Is More Money Better?

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January 7, 2010

ABSTRACT

This paper is not about the virtues of asceticism. The paper discusses the forward-looking objectives for life-cycle investing in general and target date funds in particular. Should a target date fund focus on maximizing account values at retirement ignoring how the money is likely to be invested in retirement? In other words, should a target date fund be managed ***to*** *retirement or* ***through*** *retirement? This paper demonstrates that managing a target date fund* ***to*** *retirement is generally not a good idea. Pre-retirement asset allocation should take into account post-retirement objectives and risks. As a forward-looking investment objective, the principle "the more money, the better" at retirement may lead to inefficient solutions and deliver inferior results to the investor.*

Target Date Funds and Their Objectives

Investors in target date funds (TDFs), like most investors, have had a rough ride lately. Even "near- or in-retirement" TDFs have recorded painful losses ranging from low single digits to staggering 40%-plus. A number of these TDFs have had sizable equity allocations, which in retrospect do not look appropriate for people nearing or already in retirement.

How did fund companies justify those high equity allocations? Startlingly, the justification was quite simple. Investors should expect to spend many years in retirement - a 2010 "retirement" fund is in fact a 2035 "death" fund. Therefore, investors need a considerable equity allocation with its higher expected return to fund her standard of living in retirement. Add a couple of scientifically looking Monte-Carlo simulation exhibits, and case closed.

As a result, investors in most "near- or in- retirement" TDFs experienced substantial losses that would affect the investors' standards of living for years to come. In response to this unfortunate turn of events, regulators and practitioners have been looking for ways to prevent this debacle from happening again. In particular, it has been suggested that target date funds should focus on making as much money as possible *at* retirement and let the investor worry about what to do with the money *in* retirement.

The purpose of this paper is to dispute this suggestion. The paper demonstrates that ignoring the investor's specific goals is not a good idea. Pre-retirement asset allocation should take into account post-retirement objectives and risks. Otherwise, the investor's portfolio is likely to be inefficient and deliver inferior results.

Framing the discussion as "manage-*to*-retirement" vs. "manage-*through*-retirement" is an imprudent trivialization of the problems most TDFs face. The "manage-through-retirement" approach was not the biggest problem of the "near- or in- retirement" TDFs that experienced staggering losses. Those TDFs were simply poorly designed.

The More Money, the Better, Is It Not?

Of course it is. Undoubtedly, $2 is better than $1, but worse than $3. Even if we do not know how much money one may have in the future, one would certainly prefer to pay less for a given item (*ceteris paribus*), as it leaves more money in one's possession. Clearly, everyone prefers to have more money.

This conclusion, however, is not terribly helpful to our problem, as it deals with the wrong question. The right question is the following. Does the principle "the more money, the better" make a sound *forward-looking objective* for a retirement investor? That is where the whole thing gets a bit tricky.

As we all know, most retirement investors endeavor to fund their retirements via investing in risky assets. As a result, the value of assets at retirement (a.k.a. "terminal wealth") for a typical investor is uncertain. In general, an uncertain object cannot be optimized like a conventional function. One has to optimize *risk measurements* of terminal wealth in order to find the optimal portfolio that delivers "more money." In fact, the optimization of the risk measurements of terminal wealth is a traditional problem of financial economics. The next section presents two classic approaches to the problem of terminal wealth maximization.

"More Money" as a Forward-Looking Objective

Think of an investor that has $1 invested in a portfolio of risky assets. At the end of the period, the investor's "terminal wealth" (future value) is equal to $1+R$, where R is the return generated by the portfolio for this period.

Let us define E and S as the mean and standard deviation of terminal wealth $1+R$ correspondingly. According to the approach proposed by H. Markowitz in 1952, "risk-adjusted expected wealth" U is defined as the difference between "reward" E and "penalty" $\tau \cdot S$, where τ is a non-negative risk aversion parameter:

$$U = E - \tau S \qquad (1)$$

In this approach, higher U means "more money." Portfolios that maximize "risk-adjusted expected wealth" U in (1) for risk aversion parameters $\tau \geq 0$ comprise the classic mean-variance efficient frontier.[1] Equivalently, given "risk-adjusted expected wealth" U, the objective of maximizing risk aversion parameters $\tau \geq 0$ in (1) leads to a mean-variance optimal policy. As a result, mean-variance optimization generates a set of optimal portfolios that can be considered "terminal wealth maximizers" at different levels of risk aversion.[2]

