A Tale of Two "Scandals"

Dimitry Mindlin, ASA, MAAA, Ph.D.
President
CDI Advisors LLC
dmindlin@cdiadvisors.com

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SUMMARY

The paper discusses the currently popular trend of "pricing" retirement commitments as tradable assets. The paper analyzes this trend and related issues in the "financial economics" and actuarial communities and emphasizes the need to restore established academic standards. The paper also outlines the key steps to correct this situation and offers an alternative approach.

1. ACADEMIC STANDARDS AND *FE*

"Something is rotten in the state of Denmark."
W. Shakespeare, Hamlet

The author believes that this paper will be useful for several reasons. First, the paper advocates a new approach to more efficient retirement plan management. Second, the paper offers common ground in a long-lasting debate. Third, the paper restores the intellectual integrity to major aspects of actuarial valuations. Forth, this approach may substantially improve actuarial standards of practice. Fifth, many practitioners may discover new analytical tools in this paper.

Yet, the author also believes that many readers will not like the premise of this paper. There are several reasons for this belief. First, the author himself finds the need to write this paper very unfortunate. Second, the paper challenges certain established approaches that many may consider inviolable. Third, the paper challenges certain ideas that are currently quite popular. Forth, the paper discusses the foundations of finance that many may perceive as somewhat extraneous. Fifth, all analytical tools utilized in this paper can be found in standard textbooks.

An astute reader may notice that the fifth items on these lists lead to a probing question. Is the author suggesting that many practitioners are unaware of relevant analytical tools that can be found in standard textbooks? Yes, and this is just a small part of the proverbial tip of the iceberg.

A major part of the tip is a notable deterioration of the foundation of retirement plan management. Violations of established academic standards are no longer highly unusual in leading publications. Some opinions are presented as facts. Some concepts are not properly defined and applied beyond the scope of their applicability. Some assumptions are not properly disclosed and justified. Some terminology is needlessly casual. Reflections of this trend can be found at the very top of the hierarchy of finance.[1]

The trend we are witnessing is quite remarkable. The primary concerns of the stakeholders of retirement plans – the outcomes of retirement programs – are no longer the front and center of recent developments. Too often, "rules of thumb and folklore" replace sound economic arguments.[2] Few appear to realize that this trend may lead to sub-optimal solutions that are not in the best interests of retirement plan participants.

This trend has several important aspects. The most notable aspect is the assumption that non-tradable retirement commitments should be treated and "priced" like tradable assets. Essentially, it is assumed that the principles of asset pricing can be applied to non-tradable retirement commitments. This assumption is called *the pricing assumption* in this paper.

The advocates of the pricing assumption often claim that the principles of "financial economics" necessitate the assumption. To acknowledge this claim, *the approach that is based on the pricing assumption is called FE in this paper*. It should be emphasized that we distinguish *FE* and the science of financial economics.

Another aspect of the trend is a broken link between objectives and suitable measurements. Generally, objectives are specified first, followed by the specification of suitable measurements. This sensible order of operations is reversed in *FE*: purportedly "economic" measurements of retirement commitments are mandated before objectives are defined.

Yet another aspect of this trend is the rejection of stochastic present values (SPVs). SPVs belong to the mainstream of finance, and their basics have long been published in textbooks. SPVs offer powerful analytical tools for the management of retirement commitments.

Retirement commitments may have a multitude of relevant present values. Some of these present values are necessitated by *pricing* objectives and some by *funding* objectives. Some are *deterministic* and some are *stochastic*. In *FE*, present values are deterministic "prices," so *FE* has no place for SPVs. By virtue of the pricing assumption, the uncertainties of present values are essentially "assumed away" in *FE*.

The pricing assumption is not a mere technicality. While seemingly innocuous, this assumption has momentous implications for the major practical aspects of retirement plan management. Yet, few proponents of *FE* acknowledge the need to justify this assumption. Moreover, many proponents of *FE* do not seem to realize that the pricing requirement is an assumption. As a result, currently popular practical solutions are currently based on shaky foundations. LDI and the "human capital" theory belong to this category.

Solutions based on shaky foundations are not without precedent in science. Paul Samuelson gave us an illuminating example of a somewhat similar situation in a delightful essay entitled "What Makes for a Beautiful Problem in Science?" in 1970. Samuelson (1970) observes that some of mathematical works in the 18th century and before were not in perfect compliance with the modern standards of mathematical rigor. Some solutions assumed that certain mathematical facts can be applied beyond the strict scope of their applicability.[3]

After a concise description of one such solution, Samuelson (1970) delivers a powerful and uncompromising verdict (emphasis added):

> "*All this is good enough for the brilliant eighteenth century. But by the nineteenth **it was a scandal** that a rigorous mathematical theory was still not known*."

A "scandal," no less! An assumption that allowed certain facts to be applied beyond the scope of their applicability may have been "good enough" at some point. Yet, the presence of this assumption turned into a "scandal" when it became clear that a rigorous framework without this assumption was viable.

To put this statement in its proper contemporaneous context, it should be noted that Samuelson was about to embark on his decades-long quest to design a theoretical framework that would rationalize evolving glide paths. Various aspects of theoretical framework design may have been at the top of his mind.[4]

The value of a theoretical framework critically depends on its assumptions. Any assumption is a potential limitation on the scope of applicability of the framework. Unnecessary assumptions are highly undesirable and should be avoided whenever possible.

The endeavors of countless scientists to eliminate unnecessary assumptions extend at least back to Euclid in the 4th century BC. To name a few, arguably the most renowned pronouncements on the subject belong to William of Occam, Isaac Newton, and Albert Einstein.

The 14th-century English Franciscan friar William of Occam proposed a principle (known as "Occam's razor" today) that, in a nutshell, recommends seeking the simplest explanations. William advanced the following principle:

> "*It is futile to do with more what can be done with fewer*."[5]

Occam's razor is often stated as "make no more assumptions than you absolutely need." Isaac Newton essentially restated the same idea in the following rule:

> "*We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances*."[6]

Albert Einstein concurred:

> "*It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience.*" [7]

Overall, the requirement to scrutinize assumptions is a universally recognized academic standard.

How does *FE* look in light of this standard? The pricing assumption is essentially unnecessary, unjustified, and undisclosed. The presence of this assumption serves an unspecified goal. Few if any publications scrutinoze this assumption. A rigorous framework without this assumption is not only feasible, but its basics are presented in popular textbooks. In the spirit of the abovementioned Samuelson quote, *FE* is a "scandal."

In fact, we are witnessing more than one "scandal." *FE* manifests itself differently in different industries and professional groups. In this paper, we discuss the manifestations of *FE* for the DB and DC plan industries and two professional groups – the loosely defined "financial economics" community and the actuarial community.[8]

In the actuarial community, *FE* has been highly controversial and ignited spirited debates for quite some time. Major actuarial organizations have given the proponents and opponents of *FE* numerous opportunities to make their case. Yet, *FE* has been neither incorporated in full into the actuarial model, nor dismissed as a fallacy.

Many leading actuarial publications contain a confounding blend of the conventional actuarial model and some elements of *FE*. These publications still contain the "scandal" of "riskless" risk premium-based discount rates. The relationship between discount rates and corresponding portfolio returns is often confusing at best. Given that the mathematical basics of this relationship are available, the lack of clarity in this area is especially troubling.

Unlike the actuarial community, the financial economics community has done little to promote the diversification of thought in this area. Few economists seem to realize that the lack of the diversification of thought is a problem. Even fewer seem to appreciate the need to spell out the economic principles behind *FE*. Meanwhile, CFA Institute publications have been very supportive of the proponents of *FE*. Few if any opponents of *FE* have been so fortunate.

