The Case for Stochastic Present Values

Dimitry Mindlin, ASA, MAAA, PhD
President
CDI Advisors LLC
dmindlin@cdiadvisors.com

July 2009

The defined benefits (DB) system is approaching a turning point. While the advantages of DB plans are well-known, the number of DB plans has been declining since the mid-80s. The corporate side of the system appears to be in the process of self-termination, and the battle for the survival of the system has largely shifted to the public plans arena.

The heart of the debate is in the area of reporting requirements and decision-making tools for DB plans. In recent years, this area has been the subject of extensive discussions. For public plans, these discussions have been particularly intense. Even relatively minor changes to the existing paradigm and its key component – the concept of pension "liability" – have ignited raging debates. The intensity of these debates notwithstanding, little common ground has been found between the major camps in the debate.

This paper presents a methodology that may represent such common ground. The methodology incorporates the elements of pension actuarial science and financial economics and adds another dimension to the existing paradigm – risk measurements. This author is optimistic that this methodology will eventually become an integral part of the mainstream approach to the management of retirement commitments.

Why the Existing Paradigm Needs Change

The roots of the current pension "conundrum" are as follows. A typical plan endeavors to fund its financial commitments by virtue of investing large portions of its wealth in risky assets. The plan normally has a policy portfolio and rebalances the assets on a regular basis. According to the conventional approach that has been around for decades, the plan's actuary utilizes the expected return for the policy portfolio (or something close to it) as the discount rate to calculate, among other things, present values of pension commitments and required contributions to the plan.[1]

The main problem with this approach is the *riskiness* of the policy portfolio plays no role in the present value calculations. At the same time, the *risk premium* assumption embedded in the discount rate is one of the key factors in the determination of these present values. Therefore, the conventional approach employs risk premium without risk, which makes little sense. One of the unintended consequences of this approach is the riskier the policy portfolio, the lower the pension "liabilities." Indeed, a riskier portfolio usually has higher expected return, which is equal to the discount rate. A higher discount rate means lower present values. The relationship "higher risk means lower liability" is illogical.

Another deficiency of the conventional approach is the fact that it essentially produces *median values only* (not that there is anything wrong with median values). Effectively, the plan's benefit design and contribution decisions are based on certain asset values that have only a 50-percent chance of being sufficient to fund the promised benefits. Median values are obviously important for the decision-making process, but focusing solely on median values and ignoring other risk measurements is inadequate if the goal is to make a well-informed decision.

Yet another deficiency of the conventional approach is the fact that policy portfolio decisions are artificially separated from contribution and benefit design decisions. Pension plans commonly optimize their policy portfolios based on the risk analysis in the "asset-only" space (with the help of the plans' investment consultants), while contribution and plan design decisions are made based on riskless median values (with the help of the plans' actuaries). Those investment consultants and actuaries do not talk to each other on a regular basis, to put it mildly. A unified internally consistent decision-making framework for all aspects of pension plan management appears to be nonexistent at this moment.

Overall, the existing paradigm contains a lot of room for improvement. The criticism the existing paradigm have endured in recent years has merit.

Why "Marked-to-Market" Accounting Will Not Work

In order to fix the deficiencies of the existing paradigm, several economists and actuaries have proposed to replace the existing paradigm with "marked-to-market" pension accounting. In essence, the proposal is to replace portfolio return based present values by "market values" that have no connection to the plan's policy portfolio. The proponents of the concept of "market value of liability" (MVL) appear to believe that "marked-to-market" pension accounting is the solution to the problem of prudent pension plan management.

The essence of "marked-to-market" pension accounting is the following. It is assumed that there exists a portfolio of tradable assets (usually bonds or bond-like instruments) that matches a termination-like benefit stream (i.e. this stream excludes future accruals and salary growth).[2] For public plans, in particular, this portfolio would contain Treasury bonds only (not that there is anything wrong with Treasury bonds). MVL is defined as the market value of this hypothetical portfolio.

There are a few major problems with this approach as related to on-going DB plans. These problems have been thoroughly discussed in several publications from the practical and scientific perspectives.[3] Consequently, this paper presents just a brief overview of these problems.

The biggest problem with "marked-to-market" pension accounting is its detachment from the reality in which on-going plans operate. This detachment is exemplified in the unrealistic assumptions the approach utilizes. Some of these assumptions are based on misconceptions (e.g. "the objective is to *price* the plan's financial commitment," "the matching bond portfolio always exists"). Some other assumptions are plainly unreasonable (e.g. "benefit payments are perfectly known," "the yield curve can be extended indefinitely," "future accruals and salary growth are irrelevant").

The deficiencies of these assumptions are especially evident for public funds. In reality, the objective is to *fund* the plan's commitment, not to *price* it; matching bond portfolios rarely exist; benefit payments are contingent cash flows of uncertain timing and magnitude; the yield curve cannot be extended indefinitely; future accruals and salary growth are highly relevant to prudent pension plan management.

Other problems with "marked-to-market" pension accounting include, but are not limited to, the following. As a measurement of a plan's funded status, the MVL-based funded ratio depends greatly on the twists and turns of the yield curve and may create misleading notions regarding the plan's financial health and the quality of the plan's management. The approach completely conceals the riskiness of the plan's existing portfolio and, therefore, may be exceedingly deceptive. Most importantly, numerous decision makers for public plans (including a number of elected officials) have made it clear that the concept of MVL is not helpful to their goals.[4]

Furthermore, the scientific foundation of the approach is exceedingly shaky at best. There is no sound principle of financial economics that requires valuing non-tradable, non-transferable pension benefits similar to a (non-existent) matching portfolio of tradable assets. The approach's disregard to one of the cornerstones of finance – risk premium – has no basis in either actuarial science or financial economics.[5]

To recap, "marked-to-market" pension accounting is based on unreasonable assumptions, not particularly helpful to on-going plans in general and public plans in particular, may produce deceiving results, and has a dubious scientific foundation.