A. Roy proposed a different approach – named *Safety-First* – also in 1952. For an investor that has $1 invested in a portfolio of risky assets and given (potentially "disastrous") asset value A, let us calculate probability P that the investor's terminal wealth $1+R$ is greater than A:

$$P = \Pr(1 + R > A) \qquad (2)$$

The objective of the Safety-First approach is to find a portfolio that maximizes this probability - the likelihood that the investor's terminal wealth is greater than A.[3] In other words, the objective is to maximize the chance that the investor will have "more money" than some potentially "disastrous" asset value. Equivalently, given probability *P*, the objective is to maximize asset value *A* in (2), so the lowest point of the "top *P* segment" of terminal wealth would be as high as possible. Similar to mean-variance optimization, the Safety-First approach generates a set of optimal "terminal wealth maximizers" at different levels of risk aversion.[4]

It is important to note that both mean-variance and Safety-First methodologies, by design, do not lead to a specific portfolio. Rather, each methodology generates an efficient frontier – a set of optimal portfolios that depend on the investor's risk tolerance. Also by design, these methodologies do not take into account what may happen to the assets after the period under consideration.

Now, let us make this additional step and try to figure out how to use the "more money" the investor has presumably made. For example, a sensible decision for many investors would be to buy an annuity. Naturally, the investor wishes the annuity payments to be as high as possible – this is the principle "the more money, the better" in action once again.

On the surface, the more money the investor has, the higher the annuity payments. Beneath the surface, however, this matter is not so obvious. As we just discussed, in pursuit of "more money," the investor have generated a set of efficient policies ("terminal wealth maximizers"). Would these policies, in addition to maximizing terminal wealth, deliver the highest annuity payments? The answer to this question is an emphatic "No," as demonstrated below.

The methodology utilized in this paper is a variation of the Safety-First approach. This methodology is applied to the problems of terminal wealth maximization and, consequently, annuity payment maximization. To emphasize its main message, the paper deals with simplified one-year optimization problems.

Terminal Wealth Maximization

Let us assume that the investor has $95 at the present. The investor's objective is to have future asset value (terminal wealth) to be greater than $100 with probability *P*. If $95 is insufficient to achieve this objective, the investor wishes to make the lowest contribution required at the present to make this result possible. Therefore, the asset value at the present has two components: $95 and contribution *C* that will be determined via an optimization procedure.

Assuming that the money is invested in a portfolio of risky assets that generates return *R*, the investor's terminal wealth is equal to $(95+C)(1+R)$. The investor's objective (to have more than $100 with probability *P*) is expressed in the following equation.

$$\Pr\left(\left(95+C\right)\left(1+R\right)>100\right)=P \tag{3}$$

It is convenient to re-write equation (3) in terms of stochastic present values as follows.

$$\Pr\left(\frac{100}{1+R}<95+C\right)=P \tag{4}$$

Equation (4) shows that $95+C$ is equal to the *P*th percentile of stochastic present value $100/(1+R)$. Therefore, the investor's objective is to minimize the *P*th percentile of $100/(1+R)$. For every safety level *P*, the lowest required contribution is equal to the difference between the lowest *P*th percentile of $100/(1+R)$ and existing asset value $95. *Exhibit 1* shows optimal portfolio allocations for safety levels ranging from 50% to 99%. Capital market assumptions are presented in the Appendix.