A community exhibits a constructive attitude when it openly debates all sides of an issue even if the debate lack systematic underpinnings. A community exhibits an entirely different attitude when it declares an issue "common knowledge" and summarily snubs its critics. Neither attitude is a perfectly healthy, but the latter is much more problematic than the former.

The attitudinal and other differences compel this author to present two "scandals" in separate sections. The financial economics "scandal" section is largely qualitative. In contrast, the actuarial model "scandal" section is more detailed and contains several numeric examples that illustrate the points made throughout the paper.

Apart from *FE*, what unites these two "scandals" is violations of established academic standards in leading publications. To substantiate this point, a few high-profile publications, their arguments, logic, and language are discussed in detail. An additional section outlines the key steps to correct this situation.

The main message of this paper is largely theoretical. Violations of established academic standards should be corrected. Without a proper justification and disclosure, the requirement to "price" non-tradable retirement commitments is an assumption that should be rejected. The use of the "riskless" risk premium-based discount rates should be put on a proper foundation. Joint efforts of the actuarial and financial economics communities would be required to correct the current status-quo.

The crux of the matter is "present valuing" of retirement commitments in funding problems. As we all know, "what doesn't get measured doesn't get managed." Inappropriate measurements (present values) of retirement commitments lead to inefficient asset allocation decisions that may not be in the best interests of retirement plan participants.

Consequently, *FE* and other violations of established academic standards are just the tip of the iceberg. The iceberg is *the practical implications of FE* and other violations of established academic standards in the areas of optimal portfolio (glide path) selection, financial reporting, and others. These implications impose substantial inefficiencies and "surcharges" on retirement plan participants.[9] While later sections touch on some of these implications, the details of these implications are outside of the scope of this paper. These implications will be discussed in future publications.

This paper is a call to jump-start an open debate about the foundation of retirement investing. The actuarial and financial economics communities should make joint efforts to end the *FE* and other "scandals" and reverse the erosion of the foundation of retirement plan management.

2. THE FINANCIAL ECONOMICS "SCANDAL"

"... even scholars of audacious spirit and fine instinct
can be obstructed in the interpretation of facts
by philosophical prejudices."
Albert Einstein [10]

The financial economics community has a problem.

Financial economics has a rich history of endeavors to develop better science. Yet, some currently popular approaches to retirement plan management are based on certain questionable arguments that include *FE*. These approaches are poorly rationalized, fall short of the needs of retirement investors, and – most importantly and regrettably – are likely to deliver sub-optimal outcomes.

CFA Institute supports *FE* in its publications and educational materials without reservations. Certain aspects of retirement investing presented in these publications are inadequate at best. The *FE* related "inadequacies" range from confusing to plainly unreasonable. One of the main reasons for the abundance of confusion in these publications is the inadequate level of academic rigor.

For example, Maginn (2007) and Ibbotson (2007) – popular monographs endorsed by CFA Institute – contains numerous questionable pronouncements. Some definitions are incorrect, contradictory, or non-existent. Some terms have multiple meanings.[11] Some assumptions are unreasonable and/or undisclosed. These semantic tricks are not innocuous imperfections – they are indispensable building blocks in the development of the approaches to retirement investing presented in Maginn (2007) and Ibbotson (2007). Appendix II (Maginn (2007)) and Appendix III (Ibbotson (2007)) discuss these issues in more detail.

The maxim "what gets measured, gets managed" is generally accepted. Similarly, "what gets mismeasured, gets mismanaged." In *FE*, questionable measurements, assumptions, and logic produce questionable conclusions. As a result, millions of retirement plan participants rely on investment solutions that are based on little more than questionable semantics.

The level of presentation of the foundation of retirement plan management should be elevated to the level of established academic standards. These standards include clear definitions, realistic assumptions, sensible objectives, and impeccable logic. The adherence to established academic standards is imperative for the development of rigorous quantitative models with practical applications that retirement plan participants and sponsors clearly need.

The next two sections discuss *FE* applications to DB and DC plans separately.

2.1. *FE* and DB Plans

In the DB plan industry, a group of actuaries and economists urged practitioners to "reinvent" actuarial valuations for DB plans to comply with *FE* in the early 2000s. Some practitioners were receptive to their ideas, some resisted, and the ensuing debate in the actuarial community was appropriately dubbed "The Great Controversy."

The classic pension actuarial science was created when risk models were in their infancy and computing power was mostly inaccessible. One of the vital components of the actuarial model was deterministic ("riskless") discount rates that incorporated risk premium. "Riskless" risk premium-based discount rates have always been problematic.

Today we live in a different world. Risk models are common and computing power is abundant. Yet, the actuarial community has been reluctant to revisit and modernize the foundations of actuarial science. The controversy has endured.

"The Great Controversy" framed the debate as a choice: to keep the conventional model or switch to *FE*. Having failed to realize that this choice was a "false dilemma," the actuarial community was torn: the conventional model was obviously problematic, but *FE* was fundamentally flawed.

Let us take a closer look at certain implications of *FE*. *FE* requires pension benefit payments to be discounted at a rate consistent with the riskiness of the payments. For instance, if these payments can be considered "low-risk-of-default" (e.g., for public plans), then "*FE* compliant" discount rates should be close to the U.S. Treasury yield curve. Yet the discount rates utilized by actuarial public plan valuations are substantially higher. Some economists disagree with these discount rates and advocate the U.S. Treasury yield curve-based rates. Here is a small sample of their assertions (emphasis added):

Ennis (2007): "*The most basic concept in the field of finance is that of* ***the present value*** *of a future payment, whereby the future payment is discounted at a rate that reflects the risk associated with the payment. Because public pension payments are risk free for all intents and purposes, a theoretically correct discount rate is the yield of long-term U.S. T-bonds.*"

Kohn (2008): "*While economists are famous for disagreeing with each other on virtually every other conceivable issue, when it comes to this one there is no professional disagreement: The only appropriate way to calculate* ***the present value*** *of a very-low-risk liability is to use a very-low-risk discount rate.*"[12]

Bader (2015): "*It is a truth universally acknowledged (in the universe of financial economists) that* ***the value*** *of a secure, fixed future payment is determined by discounting at a default-free rate*."

The trouble with these statements is their assumptions and objectives are not disclosed. These "technicalities" are relegated to the "every-economist-knows-this" category. Those who wish to understand the economic principles behind these statements are at liberty to guess.

These statements appear to be based on several assumptions that include the following. *The* present value is assumed to exist and be deterministic; the risk-free asset is assumed to exist; the plan sponsor's goal is assumed to be to completely de-risk its retirement commitments immediately.

These assumptions are debatable at best. Few if any objects have "*the* value" that is superior to all other values.[13] The existence of the risk-free asset assumption makes little sense outside of some closed/terminated DB plans. Most plan sponsors utilize risky assets for funding purposes and do not contemplate complete immediate de-risking today.

But arguably the most consequential problem with these statements is their implied rejection of one of the key functions of retirement plan management – the determination of how much to contribute and how to allocate assets today. Present values of retirement commitments funded by risky assets are generally SPVs that are instrumental in executing this function. There are no efficient contribution and asset allocation strategies in a world where *the* deterministic present value reigns supreme.[14]

To illustrate these points, let us consider a basic funding problem. Think of an investor that has made a commitment to make a single future payment of known magnitude and timing. Let us assume that the risk-free asset – a zero-coupon Treasury bond that matches the commitment – is available, yet the investor has chosen to take some risk and selected a portfolio of risky assets to fund the commitment. This choice implies that the shortfall probability is non-zero.

Let us also assume that the investor has calculated asset value X that should be invested in the selected portfolio at the present to have a specific shortfall probability to fund the commitment. Let us ask a few questions about present value X.

