matching "buy-and-hold" asset – the asset that will fund the commitment, i.e. will certainly make the payments $\$P(1+I_1)\ldots(1+I_k)$ for all k. The price of this "buy-and-hold" asset is $\$B$.

The investor's task is to select a *policy portfolio*, which is defined as an allocation to the "buy-and-hold" asset and a series of traditional "rebalanceable" portfolios expected to be utilized in the future years for the remaining assets.[4] Specifically, the investor is to select a fixed fraction of all payments that would be funded by an allocation to the "buy-and-hold" asset (this fraction is called *the hedged percentage* in this paper). The remaining payments would be funded by the remaining assets via investing in traditional portfolios. In other words, the investor is to select a fraction m $(0 \le m \le 1)$ such that $mP(1+I_1)\ldots(1+I_k)$ would be funded by the "buy-and-hold" asset and $(1-m)P(1+I_1)\ldots(1+I_k)$ would be funded by the remaining assets for all k.

Let us introduce the main technical tool of this paper – stochastic present values.[5] Let us assume for a moment that the policy portfolio contains traditional portfolios only, i.e. there is no allocation to the "buy-and-hold" asset (this assumption will be removed shortly). Let $R_1,\ldots,R_N$

be portfolio returns for the next N years. The stochastic present value of all payments is defined as follows.

$$RA = \sum_{k=1}^{N} P \frac{(1+I_1)\ldots(1+I_k)}{(1+R_1)\ldots(1+R_k)} \tag{1}$$

Random variable RA (which stands for *Required Assets*) is the asset value required at the present to fund the commitment. The stochastic measurements of RA (e.g. mean, standard deviation, percentiles, etc.) are of the investor's vital interest.

The investor's primary objective is to fund the commitment. *The investor's primary risk is the shortfall event* – the assets are insufficient to fund the commitment. Random variable RA is directly related to the shortfall event. Namely, the shortfall event happens if (and only if)

$$RA > A \tag{2}$$

In other words, the shortfall event happens if funding the commitment requires more money than the investor has. The challenge is to select policy portfolios that minimize the part of the distribution of RA that lies to the right of A.

Let us assume now that the allocation to the "buy-and-hold" asset is non-trivial (i.e. the hedged percentage is greater than zero). The impact of this allocation is twofold. First, this allocation reduces the commitment. Second, this allocation reduces the assets available for return generation by the price of the "buy-and-hold" asset. Hence, the overall impact of the "buy-and-hold" asset on the expected cost is unclear – both the commitment and assets are reduced.

Specifically, the required assets for the portion of the commitment not funded by the "buy-and-hold" asset is $(1-m)RA$. The asset value available to fund this portion is $A-mB$. In this case, the shortfall event happens if (and only if)

$$(1-m)RA > A-mB \tag{3}$$

In other words, the shortfall event happens if funding the remaining commitment requires more money than the investor has after paying for the selected allocation to the "buy-and-hold" asset.

To recap, this section introduces the funding problem and demonstrates that stochastic present value RA is instrumental in analyzing the investor's primary objective and risk.

Decision Factors

This section introduces quantitative decision factors that will be subsequently used to identify optimal solutions to the funding problem.

As demonstrated in the previous section, the challenge is to maximize the portion of the distribution of $(1-m)RA$ that lies to the left of $A-mB$. In general, it is desirable to have *low RA* and pull the distribution of *RA to the left*. Policy portfolios that do so improve the investor's ability to fund the commitment.

Therefore, there is a need for the risk measurements of *RA* that would quantify the effectiveness of a particular policy portfolio in moving the distribution of *RA* leftward. Such measurements are called *decision factors* in this paper. Policy portfolios that have "better" decision factors would be better positioned to fund the commitment.

MPT provide a valuable guidance in the development one of these factors. The riskiness of a portfolio is measured as its "dispersion" around the expected return, so the standard deviation of return is used as a measurement of volatility. MPT is based on the assumption that it is desirable to maximize expected return for a given level of volatility. The portfolio that generates the highest expected return (for a given level of volatility) is preferable. The portfolio expected return is one of the most important decision factors in MPT.

Similar logic applies to the CDI framework. The riskiness of a policy portfolio can be measured as its "dispersion" around the *expected cost* (the mean of *RA*), so the standard deviation of *RA* is used as a measurement of volatility. CDI is based on the assumption that it is desirable to minimize the expected cost of funding for a given level of volatility. The policy portfolio that generates the lowest expected cost (for a given level of volatility) is preferable. Hence, the standard deviation and the mean of *RA* are our first two decision factors.[6]

Next, we need a measurement of the right tail of *RA*. Similar to the VaR concept, we may consider the 95th percentile of the *RA* distribution as a measurement of the "size" of the distribution's right tail. For a given volatility, the policy portfolio that generates the lowest 95th percentile of the *RA* distribution is preferable. Hence, the 95th percentile of the *RA* distribution is our third decision factor.

Last, but not the least, one of the most important measurements of the investor's ability to fund the commitment is the probability of the shortfall event (*shortfall probability*). For a given volatility, the policy portfolio that generates the lowest shortfall probability is preferable. Hence, the shortfall probability is our fourth decision factor.

To recap, the decision factors for the selection of the optimal combination of a "buy-and-hold" allocation and conventional portfolio are the following.

1. The standard deviation of the *RA* distribution.
2. The mean of the *RA* distribution.
3. The 95th percentile of the *RA* distribution.
4. The shortfall probability.

For every decision factor 1 - 4, the general rule is "the lower, the better."

The next two sections present quantitative analysis of the decision factors for the paper's main examples of funding problems and corresponding "buy-and-hold" assets – annuities for DC plan participants and matching assets for DB plans. Subsequent sections summarize this analysis and present the key conclusions of this paper.

Example 1: DC plan participants and Annuities

This section deals with the funding problem for a just-retired DC plan participant (called *the investor* in this section). We assume that the investor wishes to fund her post-retirement spending in a reasonably risk-averse manner. In general, the investor prefers to be in full control of her retirement assets and avoid transferring a meaningful part of her assets to an annuity provider. However, the investor would consider buying an annuity if the annuity makes the funding of the investor's post-retirement standard of living safer.