Exhibit 1

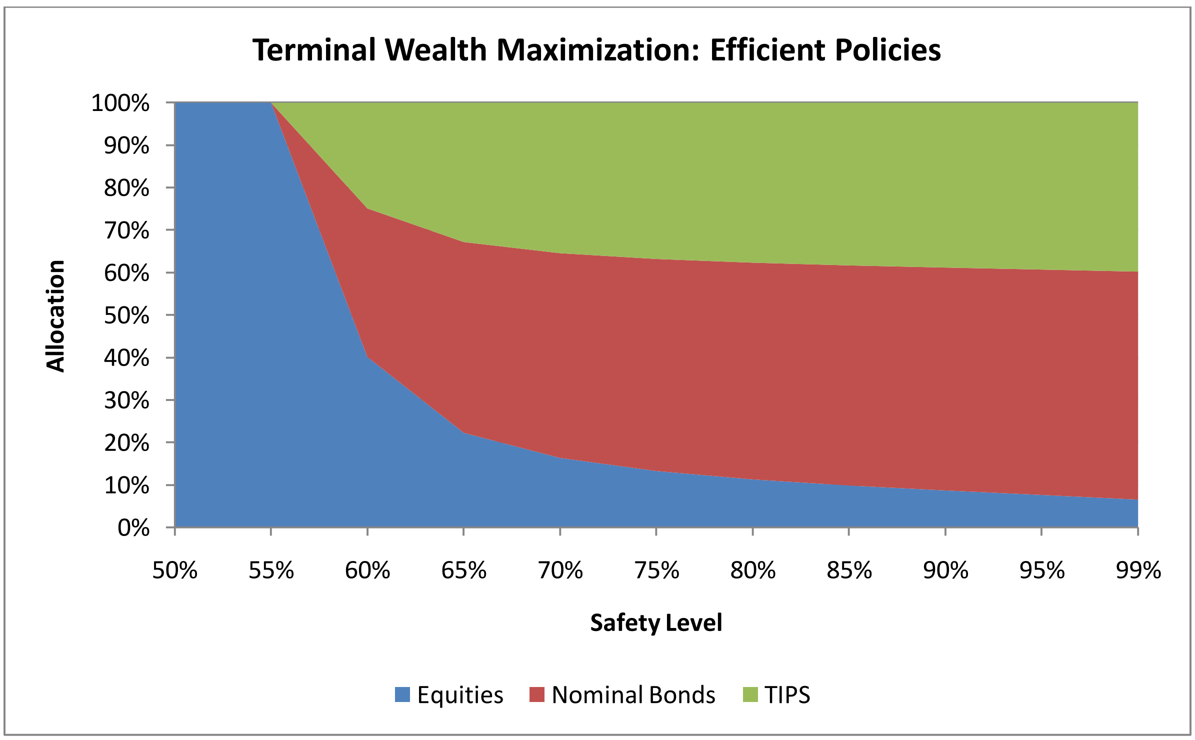
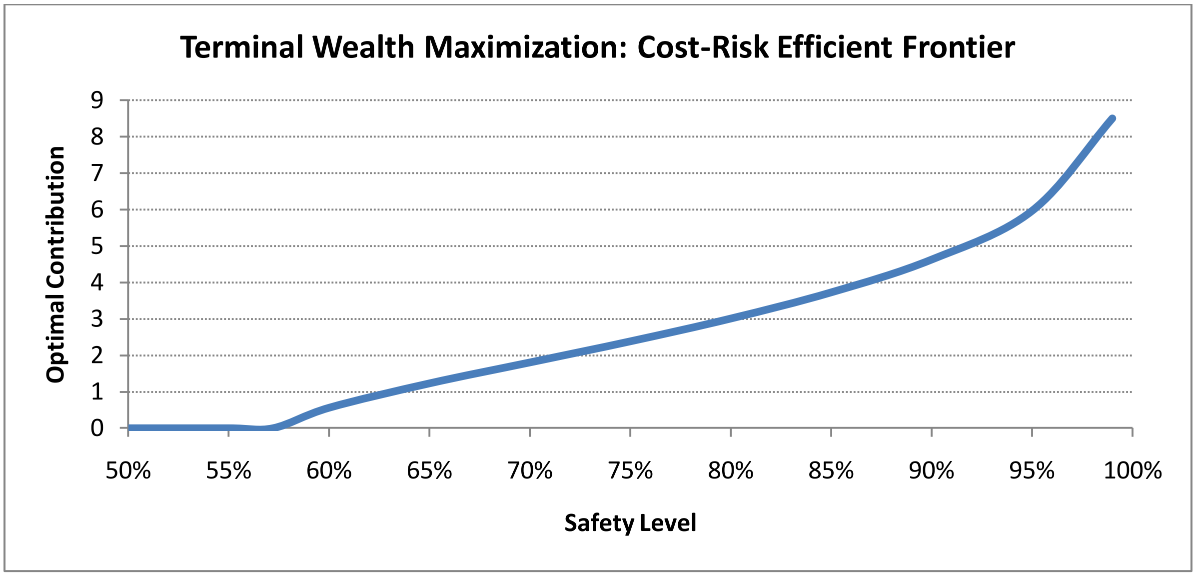


Exhibit 2 presents the optimal contribution amounts for various safety levels (*the cost-risk efficient frontier*). Note that no additional contribution required for safety levels up to 57.2%.