- *Is X equal to the price of the matching zero-coupon Treasury bond?* No.
- *Is X the same for all shortfall probabilities?* No.
- *Is X the same for all portfolios?* No.
- *Is X the same for all capital market assumptions?* No.

The investor's ability to select a portfolio of risky assets is the key feature of this example. This ability implies that there is a multitude of relevant present values. There is no present value that is superior to all other present values. "*The* present value" does not exist when investors can utilize risky assets to fund their commitments. Most retirement investors belong to this category.

The abovementioned and similar statements demonstrate a certain level of misunderstanding regarding pension plan risk, the pension plan sponsor risk, actuarial valuations, and related issues. To clarify, let us ask a few more questions.

- *Do actuarial valuations evaluate the plan sponsor's ability to make pension payments?* No. Actuarial valuations evaluate pension *plans*, not pension *plan sponsors*.

- *Are the pension* ***plan risk*** *and the pension* ***plan sponsor risk*** *"one-and-the-same"?* No. Actuarial valuations define the riskiness of pension payments in terms of the ability of the plan's assets and future contributions to fund these payments. In contrast, *FE* defines the riskiness of pension plan payments in terms of the sponsor's ability to make them.

- *Is the plan sponsor's goal to price and immediately de-risk its retirement commitments?* No. The plan sponsor's goal is to fund retirement commitments. There is a fundamental difference between *funding* and *pricing* retirement commitments. The former implies a possibility of a shortfall event; the latter does not. The former incorporates risky assets; the latter does not. The former recognizes the stochastic nature of present values; the latter does not. The former is what most pension plans do; the latter is what the proponents of *FE* want every plan to do.

- *Is the "price" of retirement commitments the only present value relevant to the plan*? No. Retirement plans have a multitude of objectives and present values associated with these objectives, so there is a multitude of present values relevant to the plan. There may be different *deterministic* present values for "budgeting," "settlement," compliance, and other purposes. There may be different *stochastic* present values for the development of optimal asset allocation and contribution strategies.

- *Is a discount rate necessary for present value calculations?* No. Generally, a discount rate is a choice, not a necessity. A deterministic present value can be calculated as a measurement (e.g., the mean or the median) of the corresponding SPV distribution. This issue is considered in more detail in later sections.

- *Is the declaration "every-economist-knows-this" a valid argument?* No. In the presence of quality academic publications on the subject, this declaration is meaningless. In the absence of quality academic publications on the subject, this declaration is meaningless as well.

- *Are there quality academic publications on the subject of FE?* Very few. The proponents of *FE* have failed to present the economic principles behind *FE*. Some authors assert that *FE is* based on the-law-of-one-price. Some authors assert that *FE is not* based on the-law-of-one-price. Some authors assert that there is no need to justify *FE*.

It is true that a proper way to price a low-risk-of-default tradable cash flow is to use a low-risk-of-default discount rate. But retirement commitments are not tradable, the goal is to fund rather than to price, and the use of a discount rate is a choice, not a necessity. Thus, *FE offers the right solution to the wrong problem*.

Most DB plans fund their commitments via investing in risky assets. *FE* offers neither conceptual nor analytical tools to meet the challenges these investors face. This observation underlines arguably the most important attribute of *FE*: *FE is not helpful to most retirement investors*.

2.2 *FE* and DC Plans

... we really ought to look into theories that don't work,
and science that isn't science.
Richard Feynman[15]

In this section, we discuss applications of *FE* to DC plans. We examine the *FE* pricing assumption as applied to retirement commitments. We demonstrate that this assumption accompanied by ambiguous objectives and confusing terminology lead to theoretical and practical conflicts with the prevailing practices in the industry as well as the wishes of some retirement plan participants. This assumption also creates substantial inefficiencies and "surcharges" on plan participants, but this issue is outside of the scope of this paper.

While applications of *FE* to DB plans have been controversial, their counterparts to DC plans have largely avoided a scrutiny. In the DC plan industry, the *FE* mindset rules unchallenged. Many publications make the *FE* pricing assumption with no discussion or disclosure. Few appear to recognize the negative impact of this assumption on the outcomes of retirement programs.

Let us look at the realities of DC plans. DC plan participants make contribution and retirement income commitments. Many employers also make (matching) contribution commitments. All these non-tradable non-binding commitments can be modified or suspended anytime. This section focuses mainly on the "pricing" of contribution commitments.[16]

Contribution commitments come from "human capital," which is defined as "a capacity to earn income or wages for the remainder of our lives." Human capital is a series of contingent payments of uncertain magnitude and timing. It plays a central role in retirement plan management.

According to the *FE* mindset, the first step is to "price" human capital, and the objectives are discussed *after* the "pricing." The order of operations here is the illogical "measure first, set objectives next." It should be noted that human capital "pricing" could be useful for some objectives, but inapplicable and even counterproductive for some others.

It is informative to ask a few questions about the nature of human capital.

- *Is human capital tradable?* No.
- *Can the human capital cash flow be replicated using conventional tradable assets?* No.
- *For most people, will the "price" of human capital be ever exercised?* No.
- *Is the objective of the human capital "pricing" to optimize outcomes?* No.

One of the key tasks of the retirement plan management is optimal glide path selection. Glide paths are one of the main features of target date funds (TDFs) – an increasingly popular and rapidly growing investment product. With over a trillion dollars in assets and attracting a sizable flow of new contributions, TDFs are well-positioned to dominate the DC plan marketplace. For most TDFs, the process of optimal glide path selection is based on the *FE* driven "human capital" theory.

The "human capital" theory is currently the only well-known approach that rationalizes evolving glide paths. The problem is the economic foundation of this theory is shaky at best and arguably non-existent. The basics of this approach are presented in Ibbotson (2007), a monograph published by CFA Institute Research Foundation.

Ibbotson (2007) offers several conflicting definitions of the term "human capital." The authors take the term "human capital" on a round trip from a series of payments to a present value and back to a series of payments. Moreover, the monograph resorts to debatable semantic games in order to achieve the dual goal of converting "the random income into a scalar" and turning "human capital" into a conventional asset class. The monograph's efforts to achieve this goal are discussed in detail in Appendix III.

Yet, after all these efforts, the authors still appear uneasy about the term "human capital". They lament the abundance of "conflicting ideas" regarding the term:

> "*The term "human capital" often conveys a number of different and, at times, conflicting ideas in the insurance, economics, and finance literature.*"[17]

This statement and especially the qualifiers "often" and "at times" reveal the chaotic state of the "human capital" theory. If the term "human capital" "*often* conveys a number of *different* ideas," then in all other cases the term conveys just one good idea. Why don't we utilize this good idea instead of a bunch of different ones? If the term "human capital" "*at times* conveys a number of *conflicting* ideas," then in all other cases the ideas it conveys are perfectly consistent. Why don't we utilize the consistent ideas instead of the conflicting ones?

The answer to these questions is simple. The qualifiers "often" and "at times" represent another example of the aforementioned semantic games. The term "human capital" conveys a number of conflicting ideas. No qualifiers, no equivocations. It is a mess that requires a thorough cleanup.

In response to this unfortunate situation, the monograph adds at least five more "conflicting ideas."

"Conflicting idea" #1: the dual use of the term "human capital" – as a series of payments and a present value.

This "idea" is discussed in detail in Appendix III.

"Conflicting idea" #2: the order of portfolios in a glide path is irrelevant.

This "conflicting idea" comes straight from the main premise of *FE* – "human capital" is "a scalar." To see why it is so, let us assume for simplicity that a retirement plan participant has made a commitment to contribute a fixed percentage p of his income to the plan until retirement. If HC is the participant's "financial economic value of human capital," then the present value of retirement contributions is equal to $p \cdot HC$. In *FE*, HC is assumed to be "a scalar," so the present value of retirement contributions $p \cdot HC$ is "a scalar" as well.