We make the following assumptions about the investor and her funding problem.

- The investor has $100 in assets.
- The investor spends $4.00 annually adjusted for inflation.[7]
- The investor utilizes only two broadly defined asset classes: stocks and bonds.[8]
- The investor portfolio is 30% stocks and 70% bonds (we call it *the base portfolio*).
- The investor employs the same portfolio in all years.[9]
- The investor will spend thirty years in retirement.
- A life annuity that makes inflation-adjusted payments is available to the investor.

The capital market assumptions and some additional assumptions are presented in the Appendix.

In addition to the base portfolio (30/70), let us consider six additional portfolios (stocks/bonds): 40/60, 50/50, 60/40, 70/30, 80/20, and 90/10. For these portfolios, *Exhibit 1* shows the means and standard deviations for portfolio returns as well as stochastic present values *RA*.[10]

Exhibit 1. Selected Portfolios and Corresponding *RA*s

	30/70	*40/60*	*50/50*	*60/40*	*70/30*	*80/20*	*90/10*
Stocks	30%	40%	50%	60%	70%	80%	90%
Bonds	70%	60%	50%	40%	30%	20%	10%
Return Expected Geometric	5.11%	5.44%	5.74%	6.03%	6.30%	6.55%	6.79%
Return Expected Arithmetic	5.29%	5.68%	6.07%	6.46%	6.85%	7.25%	7.64%
Return StDev	6.23%	7.22%	8.37%	9.60%	10.90%	12.24%	13.61%
RA Mean	85.37	82.45	80.08	78.19	76.73	75.66	74.95
RA StDev	15.69	17.24	19.12	21.23	23.53	26.00	28.67

Let us see if (partial) annuitization can be beneficial to the investor. The riskiness of the policy portfolio that consists of the base portfolio and zero annuitization is measured by the standard deviation of $RA_{30/70}$, which is 15.69 (the last row in *Exhibit 1*). If the investor wants to utilize a more aggressive portfolio P without an overall risk increase, a portion of the investor's spending must be annuitized.

If a policy portfolio utilizes portfolio *P* and hedged percentage *m*, then the required assets is $(1-m)RA_P + mB$, where RA_P is the required assets for the policy portfolio that utilizes portfolio *P* with no annuitization and *B* is the annuity price. Therefore,

$$StDev\left((1-m)RA_P\right) = StDev\left(RA_{30/70}\right) \tag{4}$$

which implies

$$m = 1 - \frac{StDev\left(RA_{30/70}\right)}{StDev\left(RA_P\right)} \tag{5}$$

For example, the hedged percentage for the 40/60 portfolio is (see the last row in *Exhibit 1*)

$$m = 1 - \frac{15.69}{17.24} = 9\%$$

Exhibit 2 shows the hedged percentages for the selected portfolios that are based on (5).

Exhibit 2. Selected Portfolios and Hedged Percentages

	30/70	*40/60*	*50/50*	*60/40*	*70/30*	*80/20*	*90/10*
Hedged %	0%	9%	18%	26%	33%	40%	45%

Note that a hedged percentage is not equal to the percentage of assets allocated to an annuity. This allocation depends on the price of the annuity.

The policy portfolios with the hedged percentages specified in *Exhibit 2* are equally risky (as measured by the standard deviation of the corresponding *RA*s). For these portfolios, decision factor 1 is the same. However, the decision factors 2 - 4 are different. These decision factors would demonstrate whether annuitization should be beneficial to the investor.

One of the key factors impacting the decision factors is the price of the annuity. To make annuity pricing transparent, this paper utilizes a simplified deterministic pricing procedure that generates three scenarios for annuity pricing. We have a series of thirty end-of-year payments adjusted for expected inflation at 2.5%. Ignoring (temporarily) the volatility of inflation for the pricing purposes and discounting this series at 3.0%, 3.5% and 4.0%, we get pricing scenarios I, II, III and correspondingly. Exhibit 3 shows the annuity prices for these scenarios.[11]

Exhibit 3: Annuity Pricing Scenarios

	Discount	Price
Annuity I	3.0%	111.38
Annuity II	3.5%	103.60
Annuity III	4.0%	96.56

Let us look at decision factor 2 – the mean of *RA* with the hedged percentages discussed above. As was noted above, the required assets for policy portfolio utilizes portfolio *P* and hedged percentage m is $(1-m)RA_P + mB$. The mean of this *RA* is equal to $(1-m)E(RA_P) + mB$. For example, the *RA* mean for the 40/60 portfolio and the annuity priced at 3.0% is equal to

$$(1-0.09)\cdot 82.45 + 0.09\cdot 111.38 = 85.05$$

The means of *RA* for the selected portfolios and annuity pricing scenarios are presented in *Exhibit 4*.

Exhibit 4: Decision Factor 2 – Means of *RA*s

	30/70	*40/60*	*50/50*	*60/40*	*70/30*	*80/20*	*90/10*
Annuity I	85.37	85.05	85.70	86.85	88.27	89.82	91.44
Annuity II	85.37	84.36	84.30	84.82	85.68	86.74	87.92
Annuity III	85.37	83.72	83.04	82.99	83.34	83.95	84.73

Exhibit 4 shows that partial annuitization generates lower decision factor 2 for

- all portfolios in pricing scenario III;
- portfolios from 40/60 to 60/40 in pricing scenario II;
- portfolio 40/60 in pricing scenario I.

In particular, it is notable that portfolio 40/60 with 9% annuitizaton is "better" than the base portfolio 30/70 with no annuitization under all three annuity pricing scenarios (according to decision factor 2).

Before we bring decision factors 3 and 4 into the picture, it should be noted that decision factors 1 and 2 – the mean and standard deviation of *RA* – can be calculated exactly.[12] In contrast, the exact calculations of decision factors 3 and 4 – the 95th percentile of the *RA* distribution and the shortfall probability – are available only in special cases (e.g. when the commitment contains just one payment). In general, a closed-form expression for the *RA* distribution is not known, and the best we can do is to estimate this distribution.