Exhibit 2



Annuity Payment Maximization

The problem presented in this section, for the most part, is similar to the problem from the previous section, but with one major difference. The investor's objective is no longer to have "more money" at the end of the year. Instead, the investor wishes to buy an annuity. For simplicity, we assume that this annuity is a flat-dollar annuity, i.e. it pays the same amount every year. Naturally, the investor would like to have the highest annuity payments possible.

Like in the previous section, the investor has $95 at the present. If the investor wanted to buy an annuity today, what would be the annuity payment? To answer this question, we have to know *the annuity factor* - the price of the annuity that pays $1. The annuity factor is equal to

$$a_x = \sum_{k=1}^{n-x} {}_k p_x v_k$$

where
x – the investor's age;
n – the highest age in the mortality table;
${}_k p_x$ – the probability that the investor survives to age $x+k$;
v_k – the discount factor generated by today's yield curve (more precisely, if i_k is the rate of return for a zero-coupon bond that pays $1 in k years, then $v_k = 1/(1+ i_k)$).

The annuity payment at the present is equal to $95/a_x$. To calculate the future annuity payment, we need the annuity factor one year from now. Assuming for a moment that the yield curve remains the same, this annuity factor is equal to the following.

$$a_{x+1} = \sum_{k=1}^{n-x-1} {}_k p_{x+1} \frac{v_k}{v_1}$$

However, the yield curve does not remain the same. Assuming that annuity factor a_{x+1} calculated using the current yield curve is equal to 10, then the annuity factor at the end of the year can be expressed as $10(1+V)$, where $(1+V)$ is a volatility factor.

The investor's objective is to accumulate enough money to buy an annuity that pays more than \$10 with probability *P*. If \$95 is insufficient to achieve this objective, the investor wishes to make the lowest contribution required at the present to make this result possible.

Assuming that the money is invested in a portfolio of risky assets that generates return *R* and *C* is yet undetermined contribution, the investor's terminal wealth is equal to $(95+C)(1+R)$. The question now is how much the annuity purchased by this amount would pay. To answer this question, we need to know the value of the annuity factor at the end of the year.

Similar to nominal bonds, the volatility of the annuity factor is largely due to the volatility of interest rates. Since the *expected* annuity factor at the end of the year is equal to 10, then the *actual* annuity factor at the end of the year can be expressed as $10(1+V)$, where $(1+V)$ is a volatility factor that has bond-like characteristics. The correlation coefficient between the volatility factor and nominal bond returns is assumed 0.9.

The investor's terminal wealth is equal to $(95+C)(1+R)$, and the annuity factor is equal to $10(1+V)$. Therefore, the annuity payment at the end of the year is equal to $\frac{(95+C)(1+R)}{10(1+V)}$.

The investor's objective (to have enough money to buy an annuity that pays more than \$10 with probability *P*) is expressed in the following equation.

$$\Pr\left(\frac{(95+C)(1+R)}{10(1+V)} > 10\right) = P \qquad (5)$$

Similar to what was done in the previous section, let us re-write equation (5) in terms of stochastic present values as follows.

$$\Pr\left(100\frac{1+V}{1+R} < 95 + C\right) = P \tag{6}$$

It is instructive to compare equations (4) and (6). The only difference between these equations is the presence of volatility factor $(1+V)$ in (6). This is a reflection of the fact that the volatility of interest rates plays no role in the process of terminal wealth maximization, but it plays a significant role in the process of annuity payment maximization. Equations (4) and (6), in particular, show that if interest rates remained the same, then terminal wealth maximization and annuity payment maximization would generate the same optimal portfolios.