Let us also assume that the participant has n years until retirement, has selected a glide path of portfolios, and R_k is portfolio return in year k. To calculate the asset value at retirement AVR, we use the following fundamental relationship between future and present values: the future value is equal to the present value times the compounded return factor (a return factor is defined as portfolio return plus one).[18]

Then the asset value at retirement AVR is equal to the product of the present value of retirement contributions $p \cdot HC$ and the compounded return factors $\prod_{k=1}^{n}(1+R_k)$:

$$AVR = p \cdot HC \cdot \prod_{k=1}^{n}(1+R_k) \qquad (2.1)$$

Since $p \cdot HC$ is a scalar and any transposition of R_k produces the same $\prod_{k=1}^{n}(1+R_k)$, the order of portfolios in the glide path is irrelevant.

<u>"Conflicting idea" #3</u>: "More stocks early, less stocks late."

The principle "more stocks early, less stocks late" is broadly accepted and adopted in the industry. The monograph's position is clearly presented in the introduction:

> *"Because human capital is usually relatively low risk (compared with common stocks), we generally want to have a substantial amount of equities in our financial portfolio early in our careers because financial wealth makes up so little of our total wealth (human capital plus financial capital)."*

This sentence is notable for several reasons. It creates a classic "apples and oranges" situation – it compares human capital (a series of contingent payments) to common stocks (a tradable asset). It continues the unfortunate trend of throwing around the term "risk" without defining it. It mistakenly suggests that, early in our careers, investing "financial capital" mostly in equities would diversify the portfolio. And it explicitly supports the principle "more stocks early, less stocks late."

Here is how *FE* rationalizes this principle. The basic premise of *FE* is human capital is a bond-like conventional asset. Young retirement investors have a lot of this bond-like asset, so they "want to have a substantial amount of equities" in their financial portfolios. As retirement investors age, the bond-like human capital diminishes, so their financial portfolios should have more bonds. Hence, "more stocks early, less stocks late."

The principle "more stocks early, less stocks late" is a "conflicting idea" for at least two reasons. First, it obviously conflicts with the previous conflicting idea "the order of portfolios in a glide path is irrelevant," which also comes from the basic premise of *FE*.

Second, this principle conflicts with the wishes of young retirement investors, who are reportedly reluctant to invest aggressively early in their careers. In an article "The Biggest Money Mistakes We Make – Decade by Decade," The Wall Street Journal reported (emphasis added):

> "*… 20-somethings don't take enough risks with investments to build up big returns. It's a conclusion* ***backed up by a number of studies****, including a 2016 analysis by Lindsay Larson … Her team sampled a group of roughly 100 millennials and found that they tended to favor retirement accounts with little stock and more guaranteed income …*"[19]

A similar theme appears in Leibowitz (2016). Discussing “a debate at TIAA-CREF,” Martin Leibowitz mentioned:

> “*On that TIAA-CREF story, we found that a large percentage of the younger professors were investing in cash. … Their allocation was not nearly as irrational as it seemed.*”

One may wonder why their conservative allocations “seemed irrational” and one of their “biggest money mistakes” in the first place. One of the few sceptics of “more stocks early, less stocks late,” Arnott-Wu (2014) was wondering too: “*Maybe a high-risk profile is unwise for young savers*.”

Mindlin (2015A) demonstrates that a glide path with an equity “take-off” – that starts conservatively, reaches a peak allocation at some point, and “glides” down afterwards – generates better outcomes than a comparable “human capital theory” based glide path. Add shorter term concerns for employment risk and emergency cash, and it is not hard to see that those young investors with conservative initial allocations are onto something.

As presented in the monograph, the principle “more stocks early, less stocks late” is premature at best and likely not in the best interests of young investors.

“Conflicting idea” #4: Optimal glide paths contain identical portfolios.

If $p \cdot HC$ is a scalar, formula (2.1) for certain glide path optimization problems leads to stationary glide paths.[20] Paul Samuelson proved a similar result in Samuelson (1969). It is well-known that Samuelson was dissatisfied with this result and called it "a failure.” Furthermore, this “idea” conflicts with the previous “ideas” as well as the common practices in the industry.

“Conflicting idea” #5: the conflict between the proponents of FE in the DB and DC industries. The authors claim that “discounting with an explicit risk premium” produces a “financial economic value.” Yet, the proponents of *FE* in the DB industry claim that “discounting with an explicit risk premium” in the conventional actuarial model violates the principles of “financial economics.” While both claims lack may be unsubstantiated, their conflict is obvious.

To recap, the core assumption of *FE* has major implications in the area of optimal glide path selection. As demonstrated in this section, this assumption leads to the following conclusions:

- The order of portfolios in a glide path is irrelevant.
- Optimal glide paths should have “more stocks early, less stocks late.”
- Optimal glide paths should contain identical portfolios.

Obviously, these conclusions conflict with each other, and the core assumption of *FE* is the main source of these conflicts. Yet, this assumption is broadly accepted in the industry. One of the few sceptics of this assumption is Arnott-Wu (2014):

> "*Too often, our industry is addicted to conventional wisdom and allergic to arithmetic and empirical testing. … Finance theory was then called upon to justify this untested conventional wisdom. Academia advanced the unexamined thesis that human capital is like a bond, so that, as we age, we should replace our diminishing human capital with bonds. Now we have an established literature which demonstrates that – assuming human capital resembles a bond – we should move from risk-tolerant to risk-averse as we age. … Now, a huge industry has been formed on the basis of conventional wisdom, backed by finance theory which is itself based on a doubtful core assumption and supported by anecdotal behavioral evidence!"*

Even though the adjectives "untested" and "unexamined" are emphasized in Arnott-Wu (2014), this statement does not go far enough. The core *FE* assumption has been tested, and it has failed – its implications are in conflict with each other and the prevailing practices in the industry.

Undeterred by these conflicts and apparently convinced that they have made a solid case for human capital as a conventional asset, the authors are ready to use a one-period portfolio-centric model to offer "lifetime financial advice." Yet, the applicability of one-period *portfolio*-centric models to *investor*-centric problems (e.g. retirement funding) has always been in question. In particular, Fischer Black explicitly rejected the goals of one-period portfolio-centric models for retirement investors in his stunning parting message to retirement investors, see Black (1995).[21]

Furthermore, the authors wish to "maximize expected utility of total wealth" for a specific utility function. The presence of this utility function represents a disputable assumption that may limit the applicability of the approach. Again, the authors refer to some undefined "economic theory":

> *"Economic theory predicts that investors make asset allocation decisions to maximize their lifetime utilities through consumption. These decisions are closely linked to human capital."*

While expected utility maximization is well-known in economic literature, its acceptance is far from universal. For instance, Roy (1952) contains the following disapproving statement:

> *"In calling in a utility function to our aid, an appearance of generality is achieved at the cost of a loss of practical significance and applicability in our results. A man who seeks advice about his actions will not be grateful for the suggestion that he maximise expected utility."*

Rabin-Thaler (2001) is even less charitable:

> *"… it is time for economists to recognize that expected utility is an ex-hypothesis, so that we can concentrate our energies on the important task of developing better descriptive models of choice under uncertainty."*

Yet despite all questionable features of the approach utilized in the monograph, its most important flaw is that it does not deal directly with the outcomes of retirement programs. It is the objectives (or lack thereof) that make this approach suspect. The stated objective is rather ambiguous:

> "*We desire to create a diversified overall portfolio at the appropriate level of risk.*"

On the surface, this objective sounds reasonable. Who would argue against something that is "diversified" and "appropriate"? Beneath the surface, however, this objective is likely to generate substantial inefficiencies for retirement investors. More to the point, the absence of any discussion regarding investment objectives is disconcerting. Fischer Black emphasized the importance of properly stated objectives in Black (1995).