Otherwise, it is a fine academic exercise.

Present Values and Risky Assets

Clearly, both conventional and "marked-to-market" approaches have substantial problems. At the heart of this conundrum lies the problem of measuring pension commitments and its key aspect – the role of risk in the calculations of present values. Conventional practices utilize risk premium without risk; the "marked-to-market" approach ignores risk premium completely;

neither of these approaches deals with risk; and these two approaches are the main alternatives the stakeholders of pension plans currently have. The need for a better approach is clear.

The main objective is to create an effective internally consistent framework for the management of the three pillars of running a pension plan: benefit package, asset allocation and contribution policies. The requirement of internal consistency for the framework is especially important. In particular, since the framework must deal with the plan's actual policy portfolio, it must also deal with uncertain portfolio returns. The volatilities of benefit payments should also be a part of the framework. One of the most important factors that connect portfolio returns and the volatility of benefit payments is inflation.

Let us go back to basics. A conventional pension plan makes financial commitments that extend for decades into the future. At the same time, the decisions regarding the plan's benefits, contributions, and the structure of the policy portfolio must be made *at the present*. In order to make prudent decisions, the decision makers may need to compare the "asset side" (future contributions and existing assets) and the "commitment side" (future benefit payments). Therefore, the decision makers need *present values* of future contributions and benefit payments, as related to the existing policy portfolio and other portfolios under consideration.

The classic principle of time value of money states that, in order to calculate a present value of a series of future payments, the payments must be discounted by the returns generated by the investor's portfolio. The present value is equal to the asset value required at the present to fund the payments. If the portfolio returns are uncertain, then the present value – the assets required to fund the payments – is uncertain as well. As a result, *the present value of pension benefit payments funded by a portfolio of risky assets is uncertain*. This is a reflection of the conventional wisdom well-known to pension practitioners: the cost of running a pension plan is uncertain.

The Key Assumption: Deterministic or Stochastic?

Yet, conventional actuarial and accounting reports do not recognize the uncertainty of present values, as these reports contain perfectly certain figures only (not that there is anything wrong with perfectly certain figures). Both the supporters of the existing paradigm and the promoters of "marked-to-market" pension accounting, as diverse as they are, utilize the following obscure but vital assumption: the present value of a financial commitment is a *deterministic object* – a number.

This assumption necessitates that benefit payments and the discounting procedure utilized in the calculation of a particular present value must be assumed deterministic as well. As an easy consequence of these assumptions, it is assumed that the discounting procedure must utilize either a single discount rate or several deterministic discount rates.[6] Since discount rates are essentially rates of return, it is effectively assumed that these rates of return are deterministic rather than stochastic.

In case of public plans, in which benefits are virtually guaranteed, these deterministic rates of return are essentially risk-free rates. The only financial instrument that delivers risk-free rates is U.S. Treasury bonds. That is the reasoning behind the call to calculate "liabilities" for public plan by virtue of discounting benefit payments by the yields of U.S. Treasury bonds of various maturities. Note that this reasoning is not based on any sound scientific principle; rather, it is based on the desire to have deterministic present values of pension commitments.

It is instructive to look into the roots of the assumption that discount rates and present values must be deterministic. This assumption was adopted decades ago when risk management and risk models were in their infancy and computing power was prohibitively expensive and inaccessible to most practitioners. It was unreasonable to inquire about the volatilities of discount rates and corresponding present values in the era when actuarial clerks had to look up the required commutation functions in thick manuals by hand. Computational convenience was the primary reason for the acceptance of the assumption that discount rates and present values must be deterministic.

Many practitioners, however, have been aware that this assumption is overly restrictive. For example, a pension mathematics textbook published in 1977 contained the following:

> "*Although it is common to find [the interest] assumption set at a constant compound rate, this is a special case of the more general assumption that would allow the rate of interest to vary over time.*"[7]

A monograph written in 1989 discussed the fundamental concepts of actuarial science and had a section named "The Rate of Interest as a Random Variable," which contained the following:

> "*Of great importance to the actuary is the rate of interest (or more generally, the rate of investment return).[…] Historically, actuaries have used deterministic models in their treatment of the time value of money, but not because they were unaware of interest rate variation. […] The difficulty has not been the lack of concern, but rather a lack of knowledge as to the complexities of interest rate variation.[…] The development of computers has opened up a range of techniques whereby interest rate variation can be modeled. It appears that* ***this is a direction in which actuarial interest and knowledge may be expected to grow***."[8]

Stochastic present values are well-known in finance. One of the most prominent examples of stochastic present values is the famous Black-Scholes formula. In its simplest version, the formula presents the value of a European call, which is equal to the *expected present value* of the excess of the asset price at expiry over the exercise price.

Stochastic present values are also well-represented in actuarial and financial literature. For example, Chapter 12 in Kellison [2009] (called “Stochastic Approaches to Interest”) explicitly recognizes the stochastic nature of present values and presents some initial facts about their volatilities. Chapters 4 and 5 in Bowers [1997] deal with stochastic present values as well. (It should be noted that Kellison [2009] and Bowers [1997] are classic textbooks that are still on the suggested reading list for the actuarial curriculum.) A number of academic researchers have analyzed the properties of stochastic present values.[9] Overall, the concept of stochastic present values is well-established and needs no introduction.

As we see from this discussion, the assumption that present values must be deterministic is based on little more than computational convenience. This assumption is a major problem in both the existing paradigm and “marked-to-market” pension accounting. One of the main goals of this paper is to demonstrate that the adoption of a much more realistic assumption – that *present values of pension commitments are stochastic* – has numerous advantages.

Why Stochastic Present Values

The biggest advantage of the concept of stochastic present values is that *this concept provides valuable tools for the comprehensive cost-risk management of retirement commitments*. The concept is instrumental to answering vital questions regarding the financial health of retirement programs.