Equation (6) shows that $95 + C$ is equal to the *P*th percentile of stochastic present value $100(1+V)/(1+R)$. The investor's objective, therefore, is to minimize this percentile. For every safety level *P*, the lowest required contribution is equal to the difference between the lowest *P*th percentile of $100/(1+R)$ and the existing asset value \$95. *Exhibit 3* shows optimal portfolio allocations for safety levels ranging from 50% to 99%.

Exhibit 3

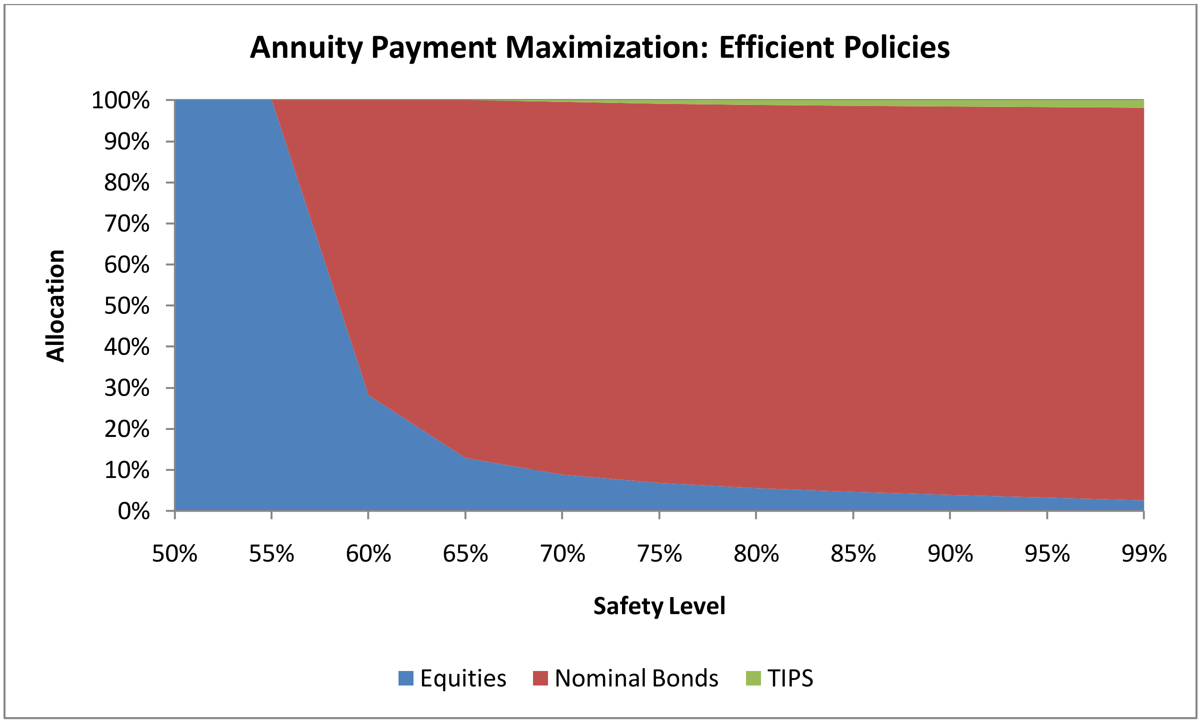
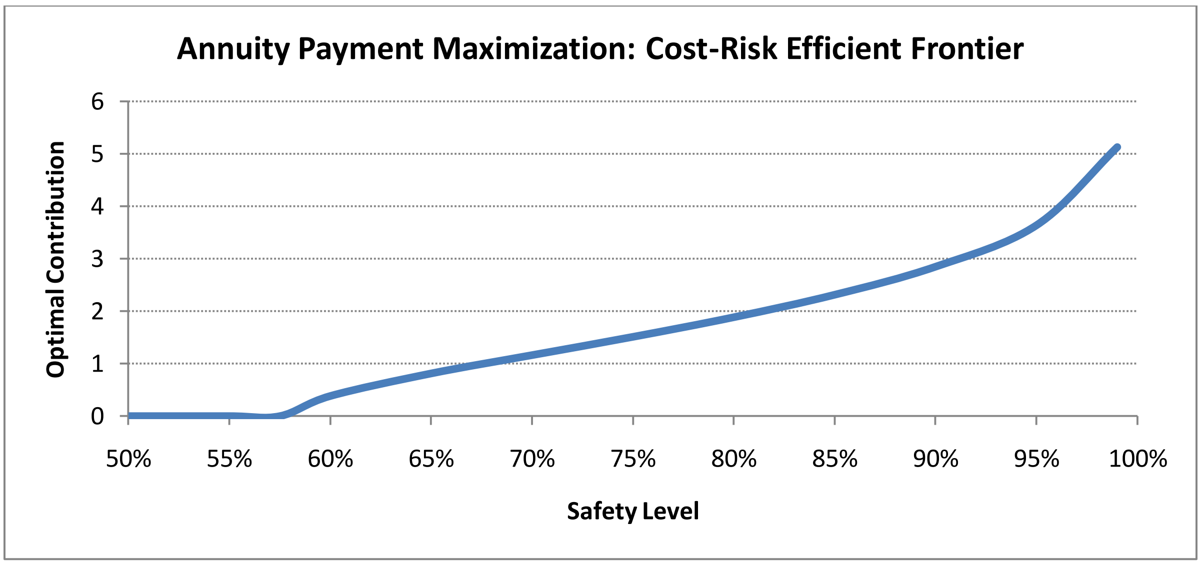