There are, however, reasonable estimates of the *RA* distributions that generate values for decision factor 3 and 4. For the purposes of this paper, once again, we use a simplified moment-matching technique to select distributions that approximate the *RA* distribution. We employ *lognormal* and *inverse gamma* distributions for this purpose.[13] The parameters of these distribution are selected to match the first *two* moments of the *RA* distribution.[14] We also include an approximation selected to match the first *four* moments of the *RA* distribution (labeled *Approximation CDI* for brevity).[15] The results of these approximations are quite similar.

Exhibit 5 shows the 95th percentile of the *RA* distribution for the selected portfolios and annuity pricing scenarios.

Exhibit 5: Decision Factor 3 – 95th percentiles of *RA*s

	30/70	*40/60*	*50/50*	*60/40*	*70/30*	*80/20*	*90/10*
Approximation Lognormal							
Annuity I	113.33	113.24	114.14	115.55	117.21	118.98	120.78
Annuity II	113.33	112.54	112.74	113.52	114.62	115.89	117.26
Annuity III	113.33	111.91	111.48	111.68	112.27	113.10	114.07
Approximation Inverse Gamma							
Annuity I	113.80	113.71	114.58	115.92	117.46	119.07	120.66
Annuity II	113.80	113.01	113.18	113.89	114.87	115.98	117.13
Annuity III	113.80	112.38	111.92	112.05	112.53	113.19	113.95
Approximation CDI							
Annuity I	113.65	113.57	114.47	115.85	117.46	119.16	120.86
Annuity II	113.65	112.87	113.07	113.82	114.87	116.07	117.34
Annuity III	113.65	112.24	111.81	111.98	112.53	113.28	114.15

Let us look at the lognormal approximation first. *Exhibit 5* shows that the partial annuitization generates lower decision factor 3 for:

- portfolios from 30/70 to 80/20 in pricing scenario III;
- portfolios from 30/70 to 50/50 in pricing scenario II;
- portfolio 40/60 in pricing scenario I.

The inverse gamma approximation generates similar results. In particular, it is notable that portfolio 40/60 with hedged percentage 9% is "better" than the base portfolio 30/70 with no annuitization under all three annuity pricing scenarios (according to decision factor 3). This conclusion is consistent with the analogous result for decision factor 2.

Exhibit 6 shows the shortfall probability for the selected portfolios and annuity pricing scenarios.

Exhibit 6: Decision Factor 4 – the Shortfall Probability

	30/70	*40/60*	*50/50*	*60/40*	*70/30*	*80/20*	*90/10*
Approximation Lognormal							
Annuity I	16.9%	16.3%	17.0%	18.4%	20.1%	22.1%	24.3%
Annuity II	16.9%	15.4%	15.2%	15.7%	16.5%	17.6%	18.9%
Annuity III	16.9%	14.6%	13.7%	13.6%	13.8%	14.3%	14.9%
Approximation Inverse Gamma							
Annuity I	16.4%	15.8%	16.3%	17.4%	18.9%	20.5%	22.4%
Annuity II	16.4%	15.0%	14.6%	14.9%	15.6%	16.4%	17.3%
Annuity III	16.4%	14.2%	13.3%	13.0%	13.0%	13.3%	13.7%
Approximation CDI							
Annuity I	16.7%	16.1%	16.7%	17.9%	19.4%	21.3%	23.2%
Annuity II	16.7%	16.1%	16.7%	17.9%	19.4%	21.3%	23.2%
Annuity III	16.7%	14.4%	13.6%	13.3%	13.4%	13.9%	14.4%

Let us look at the lognormal approximation first. *Exhibit 6* shows that partial annuitization generates lower decision factor 4 for:

- all portfolios in pricing scenario III;
- portfolios from 40/60 to 70/30 in pricing scenario II;
- portfolio 40/60 in pricing scenario I.

The inverse gamma approximation generate similar results. In particular, it is notable that portfolio 40/60 with 9% annuitizaton is "better" than the base portfolio 30/70 with no annuitization under all three annuity pricing scenarios (according to decision factor 4). This conclusion is consistent with the analogous results for decision factor 2 and 3.

Let us preset these results in a graphical format. *Exhibit 7*, *8* and *9* show the *differences* between the values of decision factors 2, 3 and 4 (correspondingly) for the selected portfolios and annuity pricing scenarios.[16] Here is how to read these exhibits. As long as these lines are below zero, the selected portfolios with the corresponding annuitization levels are beneficial to the investor.

Exhibit 7, *8* and *9* demonstrate that portfolio 40/60 along with 9% annuitization is "better" then portfolio 30/70 with no annuitization – *all decision factors are improved*. These exhibits also help to find the optimal portfolio and annuitization level according to a particular decision factor and annuity pricing scenario. For example, under annuity pricing scenario III, portfolio 60/40 with hedged percentage 26% minimize decision factor 4 (the shortfall probability). Specifically, the shortfall probability for the base portfolio is 16.9%, and the shortfall probability for portfolio 60/40 with hedged percentage 26% is 13.6%, which is 3.6% lower!

Ultimately, the choice of the policy portfolio depends on the weights the investor assigns to the decision factors utilized for the funding problem. Other economic and behavioral decision factors (e.g. the investor's reluctance to transfer a significant portion of the assets to an annuity provider) may also play an important role in the selection of optimal portfolios and annuitization levels. These matters, however, are outside of the scope of this paper.