Let us look, for comparison, at conventional and “marked-to-market” present values for a typical public plan and the information these values contain. A conventional expected-return-based “liability” (a.k.a. “accrued liability”) is the scheduled asset value the plan should have on the valuation date according to the actuarial assumptions utilized by the plan.[10] The accrued liability based funded ratio shows whether the plan is behind or ahead of the schedule. The problem with this measurement is it ignores the riskiness of the schedule.

The “marked-to-market” present value is an estimate of the cost of plan termination. The cost of termination based funded ratio shows whether the plan has enough money to terminate. The problem with this measurement is it ignores the cost of running the plan as well as a multitude of future risks.

Thus, the conventional approach ignores risk, and “marked-to-market” approach ignores cost (and risk). Neither of these approaches produces measurements that reflect the fact that the plan’s assets are invested in risky financial instruments that do not guarantee that the existing assets and currently budgeted contributions will be sufficient to make the promised benefit payments.

Let us try to ask better questions. A pension plan has made a commitment to make certain benefit payments to the plan participants. The primary objective is to fund the commitment – to ensure that the money is readily available every time a payment is due. Since the plan’s assets are normally invested in risky financial instruments, there is a possibility that the existing assets and currently budgeted contributions will be insufficient to make the promised benefit payments. The plan’s stakeholders should be keenly interested in the quantification of this possibility.

In order to assess the riskiness of the plan, its stakeholders may want to ask the following vital question. What is the probability of success – the probability that the existing assets and currently budgeted contributions will be sufficient to make the promised benefit payments? It should be emphasized that this question involves the cost of funding the commitment, the volatility of benefit payments and the riskiness of the plan’s policy portfolio.

Let us establish the relationship between the probability of success and stochastic present values. Let $B_1,\ldots,B_N$ be a series of end-of-year benefit payments, $C_1,\ldots,C_N$ be a series of contributions, $R_1,\ldots,R_N$ be a series of portfolio returns.[11] *Required Assets* (*RA*) is defined as the stochastic present value of the benefit payments less contributions.

$$RA=\sum_{k=1}^{N}\frac{B_k-C_k}{(1+R_1)\ldots(1+R_k)} \tag{1}$$

Random variable *RA* is the asset value required at the present to fund the commitment.

Let A_k be the market value of the plan assets at the beginning of year k $(1\le k\le N+1)$. The asset value after k years is

$$A_{k+1}=A_k(1+R_k)-(B_k-C_k) \tag{2}$$

Note that while A_1 is perfectly known, asset values A_k are uncertain for $k\ge 2$.

The funding objective is to make all payments $B_1, \ldots, B_N$, and, therefore, to achieve *non-negative* asset value A_{N+1} at the beginning of year N+1. Proposition 1 in the Appendix shows that the probability of this event is equal to:

$$\Pr\left(A_{N+1} \geq 0\right) = \Pr\left(RA \leq A_1\right) \tag{3}$$

Formula (3) demonstrates that the likelihood of funding the commitment depends on the position of the current asset value A_1 within the distribution of *RA*. In particular, if A_1 is equal to the *p*th percentile of *RA*, then the probability of funding the commitment is equal to *p*. This conclusion is quite intuitive – the commitment is funded if the current assets are greater than or equal to the assets required for funding. As we see from (3), stochastic present value *RA* is instrumental in the determination of the probability of success in the funding problem.

Stochastic present value *RA* defined in (1) is just an example of a variety of stochastic present values that arise from various funding problems. The advantage these stochastic present values deliver is the vital questions about the cost and risk of funding a particular commitment can usually be expressed in terms of corresponding *RA*s that subsequently can be analyzed using conventional analytical methods.

Various measurements of stochastic present values – the mean, standard deviation, percentiles, etc. – "know" something about the magnitude and volatility of the commitment as well as the risk/return properties of the policy portfolio and its ability to fund the commitment. The utilization of these measurements makes possible the analysis of the multitude of risks retirement plans face (e.g. capital markets, inflation, interest rates, longevity, etc.) in one all-inclusive framework. Moreover, as we see in the next section, these measurements may provide a quantitative framework for the optimization of asset allocation, contribution and benefit design decisions for various retirement programs.

A Case Study

This section contains initial applications of the concept of stochastic present values for a simplified retirement. We assume that the pension commitment is a stream of inflation-adjusted benefit payments.[12] Proposition 2 in the Appendix presents the formulas for the calculations of the mean and variance for the stream.