As an alternative, think of the objective "to maximize retirement income." If, given the level of contributions, we defined risk as the shortfall event and identified appropriate risk measurements, the objective would be to maximize retirement income given the contributions and risk "budget." The solutions generated by this objective are likely to deliver superior retirement income.[22]

Overall, the *FE* based ideas and techniques that incorporate contribution commitments and their "pricing" into the retirement investor's asset allocation decisions contains questionable objectives, terminology and assumptions. These the ideas and techniques may also impose substantial inefficiencies and "surcharges" on retirement plan participants. Yet, the foreword to the monograph would beg to differ:

> *"The present monograph is an unusually complete and theoretically sound compendium of knowledge on this topic."* [23]

Moreover:

> *"… the ideas and techniques described in Lifetime Financial Advice timely and necessary. We hope and expect that researchers will continue to follow this path in the future… "*

It is this author's turn to beg to differ: these ideas and techniques are built on a shaky foundation. Moreover, the endorsement of this shaky foundation as "unusually complete and theoretically sound" is a potential impediment to innovations in different directions. Confined to the straitjacket of this "complete and theoretically sound" framework, economists and asset managers have been limited to discussing somewhat peculiar issues of glide path design.[24] The primary objectives of retirement investors – maximizing retirement spending and minimizing the cost of its funding – have not received the attention they deserve.[25]

Yet, *FE* enjoys substantial institutional support for good reasons. On the DB side, the *FE* based LDI products are promoted by many asset managers and consultants. Today, *FE* is an integral part of DB plan asset management.

On the DC side, the need for *FE* is especially acute. The "under development" sign on the economic foundation of glide paths was "good enough" when the TDF industry was in its infancy. When the industry manages popular and rapidly growing retirement investment products that highlight glide paths, it had better have a "theoretically sound" approach to glide path design. The *FE* based "human capital" theory is currently the only well-known approach that rationalizes evolving glide paths. When *FE* is recognized as a fallacy and an imprudent attempt to apply the principles of asset pricing beyond the scope of its applicability, the industry would need a new approach. That would be a major challenge, as Paul Samuelson had demonstrated during his decades-long quest to find justification for glide path evolution. The industry practitioners would be well-advised to look outside the "human capital" theory for better concepts and analytics. [26]

Finally, here are several concluding take-away points of this chapter.

- All assumptions should be disclosed and discussed.
- "Pricing" of retirement commitments is a choice, not a requirement supported by a sound economic theory.
- The idea that human capital is "like any other asset class" is a debatable assumption that leads to debatable conclusions.
- Incorporating human capital into one-period portfolio-centric models may not be the best way to optimize the outcomes of retirement programs.
- *FE* may be detrimental to the best interests of DC plan participants.

3. THE ACTUARIAL MODEL "SCANDAL"

"You can't stop modern science…
… Can't stop science…
… Science just marches…"
George Costanza [27]

The pension actuarial community has a problem.

The conventional pension actuarial model has come under intense criticism in recent years. Some of this criticism has been justified, and some has not. Some have called the model "obsolete" and urged the actuarial community to "reinvent" it. Some have questioned the professional credentials and education of pension actuaries. Some of these opinions have been quite unflattering.

Some have advocated the adoption of *FE* as the "cure" to the "ills" that have plagued the model. Some have suggested that this "cure" would be worse than the "ills." Some have stated that the actuarial model does include both *FE* and the conventional model, even though actuarial standards of practice (ASOPs) and other leading actuarial publications have yet to reflect this inclusion.

Actuarial organizations deserve credit for having encouraged an open debate in this area. Yet, their practical response to the criticism has been overly hesitant and largely inconsequential. After several revisions of some ASOPs, issue briefs, and practice notes, the key problems in the actuarial model remain unresolved.

Moreover, some of the recent additions to ASOPs require a thorough cleanup. Today, some ASOPs contain questionable approaches to the key tasks of standard making (methodologies, terminology, math, numerical examples, etc.) while better approaches are readily available. The actuarial model as presented today is a "scandal."

The actuarial community is facing a major challenge. There is a glaring disconnect between the current portrayal of the actuarial model in leading publications and the foundations of actuarial science. This disconnect is the primary reason for the existence of this "scandal" and the abundant confusion it fosters. The actuarial profession has suffered and will continue to suffer as long as this "scandal" remains unresolved.

The core of the disconnect is a conflict between the stochastic nature of portfolio returns and the deterministic nature of actuarial "liabilities." This disconnect has always existed in actuarial publications. However, it is hard to blame "the founding fathers" of the actuarial model for not resolving this issue. The required conceptual and analytical tools simply were not there yet.

There are good reasons to believe that many actuaries understood the nature of the problem at the time. A monograph "Fundamental Concepts of Actuarial Science" published in 1989 by the Actuarial Education and Research Fund has a chapter "Random Variables" that discusses potential sources of uncertainty in actuarial calculations.[28] This chapter has a section "The Rate of Interest as a Random Variable" that discusses the variability of investment return. This section demonstrates that the actuarial community had been aware of the need to connect discount rates to investment returns and incorporate stochastic investment return into actuarial calculations (emphasis added):

> "*Of great importance to the actuary is the rate of interest (***or more generally, the rate of investment return***). ... Historically, actuaries have used deterministic models in their treatment of the time value of money, but not because they were unaware of interest rate variation.* ***The difficulty has not been a lack of concern, but rather a lack of knowledge*** *as to the complexities of interest rate variation. ... The development of computers has opened up a range of techniques whereby interest rate variation can be modeled. It appears that this is a direction in which actuarial interest and knowledge may be expected to grow*."

The actuarial textbook "The Theory of Interest" published in 1991 has a chapter "Stochastic Approaches to Interest" that presents the basics of SPVs generated by the variability of investment return.[29] The textbook "Actuarial Mathematics" published in 1997 by the Society of Actuaries presents SPVs generated by the variability of demographic decrements.[30] The monograph "Financial Economics" published in 1998 by the Actuarial Foundation presents financial economics with emphasis on investments, insurance and pensions – the key areas of interest for the actuarial community.[31]

By the late 1990s, the actuarial community had been making reasonable progress toward a more rigorous model. But the disconnect accompanied by a certain level of criticism was still there.

This criticism markedly intensified in the early 2000s. A group of actuaries and economists urged the actuarial community to "reinvent" conventional actuarial valuations for DB plans in the spirit of *FE*. The actuarial community promptly organized a high-profile symposium called "The Great Controversy." The symposium presented a broad range of views on the conventional actuarial practices and their links (or lack thereof) to financial economics. The main tenet of *FE* – *the "market value" of retirement commitments is superior to other present values* – was presented prominently.

To give *FE* even more prominence, the American Academy of Actuaries (AAA) and the Society of Actuaries (SOA) created the Joint Task Force on Financial Economics and the Actuarial Model. The task force was primarily comprised of the proponents of *FE*. The most notable product of the task force was "Pension Actuary's Guide to Financial Economics" published in 2006.[32] The "guide" is essentially a collection of observations from the field of corporate pension finance.[33]

Few if any objects have a single measurement that is unconditionally superior to all others. Pension plans are no exception – there are a multitude of valuable measurements that serve different goals.

Any theory that claims otherwise is likely to be a fallacy. One presenter at the symposium dedicated a large segment of his presentation to this simple observation.

It took a decade for this message to sink in. In November 2013, the AAA published an issue brief "Measuring Pension Obligations: Discount Rates Serve Various Purposes." One of the key points was "two common measurements of pension obligations have significantly different meanings." In other words, more than one number are required to "comprehensively address" the funded status of a pension plan.