Exhibit 3 stands in a stark contrast to *Exhibit 1*. While equities and TIPS take significant parts in the efficient policies throughout the frontier in *Exhibit 1*, the role of equities is much more limited in *Exhibit 3*, and TIPS play virtually no role. The reason that nominal bonds play such a dominant role in the efficient policies in *Exhibit 3* is nominal bonds represent a very good hedge for annuity prices.

Exhibit 4 presents the optimal contribution amounts for various safety levels (*the cost-risk efficient frontier*). Note that no additional contribution required for safety levels up to 57.5%.

Exhibit 4



Terminal Wealth Maximizers vs. Annuity Payment Maximizers

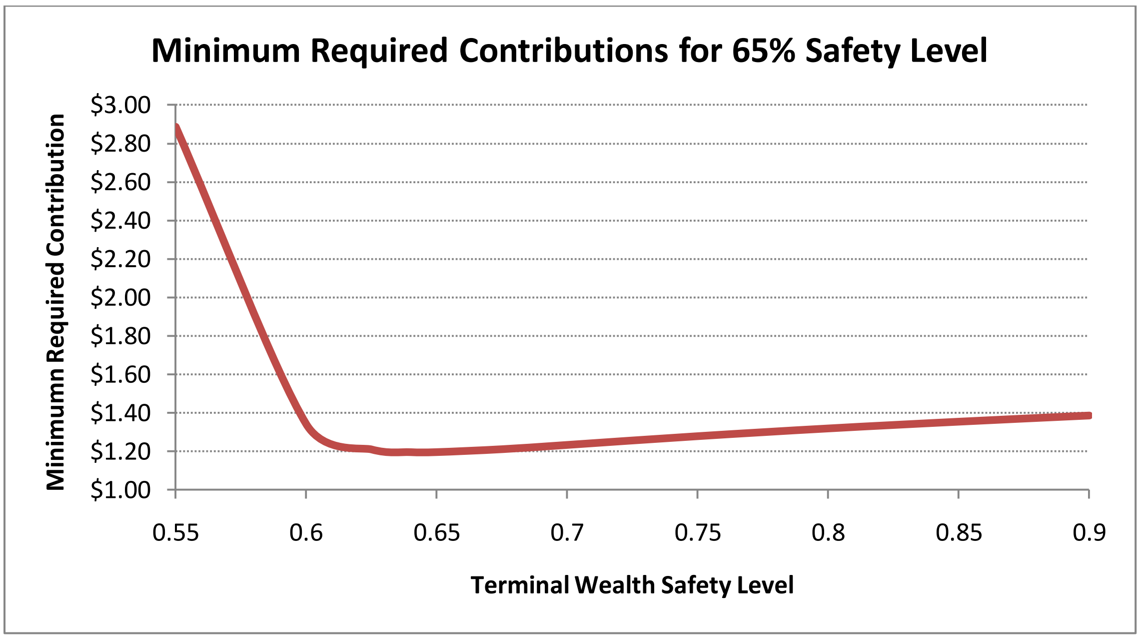
As we see in prior sections, the efficient frontiers generated by terminal wealth and annuity payment optimizations are quite different. Let us take a closer look at what the efficient portfolios generated by terminal wealth optimization can do for an investor that wishes to purchase an annuity.