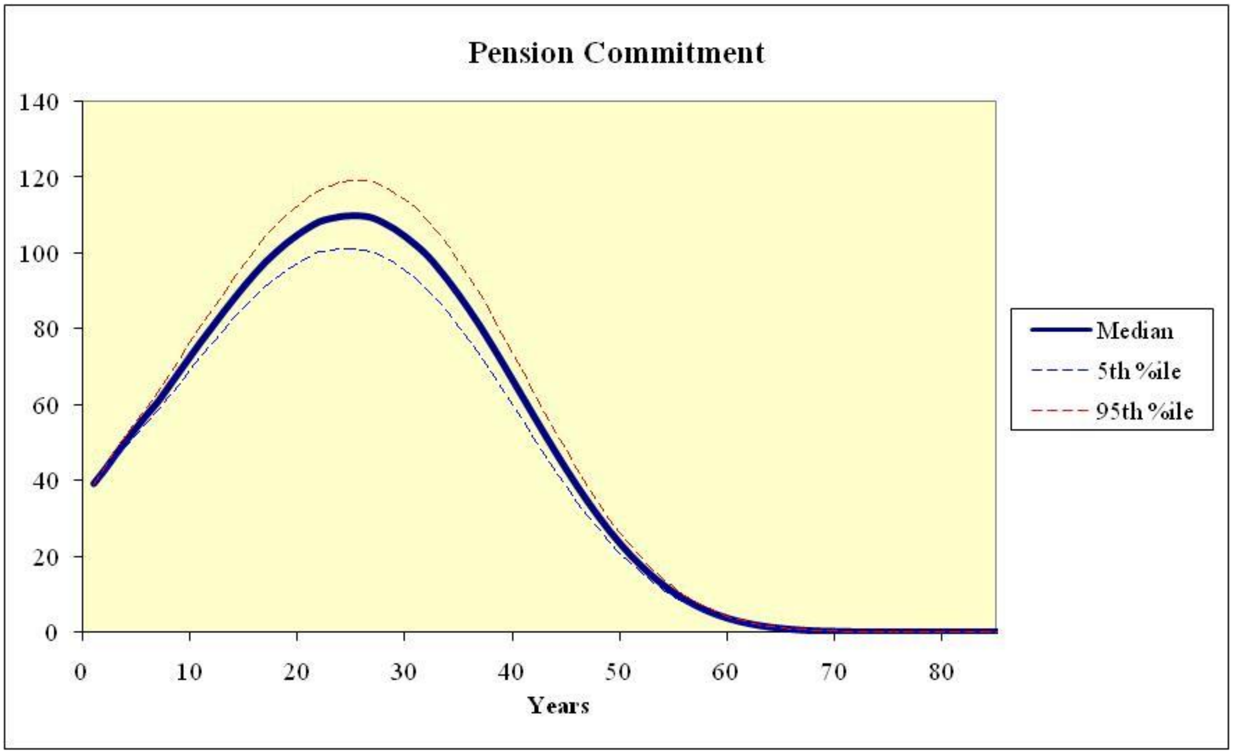
We assume that the following conditions hold for the plan.

1. The market value of assets is $1,000.
2. The plan's policy portfolio is 45% U.S. equity, 15% international equity, and 40% bonds.

3. The accrued liability (the present value of the median benefit payments discounted at 7%) is $1,000.
4. The "marled-to-market liability (the present value of the median benefit payments discounted at 5%) is $1,375.

Exhibit 1 shows the median payments as well as the 5th and 95th percentiles of the stream.

Exhibit 1



Let $B_1, \ldots, B_N$ be a series of end-of-year benefit payments, $R_1, \ldots, R_N$ be a series of portfolio returns. Stochastic present value *RA* for this plan is defined as follows.

$$RA = \sum_{k=1}^{N} \frac{B_k}{(1+R_1)\ldots(1+R_k)}$$

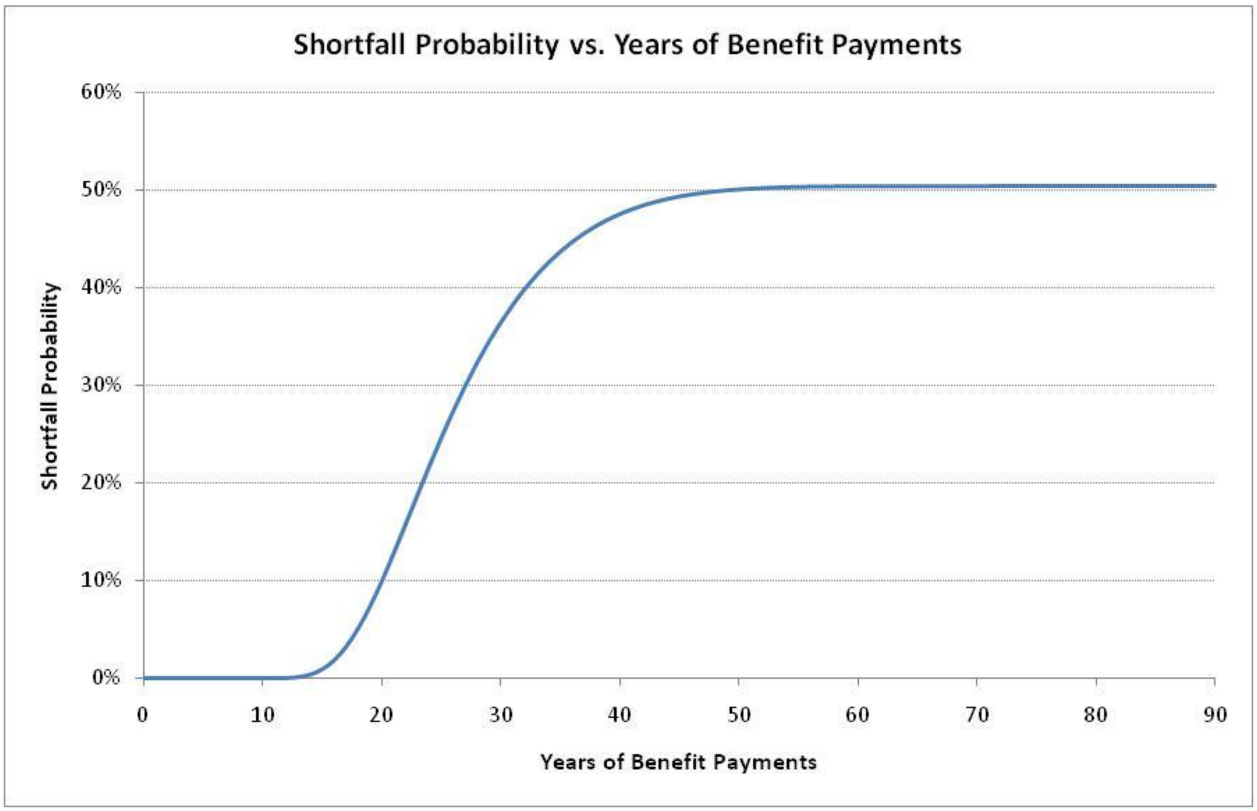
Exhibit 2 contains initial results of the cost-risk analysis of the plan. *Exhibit 3* contains shortfall probabilities for all years of benefit payments.