Another issue brief "Assessing Pension Plan Health: More Than One Right Number Tells the Whole Story" published by the AAA in July 2017 made essentially the same point. One of the key messages of this issue brief was "a single number often cannot comprehensively address an issue as complex as the obligation or funded status of a pension plan."

Even though these issue briefs offer rebuttals to *FE*, these publications should be a cause for concern. A simple common-sense observation – different portfolios generate different present values – should not require two issue briefs from the AAA. Moreover, these issue briefs use "riskless" risk premium and highlight the problem rather than offer solutions. Even if these issue briefs refuted one of the most dubious points of *FE*, there would still be no justification for the use of "riskless" risk premium. A later section analyzes these issue briefs in detail.

It appears that the progress the actuarial community has made toward a better model has slowed down since the late 1990s. Not that the progress has stopped – we have gained new knowledge and insights in the area of stochastic portfolio returns and present values in the last couple of decades. However, these new developments have largely occurred outside of the auspices of the leading actuarial organizations. These organizations should review and modify their priorities in pursuit of a better model of retirement plan valuation and management.

To rephrase the abovementioned quote from Trowbridge (1989), *the difficulty today appears to be not a lack of knowledge, but rather a lack of concern*.

3.1. The Key Issue

Here is one of the key problems of the conventional actuarial model as presented in Mindlin (2010), section "Why the Existing Paradigm Needs Change":

> "*A typical plan endeavors to fund its financial commitments by virtue of investing … in risky assets. According to the conventional approach …, the plan's actuary utilizes the expected return for the policy portfolio … as the discount rate to calculate … present values*

> *of pension commitments... The main problem with this approach is the riskiness of the policy portfolio plays no role in the present value calculations. At the same time, the risk premium assumption embedded in the discount rate is one of the key factors in the determination of these present values. Therefore, the conventional approach employs risk premium without risk, which makes little sense.*"

The proponents of *FE* emphatically highlight this problem in their quest to "reinvent" the actuarial model, and they have a point here. As currently presented in leading publications, the "riskless" risk premium is a problem in the actuarial model.

There are two logical solutions to this problem:

A. To incorporate risk in present value calculations.
B. To eliminate risk premium from present value calculations.

This is the key issue: solution A or solution B. This author advocates solution A and the use of SPVs. The proponents of *FE* advocate solution B and reject SPVs. It should be emphasized that solutions A and B are not mutually exclusive. *Everything solution B does can also be done within the framework of solution A.*

Conventional actuarial "liabilities" have no place in the framework of solution B. In contrast, conventional actuarial "liabilities" have a clear meaning in the framework of solution A – they are measurements of the corresponding SPV (e.g., medians). These measurements can be estimated directly or using conventional discount rates.

Here is the essence of solution B. There may exist a "buy-and-hold" portfolio of tradable assets that matches the retirement plan's commitments. This asset is usually called "the matching asset" or "the risk-free asset." The price of this (hypothetical) asset is what solution B designates as *the* present value of the commitment. This present value is equal to the asset value required to fund the commitment with certainty. This present value is deterministic.

Solution A is an expansion of solution B from matching to non-matching assets. Solution A produces the same deterministic present value if the matching asset is the only asset under consideration. Otherwise, solution A produces other present values related to different relevant portfolios and levels of risk tolerance. These present values are still the asset values required at the present to fund the commitment, but they are no longer deterministic. The presence of non-matching assets introduces uncertainty in the required asset values. Hence, solution A necessarily involves SPVs.

So far, the actuarial community has embraced neither solution A nor solution B. Instead of facing the problem of "riskless" risk premium directly, the actuarial community continues using "riskless" risk premium-based discount rates in a seemingly nonchalant manner. "If investment return is I% in all years, then the present value is \$$X$" appears to be the best leading actuarial publications can do today.

If I = 6%, this author believes that most actuaries would consider this statement "kindda-sortta" reasonable, even though having "investment return 6% in all years" is next to improbable. This author also believes that most actuaries would not consider a similar statement for I = 30% reasonable, even though having investment return 30% in all years is also next to improbable (estimates of relevant probabilities are presented later in the paper).

Why? Because most actuaries likely feel that 6% may be "kindda-sortta" close to the "average" return, and 30% may not. But to get rid of the "kindda-sortta" and quantify this feeling via clarifying the meaning of the "average," one would need to analyze the distribution of portfolio returns and their measurements (e.g., the arithmetic and geometric means). Inexplicably, leading actuarial publications provide little guidance in this area.

To rephrase the abovementioned quote from Samuelson (1970), all this may have been good enough for the mid-twentieth century. But by the twenty first, it is a "scandal" that all we have is an "if-then" statement for which the "if" part is next to improbable.

Collapsing the full spectrum of asset returns into a single number to calculate present values may have been good enough at some point. "The Great Controversy" highlighted the inadequacy of this approach. Yet, this inadequacy does not mean that discount rates and the conventional approach should be eliminated. The conventional approach provides useful estimates of valuable measurements of retirement plans. They should be included into an expanded model and presented according to established academic standards.

To expand the actuarial model to incorporate both matching and non-matching assets, we need to recognize and incorporate conventional policy portfolios and "buy-and-hold" assets, the full spectrum of portfolio returns and their measurements, and SPVs. Subsequent sections provide more details on these issues.

3.2. Investment Returns, Future and Present Values

To understand funding problems that involve risky assets and the role SPVs play in these problems, this section discusses the basic formulas for future and present values as well as illustrates these formulas with numerical examples.

Think of an investor that has *P* invested in a portfolio at the present. For simplicity, we assume the investor's time horizon is one period and there are no intermediate contributions or payments. The end of period value of the portfolio is *F*. Portfolio return *R* is defined as the ratio of the investment gain $F - P$ over present value *P*:

$$R = \frac{F - P}{P} \tag{3.1}$$

Definition (3.1) establishes the basic relationship between portfolio return *R*, future value *F*, and present value *P*. Utilization of this formula depends on the context of calculation and on the way the outcomes are expressed – via future or present values. It should be emphasized that a specific investment portfolio generates return *R*. *Different portfolios generate different returns*.

Most investors utilize risky assets that imply the stochastic nature of portfolio return *R*. Portfolio return *R* is commonly estimated using conventional capital market assumptions (CMAs). CMAs include expected returns, volatilities, and correlations for all asset classes under consideration.

If the investor chooses to express the outcomes at the end of the period and the goal is to estimate future value F given present value P, then a simple modification of definition (3.1) connects future value *F* to present value *P* and portfolio return *R* as follows:

$$F = P(1 + R) \tag{3.2}$$

Formula (3.2) presents a fundamental relationship between *F*, *P* and *R*: the future value is equal to the present value times the return factor.

If portfolio return *R* is uncertain, future value *F* in formula (3.2) is stochastic and should be modeled as a random variable. It should be noted that the investor's needs and choices – to solve the future value problem and utilize risky assets – require the use of stochastic future values in formula (3.2). It should also be noted that *different portfolios generate different future values*.

Example 1 illustrates the future value problem and formula (3.2). Formulas for all numeric examples in this paper can be found in Kellison (2009), a classic actuarial textbook.

Example 1: Estimate Future Values Given Present Value. An investor has $100 today and utilizes two broadly defined asset classes: stocks and bonds (for all examples in this paper, CMAs, and the technical details are presented in the Appendix I). The investor wishes to estimate the future value that is achievable with probability 70% (i.e. the 30^{th} percentile of the future value distribution). *Exhibit 1* presents these values as well as the mean and standard deviation for various portfolios.