Exhibit 7

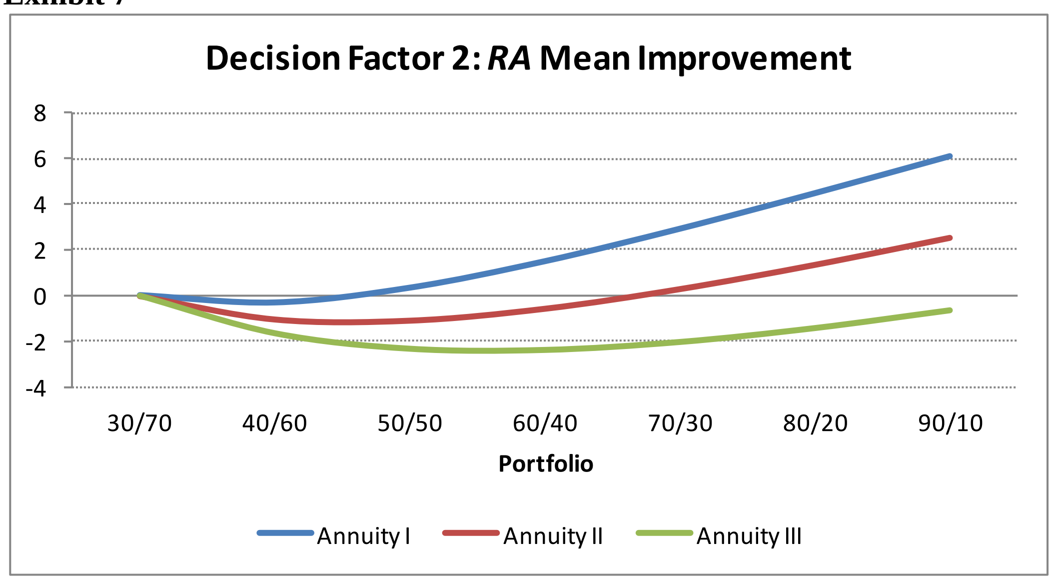


Exhibit 8

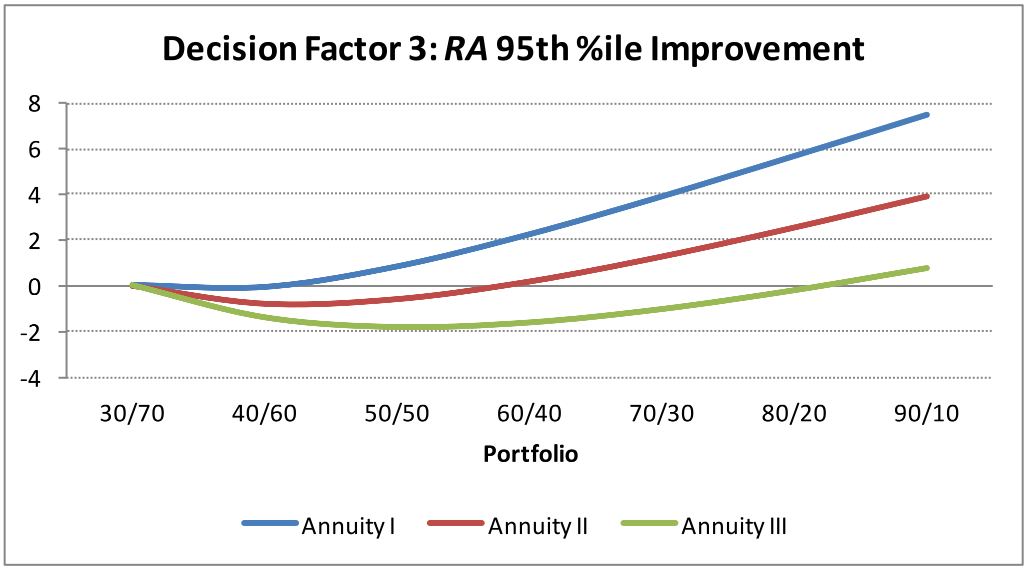
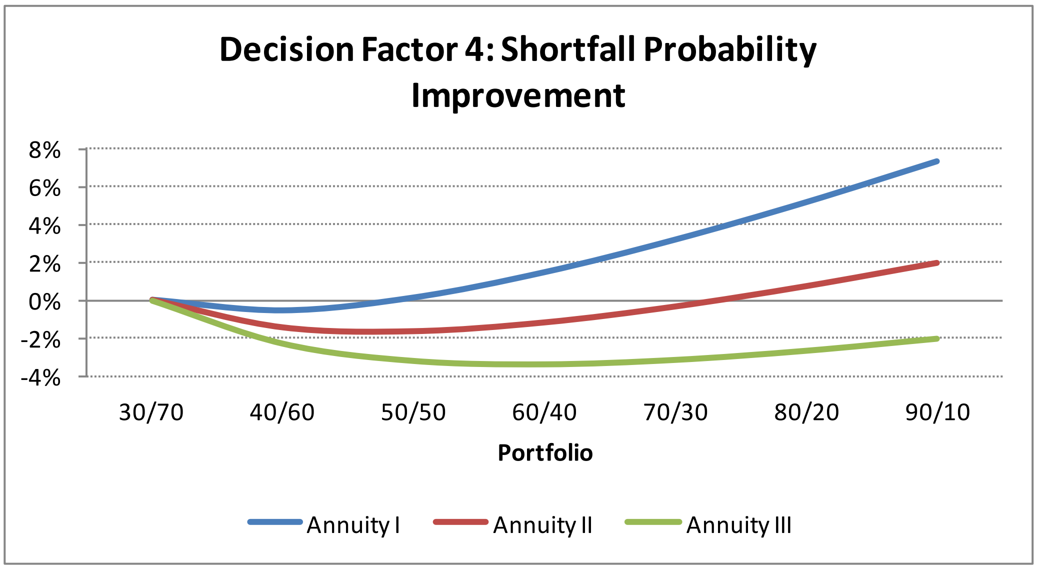


Exhibit 9



Example 2: A Frozen DB plan and Matching Assets

This section deals with the funding problem for a frozen DB plan. The methodology utilized in this section is similar to the one utilized in the previous section. We assume that the plan sponsor wishes to minimize the future contributions to the plan and, at the same time, fund the promised benefits in a reasonably risk-averse manner. The plan would be happy to "de-risk" and invest a portion of its assets in a matching asset if the asset makes the promised benefits safer.

We make the following assumptions about the plan and its funding problem.