The risk measurements presented in this section do not belong to any mandatory financial report. The usefulness of these measurements comes from the fact that they are instrumental for determining optimal benefit design, asset allocation and contribution policies. The next section provides examples of the use of these measurements as well as an outline for future developments.

Exhibit 2

Cost-Risk Analysis	
Market Value of Assets	1,000
***RA* Mean**	1,055
***RA* Median**	1,002
***RA* St Dev**	321
Shortfall Probability	50%
Shortfall Size	295
Shortfall St Dev	270

Exhibit 3



Further Developments

This section just sketches out a few areas of further developments in which the concept of stochastic present values should be valuable. The section does not attempt to present a comprehensive treatment of the applications of stochastic present values.

Asset Allocation I: The Plan Sponsor's Goal

The asset allocation decision is one of the most important responsibilities of the plan management. What is the plan sponsor's goal in the area of asset allocation? This question is, obviously, not new. Fischer Black explicitly addressed this and related questions in one of his last papers.

> "*How should defined benefit pension plan sponsors choose an investment strategy for their pension funds? How should they allocate the assets in their funds among broad asset classes such as stocks, bonds and real estate? How should they diversify within and across asset classes?*"[13]

Fischer Black's conclusion was *" a plan sponsor may want to choose an investment strategy to* ***minimize the present value of future contributions*** *to the plan.*"[14] While this recommendation is somewhat controversial, the concept of stochastic present value allows a sensible interpretation that leads to a workable solution.

If $B_1, \ldots, B_N$ are a series of benefit payments, $R_1, \ldots, R_N$ are a series of portfolio returns, and *A* is the existing asset value, then the additional asset value *RA* required to fund the commitment can be defined as the stochastic present value of the benefit payments less the existing asset value.

$$RA = -A + \sum_{k=1}^{N} \frac{B_k}{(1+R_1)\ldots(1+R_k)} \tag{4}$$

This *RA* can be viewed as the present value of future contributions to the plan. However, it is unclear how to minimize *RA*, since it is a random variable, not a conventional function.

A valuable guidance comes from the classic approach to portfolio valuation and optimization developed by Harry Markowitz.[15] For a given policy portfolio *P*, we denote E_P and S_P the expected *portfolio return* and standard deviation of *portfolio return* correspondingly. Expected return E_P can be viewed as a "reward" for risk-taking. Volatility of return S_P can be viewed as a "penalty" for risk-taking. For a given risk aversion parameter $\tau \geq 0$, the utility ("usefulness") *U* of portfolio *P* is defined as the difference between "reward" E_P and "penalty" τS_P.

$$U = E_P - \tau S_P \tag{5}$$

Since it is desirable to have *high* returns, portfolio utility *U* is subsequently *maximized* for every risk aversion parameter $\tau \geq 0$. The result of this optimization is a set of efficient policy portfolios that comprise the classic *mean-variance efficient frontier*. Even though this approach has well-known limitations, it is still very popular among many institutional and individual investors. As far as retirement plans are concerned, the biggest limitation of this approach is the financial commitment of a particular plan plays no role in the optimization procedure.

Let us take a similar approach to the valuation and optimization of retirement commitments. For a given policy portfolio *P*, we denote E_P and S_P as the mean and standard deviation of *RA* correspondingly.[16] (Proposition 2 in the Appendix contains the calculations of E_P and S_P for a simplified *RA*.) For a given risk aversion parameter $\tau \geq 0$, let us define asset value *A* as the sum of "reward" E_P and "penalty" τS_P.

$$A = E_P + \tau S_P \tag{6}$$

Note that asset value *A* is a risk measurement of the present value of future contributions that can be optimized. Since it is desirable to have *low* cost, asset value *A* is subsequently *minimized* for every risk aversion parameter $\tau \geq 0$. The result of this optimization is a set of efficient policy portfolios that comprise the *cost-risk efficient frontier*.[17]

It is important to note that the characteristics of both the policy portfolio and the plan's financial commitment play a direct role in the optimization procedure. Thus, the cost-risk optimization generates much more comprehensive solutions to the funding problem than the mean-variance optimization.[18]

Asset Allocation II: Which Policy Do You Mean?

An alternative view on the role of the policy portfolio can be expressed in terms of the safety of promised benefits. It is reasonable to believe that the role of the policy portfolio is to maximize the likelihood that the promised benefits will be delivered. As Peter Bernstein put it,

> "*the policy is to provide the investor with the highest probability of being able to pay for the groceries when the time comes*."[19]

It is informative to look at a similar problem in the "asset-only" space. Andrew Roy introduced the "Safety-First" approach to portfolio optimization in 1952.[20]

For a given portfolio *P*, let threshold return *r* be equal to the p^{th} percentile of portfolio return R_P:

$$p = \Pr(R_P < r) \qquad (7)$$

Given threshold return *r*, the objective is to minimize probability *p*. In other words, we want to find policy portfolio *P* that *minimizes* the likelihood that its return is below some potentially "disastrous" level.

Similarly, for a given portfolio *P*, let threshold cost *C* be equal to the p^{th} percentile of *RA*.

$$p = \Pr(RA < C) \qquad (8)$$

Given threshold cost *C*, the objective is to maximize probability *p*. In other words, we want to find policy portfolio *P* that *maximizes* the likelihood that the cost of funding is below some potentially "disastrous" level.

As we see from this and previous examples, the concept of stochastic present values makes possible the integration of the "commitment" side of the equation into policy portfolio optimization.[21]

Asset-Liability Valuations of DB Plans

The concept of stochastic present values allows us to evaluate DB plans from an entirely different perspective. Here is a small sample of questions regarding the cost and riskiness of a particular retirement plan the stakeholders of the plan may want to ask.