Let us assume that the investor's objective is to accumulate enough money to buy an annuity that pays more than $10 with probability 65%. Like in the previous section, the investor has $95 at the present. Then, from the data for *Exhibits 3* and *Exhibit 4*, the optimal portfolio contains 13% of equities and 87% of nominal bonds; the minimum required contribution is $0.81.

At the same time, let us assume that the investor utilizes a TDF whose objective is to maximize the investor's terminal wealth, as discussed in prior sections. Therefore, the TDF would utilize one of the "terminal wealth maximizers" presented in *Exhibit 1*. Let us see how well these portfolios would work for the investor.

First, for each terminal wealth maximizer, we calculate the additional contribution amount required for this portfolio to have a 65% chance to have an annuity that pays more than $10. *Exhibit 5* shows the results of these calculations. As we see from *Exhibit 5*, these contributions are much higher than $0.81 contribution required by the "annuity payment maximizer." The lowest among these contributions is equal to $1.20; it is generated by the "terminal wealth maximizer" at safety level 64%. Consequently, *the utilization of terminal wealth maximizers for our annuity investor increases the investor's cost by at least 47%.*

Exhibit 5

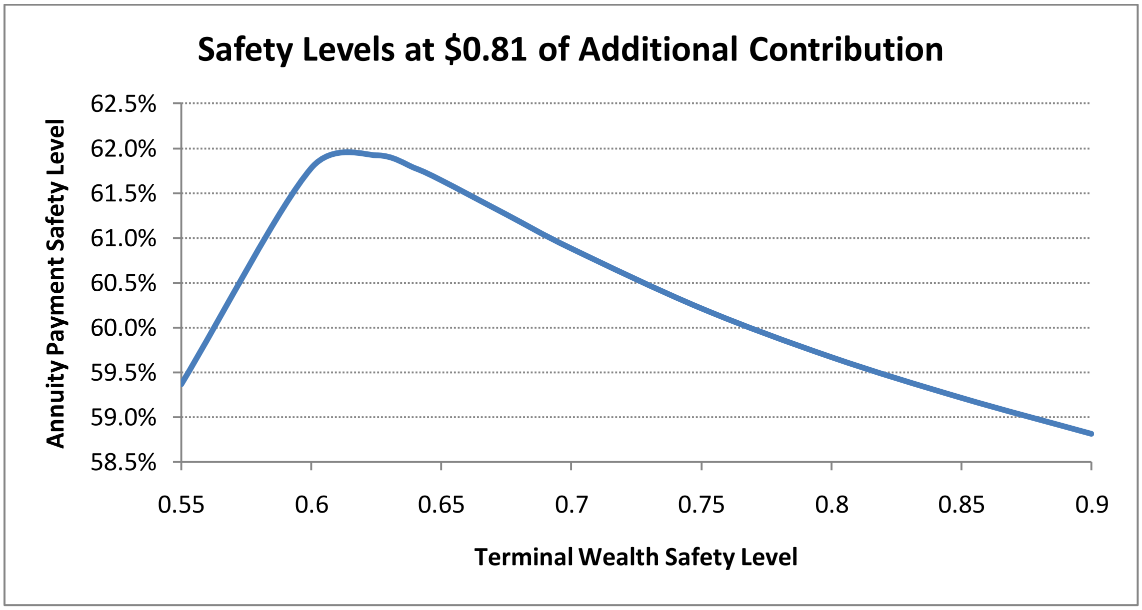


Next, for each terminal wealth maximizer, we calculate the annuity payment "safety level" – the probability that the investor's annuity payments will be greater than $10, given that the additional contribution is equal to the optimal "annuity payment maximizer" contribution $0.81. *Exhibit 6* shows the results of these calculations. As we see from *Exhibit 6*, these safety levels are considerably lower than the safety level delivered by the "annuity payment maximizer" (65%). The highest safety level delivered by a "terminal wealth maximizer" is equal to 61.9%.