***Exhibit 1*. Future Value Estimates**

	100% Stocks 0% Bonds	50% Stocks 50% Bonds	0% Stocks 100% Bonds
FV Mean	108.16	106.14	104.12
FV Median	107.00	105.78	104.00
FV St Deviation	16.00	8.85	5.00
FV 30th %ile	99.06	101.26	101.42

Let us put definition (3.1) in the context of a retirement funding problem. Given a DB plan's benefit stream or a DC plan participant's retirement income commitment to fund, the investor's challenges include the development of asset allocation and contribution strategies *at the present*. It stands to reason that the analysis of present values would be instrumental in meeting these challenges.

If the investor chooses to express the outcomes at the present and the goal is to estimate present value P given future value F, then a simple modification of definition (3.1) connects present value *P* to future value *F* and portfolio return *R* as follows:

$$P = \frac{F}{1+R} \tag{3.3}$$

Formula (3.3) defines *a discounting procedure*. If return *R* is uncertain, present value *P* in formula (3.3) is generally stochastic and should be modeled as a random variable. The investor's needs and choices – to solve the present value problem and utilize risky assets – necessitate the use of SPVs in formula (3.3). It should be noted that *different portfolios generate different present values*.

Even though present value *P* in formula (3.3) is generally stochastic, the investor may need deterministic measurements of present values. For instance, solvency, financial reporting, and budgeting considerations generally require deterministic present values. In this case, the investor may consider estimates of various measurements of the corresponding SPV (e.g. the mean or the median) as "deterministic" present values. Yet, traditionally, a deterministic present value is calculated using a measurement of portfolio return (e.g. the mean or the median) as a discount rate. This present value is an estimate of the corresponding SPV.

Example 2 illustrates the present value problem and formula (3.3).

Example 2: Estimate Present Values Given Future Value. An investor has a commitment to accumulate $100 at the end of the year. As in *Example 1*, the investor utilizes two broadly defined asset classes: stocks and bonds. The investor wishes to estimate the present value that has a probability 70% to fund the commitment. In other words, the investor wishes to estimate the 70th percentile of the present value distribution. *Exhibit 2* presents these values as well as the mean and standard deviation of present values for various portfolios.

***Exhibit 2*. Present Value Estimates**

	100% Stocks 0% Bonds	50% Stocks 50% Bonds	0% Stocks 100% Bonds
PV Mean	94.47	94.87	96.26
PV Median	93.46	94.54	96.15
PV St Deviation	13.98	7.91	4.62
PV 70th %ile	100.95	98.76	98.60

Here are the key lessons to be learned from these examples:

- There is no discounting without investing.
- The investor's goals determine the point of expressing outcomes, e.g. in the future or at the present.
- If risky assets are used to fund the investor's commitments, then future and present values are generally stochastic. In this case, present and future values should be modeled as random variables. Random variables generally require "more than one number to tell the story."
- Different portfolios generate different future and present values.
- Different levels of risk tolerance generate different future and present values.
- Multiple portfolios and levels of risk tolerance may be relevant to the plan.
- Multiple present and future values may be relevant to the plan.
- Generally, there is no present value that is unconditionally superior to other present values.
- The investor may have valid reasons to assume that present values are deterministic. This assumption should be disclosed and justified.

These important points should be thoroughly discussed in the actuarial and financial economics communities.

3.3. 2013-2017 AAA Issue Briefs

In November 2013, the American Academy of Actuaries (AAA) published the issue brief "Measuring Pension Obligations: Discount Rates Serve Various Purposes." In July 2017, the AAA published another issue brief "Assessing Pension Plan Health: More Than One Right Number Tells the Whole Story" essentially on the same subject. The current state of affairs in the actuarial model is on display in these briefs.

The two briefs have several things in common. First, the main point of both briefs is "more-than-one-right-number-tells-the-whole-story." Second, the numerical examples presented in the briefs are funding problems, i.e. the goal is to determine the contribution amount and the optimal portfolio. Third, both briefs avoid examples of *retirement* funding problems. Forth, the numerical examples in the briefs use conventional "riskless" risk premium-based discount rates.

Fifth, and most importantly, both issue briefs utilize an inferior framework when a superior framework is readily available. Several questionable pronouncements in these briefs clearly demonstrate the troubles in the current actuarial model. Such pronouncements should be dispelled.

In short, the logic of both issue briefs is the following: different portfolios may be relevant to a funding problem; different portfolios may require different asset values at the present; different present values may be relevant. Hence, "more-than-one-right-number-tells-the-whole-story."

The issue briefs have two major problematic areas: portfolio returns and present values. This section presents a detailed analysis of these areas and offers alternative concepts/calculations for problematic concepts/calculations. The two issue briefs are considered separately.

The 2013 AAA issue brief starts with several key points. The first key point states:

> "*Two common measurements of pension obligations have significantly different meanings*."

This point is incorrect. The "two common measurements" – "the solvency value" and "the budget value" – represent the asset values to be invested at the present in the matching portfolio of "default free securities" and "a diversified portfolio of assets" correspondingly. Thus, the portfolios and the default probabilities are different, but the meaning – the asset value to be invested at the present – is the same for both values.

The second key point states:

> "*Market-based methods use a discount rate based on observable data from the financial markets. Expected return-based methods use a discount rate based on the estimated return of the plan's investment portfolio*."

As a "point," this statement is incoherent. As a definition, this statement is meaningless. "Expected return-based methods" are based on capital market assumptions, which, in turn, are based on "observable data from the financial markets." So, "expected return-based methods" are "market-based." "Market-based methods" are based on "observable" prices of various "buy-and-hold" portfolios (e.g. matching bond portfolios, group annuity contracts), which can be default-free or otherwise. These portfolios do have expected returns implied by their market prices. So, "market-based methods" are "expected return-based."

The separation of "expected return-based" are "market-based" methods is unhelpful because "expected return-based methods" are "market-based" and "market-based methods" are "expected return-based." The correct way to distinguish "the budget" "and the solvency" values is to distinguish conventional policy portfolios of risky assets and "buy-and-hold" assets.

After a less than promising start, the issue brief presents a simplified example of the commitment to pay $1,000,000 in 10 years. The brief considers two portfolios to fund this commitment:

A. "a 10-year zero coupon Treasury note" with a certain "effective return of 3%";
B. "a diversified portfolio of assets" with an uncertain return.

Since portfolio A returns are certain, the present value of the commitment is certain as well. "The solvency value" invested in portfolio A today would be "the matching asset" that would fund the commitment with certainty.

Unlike portfolio A, portfolio B returns are uncertain, and the brief is needlessly vague about the investor's expectations of these returns. The brief offers several ambiguous allusions of these expectations. Let us look at some of them.

"Return 6 percent" makes its first appearance in a somewhat casual manner:

> "*If you could reasonably expect the portfolio to return 6 percent, an investment of just $558,000 would be expected to fund the debt …*"

The trouble is no one "could reasonably expect the portfolio to return 6 percent" over 10 years. The probability of this event is close to zero. Yet, the authors continue to allude to "6 percent":

> "*The portfolio might earn more or less than 6 percent over the 10 years.*"

This is true, but the same is true for 5 percent, 7 percent, or any other value.

The next target of the authors' allusions is the term "expected return":

> "*The expected return on assets often represents the median or the average of an array of estimated rates based on the potential variability of the return of the portfolio.*"

It is nice to know what the term represents "often," but it would be much nicer to know the meaning of the term. Is it "the median" or "the average"? It would also be nice to know the meaning of "an array of estimated rates based on the potential variability of the return of the portfolio."

Almost immediately, the authors add another "allusion" to the term "expected return":

> "*The expected return on assets is often set as the median expected return of a wide range of possible outcomes.*"

Again, it is nice to know what the term "expected return" is "often set as," but would be much nicer to know what the term is "set as" in this example. The trouble is the term "the median expected return" is the authors' brainchild. Textbooks distinguish the terms "median return" and "expected return."