- The plan has $15 in assets.

- The pays out flat \$1.00 annually (*not* adjusted for inflation).
- The plan utilizes only two broadly defined asset classes: stocks and bonds.
- The plan's portfolio is 30% stocks and 70% bonds (we call it *the base portfolio*).
- The plan employs the same portfolio in all years.[17]
- The plan's financial commitment is to make thirty payments of \$1.00.
- A bond portfolio that makes thirty payments of \$1.00 is available to the plan (we call it *the matching asset;* an allocation to the matching asset is called *matching allocation*).

The capital market assumptions and some additional assumptions are presented in the Appendix.

Now, let us see whether an allocation to the matching asset be beneficial to the plan.

Similar to the previous section, we consider the base portfolio (30/70) and six additional portfolios (stocks/bonds): 40/60, 50/50, 60/40, 70/30, 80/20, and 90/10 . For these portfolios, *Exhibit 1* shows the means and standard deviations for portfolio returns as well as stochastic present values *RA*.

Exhibit 10. Selected Portfolios and Corresponding *RA*s

	30/70	*40/60*	*50/50*	*60/40*	*70/30*	*80/20*	*90/10*
Stocks	30%	40%	50%	60%	70%	80%	90%
Bonds	70%	60%	50%	40%	30%	20%	10%
Return Expected Geometric	5.11%	5.44%	5.74%	6.03%	6.30%	6.55%	6.79%
Return Expected Arithmetic	5.29%	5.68%	6.07%	6.46%	6.85%	7.25%	7.64%
Return StDev	6.23%	7.22%	8.37%	9.60%	10.90%	12.24%	13.61%
RA Mean	15.51	15.04	14.67	14.37	14.13	13.96	13.85
RA StDev	2.51	2.80	3.13	3.50	3.90	4.32	4.78

Let us see if (partial) allocation to the matching asset can be beneficial to the plan. The riskiness of the policy portfolio that consists of the base portfolio and zero matching allocation is measured by the standard deviation of $RA_{30/70}$, which is 2.51 (the last row in *Exhibit 10*). If the investor wants to utilize a more aggressive portfolio *P* without an overall risk increase, a portion of the plan's payments must be offset by a matching allocation.

The exact hedged percentage is calculated similar to formulas (4) and (5) from the previous section. For example, the hedged percentage for the 40/60 portfolio is (see the last row in *Exhibit 10*)

$$m = 1 - \frac{2.51}{2.80} = 10\%$$

Exhibit 11 shows the matching allocations for the selected portfolios that are based on (5).

Exhibit 11. Selected Portfolios and Hedged Percentages

	30/70	*40/60*	*50/50*	*60/40*	*70/30*	*80/20*	*90/10*
Hedged %	0%	10%	20%	28%	36%	42%	47%

Note that a hedged percentage is not equal to the percentage of assets allocated to the matching asset. This allocation depends on the price of the matching asset.

The policy portfolios with the hedged percentages specified in *Exhibit 11* are equally risky (as measured by the standard deviation of the corresponding *RA*s). Decision factor 1 is the same for all these portfolios. However, the decision factors 2 - 4 are different. These decision factors would demonstrate whether matching allocations should be beneficial to the plan.

One of the key factors impacting the decision factors is the price of the matching asset. To make the matching asset pricing transparent, this paper utilizes a simplified deterministic pricing procedure that generates three scenarios. We have a series of thirty end-of-year payments of $1.00. Discounting this series at 3.0%, 3.5% and 4.0%, we get pricing scenarios I, II, III and correspondingly. Exhibit 3 shows the matching asset prices for these scenarios.[18]

Exhibit 12: Matching Asset Pricing Scenarios

	Discount	Price
Match Asset I	3.0%	19.60
Match Asset II	3.5%	18.39
Match Asset III	4.0%	17.29

Let us calculate decision factor 2 – the mean of *RA* with the hedged percentages discussed above. For example, the *RA* mean for the 40/60 portfolio and the matching asset priced at 3.0% is equal to

$$(1-0.10)\cdot 15.04+0.10\cdot 19.60=15.51$$

The means of *RA* for the selected portfolios and matching asset pricing scenarios are presented in *Exhibit 13*.

Exhibit 13: Decision Factor 2 – Means of *RA*s

	30/70	*40/60*	*50/50*	*60/40*	*70/30*	*80/20*	*90/10*
Match Asset I	15.51	15.51	15.65	15.85	16.08	16.33	16.58
Match Asset II	15.51	15.39	15.41	15.51	15.65	15.82	16.00
Match Asset III	15.51	15.27	15.19	15.20	15.26	15.36	15.48

Exhibit 13 shows that partial matching allocation generates the same or lower decision factor 2 for:

- all portfolios in pricing scenario III;
- portfolios from 40/60 to 60/40 in pricing scenario II;
- portfolio 40/60 in pricing scenario I.

In particular, it is notable that portfolio 40/60 with hedged percentage 10% is the "same" or "better" than the base portfolio 30/70 with no hedging under all three pricing scenarios (according to decision factor 2).

To estimate decision factors 3 and 4 – the 95th percentile of the *RA* distribution and the shortfall probability – we employ lognormal and inverse gamma distributions. Just like in the previous section, the parameters of these distribution are selected to match the first two moments of the *RA* distribution. The approximation selected to match the first *four* moments of the *RA* distribution is included as well. The results of these approximations are similar.

Exhibit 14 shows the 95th percentile of the *RA* distribution for the selected portfolios and matching asset pricing scenarios.