These results clearly demonstrate that the utilization of a poorly chosen forward-looking objective may increase both the cost and riskiness of a particular saving and investment program. As was discussed above, a portfolio of 13% equities and 87% bonds requires $0.81 of additional contributions to have a 65% chance that the annuity payment is greater than $10. A similar contribution for a much better diversified portfolio of 24% equities, 44% nominal bonds and

32% TIPS is $1.20, which is 47% more! That is despite the fact that the former portfolio has a lower expected return (5.51% vs. 5.83%) and higher standard deviation of return (5.18% vs. 5.04%), so the latter portfolio has superior properties in the "asset-only" space. As we see, the portfolio analysis in the "asset-only" space is clearly insufficient for the determination of efficient policies in the "asset-commitment" space.

Exhibit 6



The main advantage of the former portfolio is this portfolio provides much better hedge for the interest rate risk, which is one of the primary risks for annuity investors. In contrast, the latter portfolio was designed with complete disregard to how the money would be used, and, therefore, "knows" nothing about the interest rate risk. As a result, the latter portfolio delivers substandard investment solution to the investor despite this portfolio's superior risk/return characteristics.

Conclusion

It is safe to say that many TDFs have not served their investors well lately. Naturally, it would be highly desirable to improve the ways TDFs operate to avoid this kind of debacle in the future. One of the biggest problems TDFs face is the economic theory of life-cycle investing is in its infancy. Most TDFs do not incorporate the standard of living in retirement in their glidepath design methodologies, and this is the main reason for the poor performance. This problem will not be resolved by simply requiring TDFs to manage assets *to* retirement, as opposed to *through* retirement. This "cure" may very well be worse than the "disease."

Everyone knows the more money, the better. Or that is what is what everyone hopes to achieve. Hope, however, cannot replace a disciplined investment strategy that takes into account *all* the needs of an investor before and after retirement. Investors may change service providers, asset allocation, and other aspects of their finances *at* retirement, but they do not stop being investors with their unique objectives and preferences *in* retirement. Ignoring these objectives and preferences *before* retirement in pursuit of vaguely defined "more money" may lead to suboptimal investment solutions and, ultimately, deliver an inferior standard of living in retirement to the investor.

REFERENCES

Markowitz, H. M. [1952]. Portfolio Selection, *Journal of Finance*, Vol. VII, No. 1, March.
Roy, A. D. [1952]. Safety First and the Holding of Assets, *Econometrica*, Vol. 20, July.

APPENDIX

There are three asset classes under consideration: stocks, nominal bonds, and TIPS (Treasury Inflation-Protected Securities). *Exhibit 7* contains the capital market assumptions for these asset classes.

Exhibit 7

Capital Market Assumptions

Risk/Return

	Geometric Return	Arithmetic Return	St Dev of Return
Equities	8.00%	9.15%	16.00%
Nominal Bonds	5.00%	5.12%	5.00%
TIPS	4.50%	4.67%	6.00%

Asset Class Correlations

	Equities	Nominal Bonds	TIPS
Equities	1	0.2	-0.1
Nominal Bonds	0.2	1	0
TIPS	-0.1	0	1

Additional assumptions regarding portfolio returns and the volatility of annuity prices are as follows.

1. Investment returns of all portfolios under consideration have lognormal distributions.
2. Volatility factor $(1+V)$ has lognormal distribution.
3. Volatility factor $(1+V)$ has median 1 and standard deviation 5.00%.
4. Volatility factor $(1+V)$ has correlation with nominal bonds 0.9, equities 0.2, and TIPS 0.0.

Endnotes

[1] See Markowitz [1952].
[2] To be precise, the classic mean-variance methodology deals with portfolio return R_P rather than future value $1+R_P$. This distinction between R_P and $1+R_P$, however, is immaterial - the addition of constant 1 doesn't affect the methodology in any way.
[3] See Roy [1952].
[4] To be precise, the classic Safety-First approach deals with portfolio return R_P rather than future value $1+R_P$. The addition of constant 1 doesn't affect the approach in any way.

Important Information