1. What is the probability of success – the probability that the existing assets, contribution and asset allocation policies will be sufficient to make the promised benefit payments?
2. What is the policy portfolio that maximizes the probability of success given the currently budgeted contributions?
3. If this probability is insufficient for the plan's stakeholders, what is the minimal increase in contributions that provides the desired probability?
4. What is the policy portfolio that delivers the desired probability of success?

The answers to these and related questions are vital to the plan's risk management. A valuation methodology that deals with cost-risk management a retirement program, including the plan's policy portfolio, is called an "asset-liability valuation" (ALV). While a conventional actuarial valuation emphasizes compliance issues, an ALV emphasizes risk management. Note that the term "liability" in ALV does not specify any particular accounting figure. It merely indicates that both "asset" and "liability" sides are incorporated into the valuation.

Asset-Liability Valuations of DC Plans

The concept of stochastic present values allows us to evaluate the sustainable standard of living in retirement provided by a particular DC plan. A typical DC plan currently has, at best, a vague idea regarding the standard of living its participants should expect in retirement. As a result, the design of a typical DC plan is likely to have a lot of room for improvement. It appears that this is a direction in which the need for actuarial expertise may be expected to grow.

The Time Diversification Problem

The time diversification problem is one of the most fascinating unresolved problems in finance. In its simplest form, the problem is to determine whether rational investors should expect to reduce the riskiness of their portfolios as the investors get older. The intense discussions of this problem have been going on for several decades. Some of the finest academics and practitioners in finance have contributed to the discussion. Yet, the problem remains unresolved.

The concept of stochastic present values provides the analytical tools that may rationalize the evolution of portfolios through an investor's lifetime. Initial applications of the concept have demonstrated that a rational investor should modify her portfolio for a simplified retirement plan.[22] This author expects the analysis of stochastic present values to play a major role in further research.

Matching Assets

The existence of the matching asset (aka a reference portfolio) for any pension plan is one of the cornerstones of the "financial economics" approach to pensions. While perfectly matching assets rarely exist, the concept of stochastic present values provides the analytical tools for the analysis of the "mismatch."

Any "buy-and-hold" asset (e.g. a bond portfolio or an annuity) provides a future cash flow in exchange for a lump sum L at the present, while the rest of the assets are invested in the plan's policy portfolio. If $B_1,\dots,B_N$ are a series of benefit payments, $R_1,\dots,R_N$ are a series of returns generated by the policy portfolio, $F_1,\dots,F_N$ is a cash flow generated by a "buy-and-hold" asset, and lump sum L is the price paid for the "buy-and-hold" asset, then corresponding RA is defined as

$$RA = L + \sum_{k=1}^{N} \frac{B_k - F_k}{(1+R_1)\dots(1+R_k)} \tag{9}$$

If cash flows $B_1, \dots, B_N$ and $F_1, \dots, F_N$ are identical in terms of timing, magnitude and likelihood (i.e. $B_k = F_k$ for all k), then the volatility of RA is equal to zero: $RA = L$. Any "mismatch" results in a nontrivial RA. Subsequent analysis of RA may reveal the advantages of investing in this "buy-and-hold" asset, namely the ability of the remaining assets to fund remaining payments $B_k - F_k$, $1 \le k \le N$. Thus, stochastic present values may be helpful to the "financial economics" approach to retirement programs.

Where Do We Go From Here?

As far as public pension plans are concerned, there have been two major camps in this debate: "traditionalists" and "financial economists." "Financial economists" have advocated the "marked-to-market" approach, while "traditionalists" have criticized the deficiencies of this approach.

Both camps have serious problems. "Financial economists" emphasize pension accounting and promote a deeply flawed approach to pension plan management. "Traditionalists" offer little beyond their criticism of "marked-to-market" pension accounting, even though the traditional approach has its own shortcomings.

This author believes that the focus of this debate has been somewhat misplaced. Instead of focusing on actual practices of pension plans in the areas of asset allocation, contributions and plan design, most of the debate's participants focus on the reflections of these practices in accounting statements. Yet, the actual practices of pension plans require utmost attention and need considerable improvements. This task represents a major challenge. Compared to this challenge, the problems with pension accounting are relatively easy to resolve.

A "marked-to-market" measurement of the benefits accrued up-to-date, as a key component of the solvency disclosure, may not be problematic if the purpose of the disclosure is to inform, not to regulate. In other words, the solvency disclosure may have a place in pension accounting for public plans *as long as the plans are not required to take actions based on this disclosure* and this figure is clearly labeled as "the cost of termination estimate."[23] A sensible way to resolve this quandary may be to disclose the stream of benefit payments that excludes future accruals and salary growth. Then, any stakeholder of a particular pension plan would be able to discount this stream in any way and "mark" this stream to any "market" the stakeholder wishes.

The intensity of this debate notwithstanding, the role of "marked-to-market" accounting will be eventually sorted out. When this debate is finally over, pension practitioners still have to make asset allocation, contribution and benefit design decisions, for which accounting is of little help. Most importantly, pension practitioners have to deal with the following facts.

- Pension commitments are funded by virtue of investing in risky assets.
- Risk premium is an indispensable part of the world of risky assets.
- Risk premium is inseparable from a multitude of risks.
- The cost of running a pension plan is uncertain.

Cost-risk measurements have to become an integral part of pension plans' decision-making process. Who will provide these risk measurements? How will a standard cost-risk measurement reporting package look like? These questions do not allow simple answers and, most likely, require significant efforts in the areas of actuarial science, financial economics, education and communications.

However, one aspect of this challenge is perfectly clear to this author: the analysis of stochastic present values will play a key part in the development of cost-risk measurements. The first step should be to recognize the stochastic nature of present values. As soon as this recognition takes place, we can start the process of incorporating the cost and riskiness of funding financial commitments into the decision-making process of retirement plans. Subsequently, actuarial science, financial economics, other scientific disciplines and common sense will guide the development of the most efficient asset allocation, contribution and benefit design policies retirement plans need.