Exhibit 14: Decision Factor 3 – 95th percentiles of *RA*s

	30/70	*40/60*	*50/50*	*60/40*	*70/30*	*80/20*	*90/10*
Approximation Lognormal							
Annuity I	16.9%	16.3%	17.0%	18.4%	20.1%	22.1%	24.3%
Annuity II	16.9%	15.4%	15.2%	15.7%	16.5%	17.6%	18.9%
Annuity III	16.9%	14.6%	13.7%	13.6%	13.8%	14.3%	14.9%
Approximation Inverse Gamma							
Annuity I	16.4%	15.8%	16.3%	17.4%	18.9%	20.5%	22.4%
Annuity II	16.4%	15.0%	14.6%	14.9%	15.6%	16.4%	17.3%
Annuity III	16.4%	14.2%	13.3%	13.0%	13.0%	13.3%	13.7%
Approximation CDI							
Annuity I	16.7%	16.1%	16.7%	17.9%	19.4%	21.3%	23.2%
Annuity II	16.7%	16.1%	16.7%	17.9%	19.4%	21.3%	23.2%
Annuity III	16.7%	14.4%	13.6%	13.3%	13.4%	13.9%	14.4%

Let us look at the lognormal approximation first. *Exhibit 14* shows that partial matching allocation generates lower decision factor 3 for:

- portfolios from 40/60 to 70/30 in pricing scenario III;
- portfolios from 40/60 to 50/50 in pricing scenario II.

Partial matching allocation generates higher decision factor 3 for all portfolios in pricing scenario I. The inverse gamma approximation generate similar results.

Exhibit 15 shows the shortfall probability for the selected portfolios and matching asset pricing scenarios.

Let us look at the lognormal approximation first. *Exhibit 15* shows that partial matching allocation generates lower decision factor 4 for:

- all portfolios in pricing scenario III;
- portfolios from 40/60 to 60/40 in pricing scenario II;
- portfolio 40/60 in pricing scenario I;

Exhibit 15: Decision Factor 4 – the Shortfall Probability

	30/70	40/60	50/50	60/40	70/30	80/20	90/10
Approximation Lognormal							
Match Asset I	55.0%	54.6%	56.5%	59.4%	63.0%	67.0%	71.3%
Match Asset II	55.0%	52.6%	52.5%	53.6%	55.5%	58.0%	60.8%
Match Asset III	55.0%	50.8%	48.8%	48.4%	48.9%	49.9%	51.5%
Approximation Inverse Gamma							
Match Asset I	54.1%	53.6%	55.4%	58.5%	62.3%	66.8%	71.8%
Match Asset II	54.1%	51.5%	51.3%	52.4%	54.3%	57.0%	60.2%
Match Asset III	54.1%	49.7%	47.6%	47.0%	47.4%	48.4%	50.0%
Approximation CDI							
Match Asset I	54.4%	53.9%	55.7%	58.7%	62.4%	66.7%	71.5%
Match Asset II	54.4%	53.9%	55.7%	58.7%	62.4%	66.7%	71.5%
Match Asset III	54.4%	50.0%	48.0%	47.4%	47.8%	48.9%	50.4%

The other two approximations generate similar results, which are consistent with the analogous results for decision factor 2.

Let us preset these results in a graphical format. *Exhibit 16*, 17 and 18 show the *differences* between the values of decision factors 2, 3 and 4 (correspondingly) for the selected portfolios and matching asset pricing scenarios.[19] As long as these lines are below zero, the selected portfolios with the corresponding matching asset levels are beneficial to the plan.

Exhibit 16, *17* and *18* help to find the optimal portfolio and annuitization level according to a particular decision factor and annuity pricing scenario. For example, under matching asset pricing scenario III, portfolio 60/40 with hedged percentage 26% minimize decision factor 4 (the shortfall probability). Specifically, the shortfall probability for the base portfolio is 6.6% higher than the shortfall probability for portfolio 60/40.

Exhibit 16

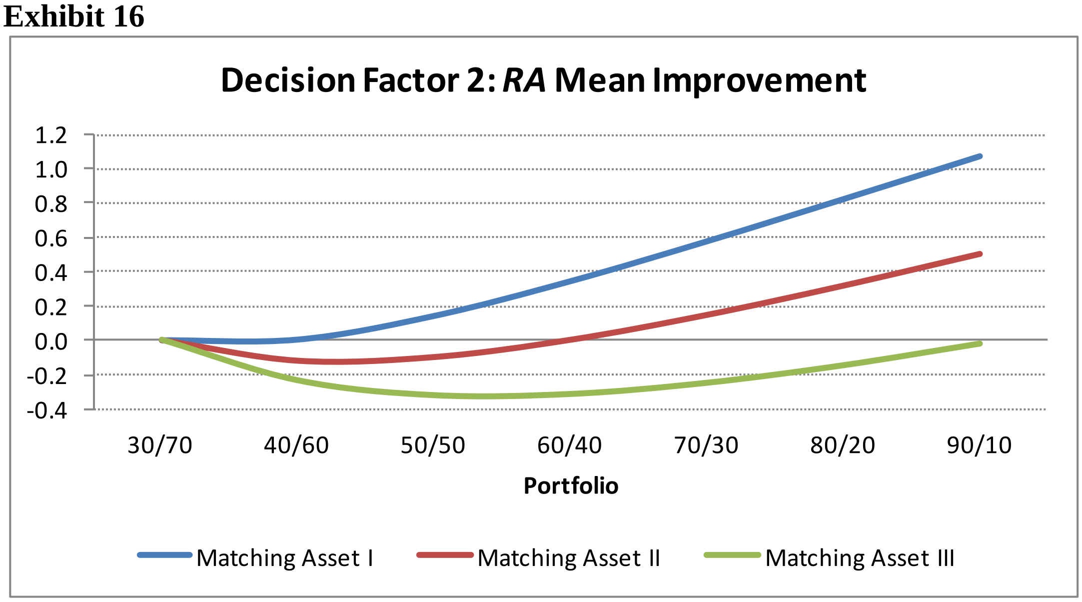


Exhibit 17

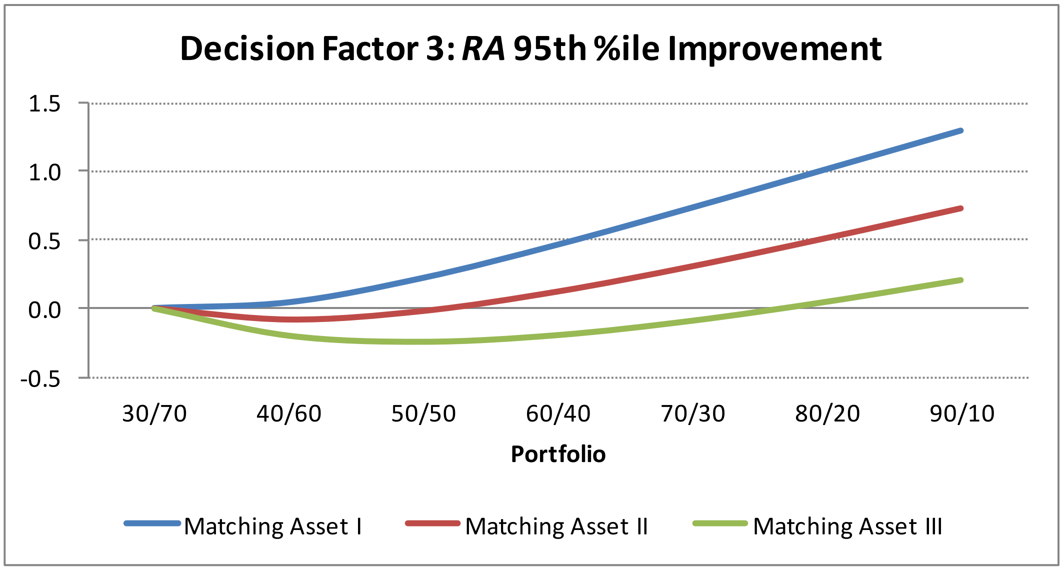
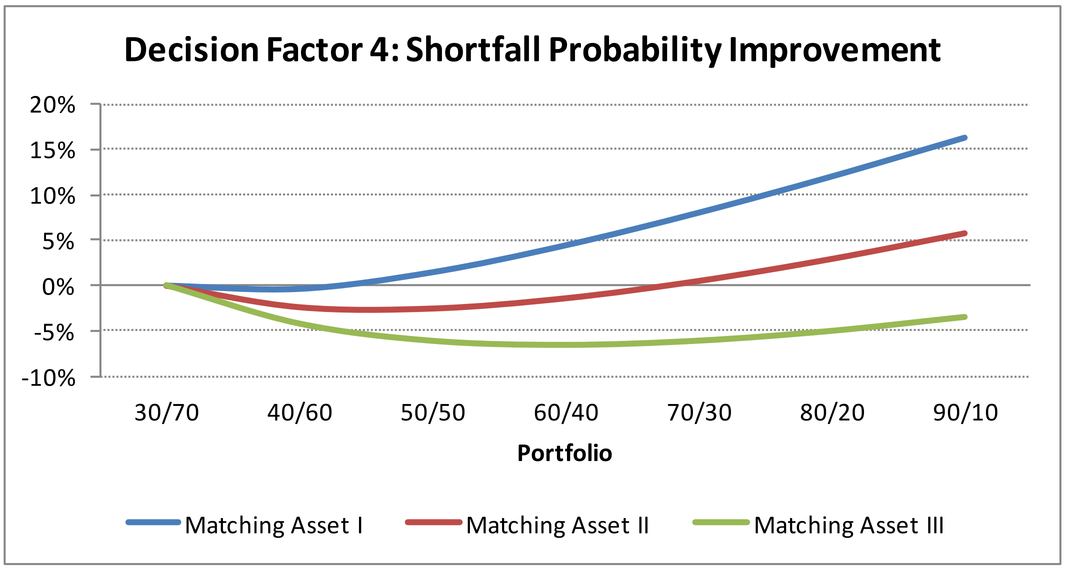


Exhibit 18



As in the previous section, the ultimate choice of the policy portfolio depends on the weights the investor assigns to the decision factors utilized for the funding problem. Other economic and behavioral decision factors (e.g. the plan's desire to de-risk) may also play an important role in the selection of optimal portfolios and matching allocation levels. These matters, however, are outside of the scope of this paper.

Lessons Learned

This section presents a partial list of inferences and implications of the analysis presented in this paper.

1. The expected return of a "buy-and-hold" asset is usually substantially lower than the expected returns of traditional portfolios considered in this paper. Therefore, it is instructive to ponder the following question. *Why would an allocation to a lower returning "buy-and-hold" asset be beneficial to the investor?*

   The answer is a "buy-and-hold" asset is *not a return enhancer,* it is *a risk reducer*. Any allocation to the "buy-and-hold" asset reduces the commitment and, consequently, the riskiness of the total portfolio measured by the standard deviation of *RA*. As a result, the remaining assets can be invested more aggressively without overall increase in the riskiness of the portfolio.

2. The presence of a potential "buy-and-hold" allocation adds another dimension to the process of optimal policy portfolio selection and may improve the investor's ability to fund the commitments. In this paper, a particular traditional "base portfolio" has served as the starting point for the discussion. Similar technique should be applied to other portfolios.

3. The price of the "buy-and-hold" asset is one of the most important inputs for the optimal portfolio selection. The relationship between the long-term expectations of asset returns and today's market prices of various assets is of paramount importance. Bond markets, in particular, may present attractive opportunities to de-risk retirement plans and, therefore, should be monitored on a regular basis.

4. "Buy-and-hold" assets do not have to match the investor's financial commitments. (In this paper, "buy-and-hold" assets are matching for technical convenience only.) A "buy-and-hold" asset may offset short-term commitments, long-term commitments or something in-between. The advantages of a particular "buy-and-hold" asset depend on the ability of the remaining assets to fund the remaining commitments.

5. For a DB plan, underfunding should not necessarily be an impediment to investing in "buy-and-hold" assets. In the example of a frozen DB plan presented in this paper, the plan is underfunded even in the most favorable scenario III (assets $15.00, "liability" $17.29, funded ratio 87%). Yet, as we saw in the example, partial allocation to the matching asset was beneficial to the investor. The issue is not necessary the funded status, but the investor's risk tolerance.