APPENDIX

Proposition 1. If random variable RA is defined as

$$RA = \sum_{k=1}^{N} \frac{B_k - C_k}{(1+R_1)\ldots(1+R_k)}$$

where $B_1,\ldots,B_N$ be a series of end-of-year benefit payments, $C_1,\ldots,C_N$ be a series of contributions, $R_1,\ldots,R_N$ be a series of portfolio returns, and A_k is the market value of the plan assets at the beginning of year k $(1 \le k \le N+1)$, then

$$\Pr(A_{N+1} \ge 0) = \Pr(RA \le A_1)$$

Proof.

$$\Pr(A_{N+1} \ge 0) = \Pr\left(A_N(1+R_N) - (B_N - C_N) \ge 0\right) = \Pr\left(A_N \ge \frac{B_N - C_N}{1+R_N}\right) =$$

$$= \Pr\left(A_{N-1}(1+R_{N-1}) - (B_{N-1} - C_{N-1}) \ge \frac{B_N - C_N}{1+R_N}\right) =$$

$$= \Pr\left(A_{N-1} \ge \frac{B_N - C_N}{(1+R_{N-1})(1+R_N)} + \frac{B_{N-1} - C_{N-1}}{(1+R_{N-1})}\right) = \ldots =$$

$$= \Pr\left(A_1 \ge \frac{B_N - C_N}{(1+R_1)\ldots(1+R_N)} + \frac{B_{N-1} - C_{N-1}}{(1+R_1)\ldots(1+R_{N-1})} + \ldots + \frac{B_1 - C_1}{(1+R_1)}\right) =$$

$$= \Pr(A_1 \ge RA)$$

Proposition 2. Let $\{B_k\}$ be a series of fixed numbers, $\{R_k\}$ be independent identically distributed returns generated by a policy portfolio, and $\{V_k\}$ be independent identically distributed volatilities. Random variable RA is defined as

$$RA = \sum_{k=1}^{n} B_k \frac{(1+V_1)\ldots(1+V_k)}{(1+R_1)\ldots(1+R_k)}$$

Let's define the following values.

$$m_1 = E\left(\frac{1+V_k}{1+R_k}\right)$$

$$m_2 = E\left(\left(\frac{1+V_k}{1+R_k}\right)^2\right)$$

$$S_1 = \sum_{k=1}^{n} B_k m_1^{\ k}$$

$$S_2 = \sum_{k=1}^{n} B_k^{\ 2} m_1^{\ k}$$

$$S_3 = \sum_{k=1}^{n} B_k^{\ 2} m_2^{\ k}$$

$$L_k = \left(B_k - B_1\right) m_1^{\ k-1} + \left(B_1 - B_2\right) m_1^{\ k-2} m_2 + \ldots + \left(B_{k-1} - B_k\right) m_2^{\ k-1}$$

$$S_4 = \sum_{k=1}^{n} B_k L_k$$

Then the mean of *RA* is

$$E(RA) = S_1$$

the variance of *RA* is

$$Var(RA) = \frac{m_2 + m_1}{m_2 - m_1} S_3 - \frac{2m_2}{m_2 - m_1} S_2 + \frac{2m_1 m_2}{m_2 - m_1} S_4 - S_1^{\ 2}$$

If volatility V_k has lognormal distribution with mean r_1 and standard deviation s_1, policy return R_k has lognormal distribution with mean r_2 and standard deviation s_2, and correlation between V_k and R_k is γ, then

$$m_1 = E\left(\frac{1+V_k}{1+R_k}\right) = \frac{\left(1+r_1\right)^2}{\left(1+r_1\right)\left(1+r_2\right) + s_1 s_2 \gamma}\left(1 + \frac{s_2^{\ 2}}{\left(1+r_2\right)^2}\right)$$

$$m_2 = E\left(\left(\frac{1+V_k}{1+R_k}\right)^2\right) = \frac{\left(1+r_1\right)^2}{\left(1+r_2\right)^2}\left(1 + \frac{s_1^{\ 2}}{\left(1+r_1\right)^2}\right)\left(1 + \frac{s_2^{\ 2}}{\left(1+r_2\right)^2}\right)^3 \left(\frac{\left(1+r_1\right)\left(1+r_2\right)}{\left(1+r_1\right)\left(1+r_2\right) + s_1 s_2 \gamma}\right)^4$$

The proof of this proposition is outside of the scope of this paper.

REFERENCES

Bernstein, P. [2003]. Which Policy Do You Mean? *Economics and Portfolio Strategy*, Aug. 15, 2003.

Bernstein, P. [2005]. Capital Ideas, *Free Press*, 2005.

Black, F., [1995]. The Plan Sponsor's Goal, *Financial Analysts Journal*, July-August 1995.

Blake, D., Khorsanee, Z. [2005]. Discussions of "Pensions and Capital Structure: Why Hold Equities in the Pension Fund?", *North American Actuarial Journal*, Vol. 9, No. 4, October 2005.

Bodie, Z., Kane, A., Marcus, A.J. [1999]. Investments, *McGraw-Hill Irwin*, 1999.

Bowers, N.L. et. al.[1997]. Actuarial Mathematics, Second Edition, *Society of Actuaries*, 1997.

Dhaene, J., Denuit, M., Goovaerts, M.J., Kaas, R. & Vyncke, D., [2002a]. The concept of comonotonicity in actuarial science and finance: Theory, *Insurance: Mathematics & Economics*, 31(1), 3-33, 2002.

Dhaene, J., Denuit, M., Goovaerts, M.J., Kaas, R. & Vyncke, D., [2002b]. The concept of comonotonicity in actuarial science and finance: Applications, *Insurance: Mathematics & Economics*, 31(2), 133-161, 2002.