6. The implementation of the CDI framework involves *monitoring* and *analyzing* the advantages of various investment products. However, the implementation of the CDI framework does not necessarily involve investing in specific investment products. There are neither "CDI products" nor "CDI managers."[20]

7. The CDI framework, viewed from a different angle, generates trigger points for investing in "buy-and-hold" assets. The decision factors utilized in the framework indicate whether the price of a particular "buy-and-hold" asset is attractive as related to funding the investor's commitment. In other words, the CDI framework helps to determine whether a particular

"buy-and-hold" asset along with a corresponding policy portfolio present an attractive opportunity to "de-risk" a retirement plan. Conversely, the CDI framework can be helpful in determining an attractive opportunity to "re-risk" the retirement plan (i.e. to sell the "buy-and-hold" asset and utilize traditional portfolios only).

Conclusion

"Buy-and-hold" assets – like annuities and matching bonds – can be exceedingly valuable in providing secure retirements for DC and DB plan participants. The objective of this paper was to simplify the analysis of "buy-and-hold" assets and make this analysis accessible to a broad range of plan participants, sponsors, consultants, advisors and others.

This paper presents a quantitative methodology as well as analytical and decision-making tools that demonstrate the utility of "buy-and-hold" assets. This methodology may help to evaluate a particular "buy-and-hold" asset, determine the optimal allocation to the asset and the optimal allocation of the remaining assets.

The paper demonstrates that "buy-and-hold" assets can be beneficial even if they are not expected to produce high rates of return. An allocation to a lower-returning "buy-and-hold" asset may enhance the cost-risk characteristics of the total portfolio, move the portfolio beyond the cost-risk efficient frontier that utilizes only traditional portfolios, and, ultimately, improve the investor's ability to fund the commitment.

This author is optimistic that this framework will be useful to a broad range of practitioners involved in investing retirement assets.

APPENDIX

Capital Market Assumptions

Expected Return/Risk

	Expected Geometric	Expected Arithmetic	Standard Deviation
Stocks	7.00	8.03	15.00
Bonds	4.00	4.12	5.00
Inflation	2.50	2.50	1.00

Correlation Matrix

	Equities	Bonds	Inflation
Stocks	1.0	0.2	0.0
Bonds	0.2	1.0	-0.2
Inflation	0.0	-0.2	1.0

All portfolios and volatilities are assumed to be lognormally distributed.

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ENDNOTES

[1] To request the worksheet, contact the author at *dmindlin@cdiadvisors.com*.
[2] The CDI framework includes a powerful and flexible multi-period glide path optimizer and can accommodate any type of illiquid "buy-and-hold" assets. However, the most advanced features of the framework are outside of the scope of this paper.
[3] The methodology presented in this paper can be utilized to the analysis of other non-matching illiquid "buy-and-hold" assets, e.g. private equity, real estate, hedge funds, infrastructure, etc. These matters, however, are outside of the scope of this paper.
[4] A series of portfolios expected to be utilized in the future years is often called *a glide path.*
[5] For more details about stochastic present values, see Kellison [2009], Milevsky [2006], Mindlin [2009b].
[6] Mindlin [2009a] contains more details about the similarities between MPT and CDI. In particular, a portfolio is *MPT-efficient* if it has the highest expected return for a given volatility. A set of MPT-efficient portfolios comprise the classic *mean-variance efficient* frontier. Similarly, a policy portfolio is *CDI-efficient* if it has the lowest expected cost for a given volatility. A set of CDI-efficient portfolios comprise the *cost-risk efficient* frontier.
[7] In other words, the participant subscribes to the conventional "4% rule" recommended by many financial advisors.
[8] This is another assumption to simplify the technical aspects of the funding problem.
[9] The assumption "the same portfolio in all years" is made primarily to simplify the technical aspects of the funding problem in this paper. This author believes that most DC plan investors should have evolving policy portfolios (glide paths).
[10] *Exhibit 1* was produced using proprietary tools developed at *CDI Advisors*. For the calculations of the expected geometric and arithmetic returns, see Mindlin [2011a]. For the calculations of the mean and standard deviation of *RA* for the simplified examples considered in this paper, see Kellison [2009], chapter 12. For a more realistic case, see Mindlin [2009b].
[11] For example, the price of the annuity using discounting at 3% is equal to $4\frac{j^{31}-j}{j-1}=111.38$, where $j=\frac{1.025}{1.03}$.
[12] See Kellison [2009], chapter 12, for a detailed description of these calculations.
[13] See Klugman [1998] for more details about lognormal and inverse gamma distributions.
[14] If m_1 and m_2 are the first two moments of the *RA* distribution, then the parameters of the lognormal and inverse gamma distributions that match these moments are calculated as follows.

Lognormal: $\mu = \ln\left(\frac{m_1^2}{\sqrt{m_2}}\right)$, $\sigma^2 = \ln\left(\frac{m_2}{m_1^2}\right)$.

Inverse Gamma: $\alpha = \frac{m_1^2}{m_2 - m_1^2} + 2$, $\theta = \frac{m_1 m_2}{m_2 - m_1^2}$.

[15] This approximation (developed at *CDI Advisors*) is more precise than the two moment matching methodology used for the lognormal and inverse gamma approximations. The technical details of this approximation are outside of the scope of this paper.

[16] *Exhibits 8* and *9* shows the results for the lognormal approximation. The results for the inverse gamma approximation are similar.

[17] Once again, the assumption "the same portfolio in all years" is made primarily to simplify the technical aspects of the funding problem in this paper. This author believes that most DB plans should have evolving policy portfolios (glide paths).

[18] For example, the price of the matching asset using discounting at 3% is equal to $\frac{j^{31} - j}{j - 1} = 19.60$, where $j = \frac{1}{1.03}$.

[19] *Exhibits 17* and *18* shows the results for the lognormal approximation. The results for the inverse gamma approximation are similar.

[20] The situation with *Liability Driven Investing* (LDI) – a popular marketing strategy for bond-like investment products – is diametrically opposite. There is no LDI implementation without investing in LDI products. For more details regarding the relationship between CDI and LDI, see Mindlin [2011b].