Dufresne, D., [1990]. The distribution of a perpetuity with applications to risk theory and pension funding, *Scandinavian Actuarial Journal*, 9, 39-79, 1990.

Dufresne, D., [2007]. Stochastic Life Annuities, *North American Actuarial Journal*, Vol. 11, No.1, January 2007.

Findlay, G. [2008] An ASOP' fable: The illusion of market value, *Pensions &Investments*, Nov. 10, 2008.

Kellison, S.G. [2009]. The Theory of Interest, Third Edition, *McGraw-Hill Irwin*, 2009.

Klieber, E. [2003]. Sources and Measurement of Risk in Defined Benefit Pension Plans, *SOA Monograph M-RS04-1*, The Great Controversy: Current Pension Actuarial Practice in Light of Financial Economics Symposium, Society of Actuaries, 2004.

Klieber, E. [2002]. Pension Valuation Needs More Disclosure, Not a New Formula, *Contingencies*, American Academy of Actuaries, September/October 2002.

Markowitz, H. M. [1952]. Portfolio Selection, *Journal of Finance*, Vol. VII, No. 1 (March), 1952.

Milevsky, M.A. [2006]. The Calculus of Retirement Income, *Cambridge University Press*, 2006.

Mindlin, D. [2006] The Myth of the Liability Benchmark, *Global Pensions*, October 2006, http://www.cdiadvisors.com/papers/CDIMythofLiabilityBenchmarkGP0610.pdf

Mindlin, D. [2007] Windmill Fighters in Potemkin Villages, *Contingencies*, American Academy of Actuaries, January/February 2007, http://www.cdiadvisors.com/papers/CDIWindmillFighters.pdf

Mindlin, D. [2008a] In Support of the Weatherman, *Contingencies*, American Academy of Actuaries, May/June 2008, http://www.cdiadvisors.com/papers/CDIInSupportofWeathermanContingencies.pdf

Mindlin, D., [2008b]. The Good, the Bad, and the Ugly of Pension Accounting, *Pension Section News*, *Society of Actuaries*, Issue No. 68, September 2008, http://www.cdiadvisors.com/papers/CDIGoodBadUglyofPensionAccounting.pdf

Mindlin, D., [2009a]. Commitment Driven Investing, Part I: From Portfolio Returns to Required Assets, forthcoming, *CDI Advisors Research*, CDI Advisors LLC, 2009.

Mindlin, D., [2009b]. Commitment Driven Investing and Time Diversification, *CDI Advisors Research*, CDI Advisors LLC, 2009, http://www.cdiadvisors.com/papers/CDIandTimeDiversification090406.pdf

Mindlin, D., [2009c]. "The Plan Sponsor's Goal" Revisited, *CDI Advisors Research*, CDI Advisors LLC, 2009, http://www.cdiadvisors.com/papers/CDIThePlanSponsorsGoalRevisited091124.pdf

Roy, A. D. [1952]. Safety First and the Holding of Assets, *Econometrica*, Vol. 20, July 1952.

Trowbridge, C.L. [1989]. *Fundamental Concepts of Actuarial Science*, *Centennial Edition*, Actuarial Education and Research Fund, 1989.

Winklevoss, H.E. [1977]. Pension Mathematics with Numerical Illustrations, *Irwin*, 1977.

ENDNOTES

[1] This approach is no longer required for corporate plans, but widely used for public plans.
[2] The matching is assumed to be in terms of timing, magnitude and likelihood.
[3] See, for example, Klieber [2002], Klieber [2003], Blake, Khorsanee [2005], Mindlin [2006], Mindlin [2007], Mindlin [2008a], Mindlin [2008b], Findlay [2008].
[4] For more details, see written statements submitted to the American Academy of Actuaries' Public Interest Committee for the Public Pension Plan Forum, Sept. 4, 2008, at .http://www.actuary.org/events/2008/forum_statements.asp.
[5] For example, see Bodie [1999]: "One of the central concerns of finance theory is the measurement of risk and the determination of the risk premiums" (page 150).
[6] See Mindlin [2008b] for more details regarding discounting procedures and rates.
[7] See Winklevoss [1977], page 26.
[8] See Trowbridge [1989], page 20.
[9] See, for example, Dufresne [1990], Dhaene J., et al. [2002a] and [2002b], Milevski [2006], Dufresne [2007].
[10] These assumptions include, but are not limited to, the funding method, discount rate(s), salary growth rate(s), demographics, etc.
[11] Note that portfolio returns are stochastic. The benefit payments and payroll values are contingent end-of-year values of uncertain timing and magnitude.
[12] The pension commitment for a conventional retirement plan usually contains several streams that may be salary growth- and inflation-adjusted. The number and the structure of these streams depend on the plan's population. The main reason behind the simplification utilized in the section is the author's desire to make this paper self-contained. Proposition 2 presented in the Appendix is essentially sufficient to reproduce the results of the case study.
[13] See Black [1995].
[14] See Mindlin [2009c] for a detailed discussion of Black [1995].
[15] See Markowitz [1952].
[16] Policy portfolio *P* may contain different portfolios in different years.

[17] The multiperiod portfolio optimization in this approach involves the determination of the investor's risk aversion evolution and, subsequently, the development of a Nash equilibrium solution to the optimization problem. A detailed description of the optimization procedure is outside of the scope of this paper.
[18] See Mindlin [2009a] for more details.
[19] See Bernstein [2003].
[20] While H. Markowitz's mean-variance optimization is renowned, A. Roy's Safety-First approach had been virtually forgotten for quite some time. According to Bernstein [2005], the most plausible explanation for this story is "*just plain bad timing*."
[21] A detailed description of this optimization procedure is outside of the scope of this paper.
[22] See Mindlin [2009b] for more details.
[23] Needless to say, this author believes that allowing pure "marked-to-market" accounting to direct plan sponsors' actions (e.g. regulate contributions) is not a good idea.