It would also be nice if ASOP 27 provided some guidance regarding the term "expected return," and ASOP 27 seemingly does so. ASOP 27 instructs actuaries to distinguish between "forward looking expected arithmetic and geometric returns," not between "the median or the average" returns. So, technically, the issue brief ignores a direct instruction from ASOP 27.

Unfortunately, ASOP 27 does not define the terms "forward looking expected arithmetic and geometric returns." Moreover, these terms do not exist outside of this ASOP – they are brainchildren of the contemporaneous authors of this ASOP.

While the meaning of the "forward looking expected arithmetic return" is not hard to guess (it is the arithmetic mean), the meaning of the "forward looking expected geometric return" is much more mystifying. Without defining the term, ASOP 27 presents its calculations in a numeric example. Those who thought that the "forward looking expected geometric return" may be equal to the geometric mean would be disappointed – their values are different in this example.

The whole point of these "allusions" and other exercises in semantics is to justify calculating "the solvency value" and "the budget value" similarly – via fixed rate discounting. "The solvency value" $744,000 is equal to the commitment discounted at 3%: $744,000 = $1,000,000 / (1+3%)[10] (the values in the brief are rounded to the nearest thousand; to be consistent, we follow the same convention in this discussion). This calculation is reasonable because returns *are* 3% in all years. In other words, the calculation is reasonable because the key assumption is reasonable.

“The budget value” \$558,000 is equal to the commitment discounted at 6%: \$558,000 = \$1,000,000 / $(1+6\%)^{10}$. The meaning of this calculation is the following: “the budget value” is equal to \$558,000 if portfolio B returns are 6% in all years. “The budget value” as presented in this calculation is problematic because the key assumption is problematic – the probability of these returns is close to zero. This and similar probabilities are estimated in *Example 3*.

Example 3: Probability of Fixed Return over N Years. Technically, the probability of “returns *I*% in all years” is equal to zero (for continuous return distributions). Still, we may want to estimate the probability of returns to be close to 6%, e.g., the probability of returns between 5.9% and 6.1% in all years. Also, as discussed earlier in the paper, it is informative to estimate the probability of returns to be close to 30% – e.g., the probability of returns between 29.9% and 30.1% in all years.

To be consistent, this example utilizes the same CMAs as in other numerical examples. Note that a portfolio of 57.8% stocks and 42.2% bonds has geometric mean 6.00% and standard deviation 9.88% (see the Appendix I for more details). *Exhibit 3* presents the probabilities of portfolio returns to be in the abovementioned ranges over *N* years for this portfolio.

***Exhibit 3*. Probability of Return in Ranges over *N* Years**

Range	$N = 1$	$N = 10$	$N = 80$
Between 5.9% and 6.1%	$8.13 \cdot 10^{-3}$	$1.25 \cdot 10^{-21}$	$6.15 \cdot 10^{-168}$
Between 29.9% and 30.1%	$5.85 \cdot 10^{-4}$	$4.69 \cdot 10^{-33}$	$2.35 \cdot 10^{-259}$

Exhibit 3 shows that “the budget value” as presented in the brief’s calculation is virtually meaningless because the probability of returns close to 6% in all years is just $1.25 \cdot 10^{-21}$. Moreover, conventional actuarial valuations routinely assume constant returns for 80 years or more. *Exhibit 3* quantifies the point made earlier in the paper: the probabilities of returns close to 6% and 30% in 80 years are next to improbable – they are $6.15 \cdot 10^{-168}$ and $2.35 \cdot 10^{-259}$ correspondingly.

Yet, “the budget value” is a valuable measurement of the asset value to be invested in portfolio B. *Example 4* demonstrates that “the budget value” is the median of the SPV for portfolio B.

Example 4: SPVs for Portfolios A and B. The mean, the standard deviation, and the median of the SPVs for portfolios A and B are presented in *Exhibit 4* (see the Appendix I for technical details).

***Exhibit 4*. SPVs of $1,000,000 in Ten Years**

	SPV Mean	SPV St Dev	SPV Median	Solvency Value	Budget Value
Portfolio A	744,000	0	744,000	744,000	
Portfolio B	583,000	174,000	558,000		558,000

Examples 3 and *4* demonstrate that the calculation of "the budget value" as the commitment discounted at 6% under the assumption "portfolio B returns are 6% in all years" makes little sense. Yet, the same calculation of "budget value" makes sense as an estimate of the SPV median. The risk premium embedded in these calculations is not "riskless"– the volatility of portfolio B return is an indispensable part of the calculations. It should be noted that these calculations do not use a discount rate – measurements of SPV are calculated directly from the CMAs. Again, the use of a discount rate is a choice, not a necessity.

If needed, the discount rate implied by the median SPV $558,000 can be calculated now, and this discount rate happens to be the median return 6%. Generally, the median return set as the discount rate *does not* generate the median SPV. It does in this example because the commitment consists of just one payment. More on this issue to come later in this section.

Other measurements of SPVs can be useful as well. SPVs are directly related to the ability of a portfolio (or, more generally, a glide path) to fund the commitment. For instance, the *P*th percentile of SPV is equal to the asset value required today to have probability of funding *P*%. *Example 5* presents selected SPV percentiles and demonstrates their usefulness for funding problems.

<u>Example 5</u>: Selected SPV percentiles for Portfolios A and B. Selected SPV percentiles for portfolios A and B are presented in *Exhibit 5* (see the Appendix I for technical details).

***Exhibit 5*. Selected SPV Percentiles**

	Median	60th %ile	70th %ile	80th %ile	90th %ile
Portfolio A	744,000	744,000	744,000	744,000	744,000
Portfolio B	558,000	601,000	651,000	715,000	813,000

Exhibit 5 adds valuable information to the investor's choices presented in the brief: to invest $558,000 in portfolio A with 50% probability of funding or $744,000 in portfolio B with 100% probability of funding. In other words, the brief's framework allows *two* probabilities of funding (50% and 100%) along with required present values. The use of SPVs allows *all* probabilities along with required present values. For instance, if the investor required 70% probability of funding, then $651,000 invested in portfolio B would be the lowest required present value. If the investor requires 90% probability of funding the commitment, then $744,000 invested in portfolio

A would be the lowest required present value. This information is not available within the confines of the framework utilized in the brief.

In July 2017, the AAA published another issue brief essentially on the same subject. The main message – "More Than One Right Number Tells the Whole Story" – is emphasized in the brief's title. The 2017 issue brief continues and amplifies the shortcomings of the 2013 issue brief and the contemporaneous version of ASOP 27.

The 2017 brief continues the unfortunate trend of avoiding examples of retirement funding problems – it presents a numerical example of a *higher education* funding problem. While the financial commitment considered in the 2013 brief consists of one certain payment, the financial commitment considered in this funding problem consists of four uncertain payments of $10,000 adjusted for (educational) inflation payable 10, 11, 12, and 13 years from now.[34]

The parents expect "increases of 5 percent per year from now until graduation." The "parents … want to calculate an "obligation" for their daughter's college to include in their financial planning." The exact purpose of this "obligation" is unclear, but it appears that this "obligation" represents the asset value required today to fund the commitment.

The three portfolios considered in this example generate three different "obligations" as follows:

A. *"Very high credit quality zero-coupon bonds" with terms of 10, 11, 12, and 13 years at 6%*. The corresponding "obligation" (calculated using a discount rate of 6%) is $35,871, i.e. $35,871 invested in this portfolio "will generate the expected tuition payments." This portfolio may generate a shortfall or surplus due to the inflation volatility even though the investment return is a fixed 6%. Note that the inflation volatility plays no role in the calculations.
B. *A diversified investment portfolio of risky assets that has a median return 8%*. The corresponding "obligation" (calculated using a discount rate of 8%) is $28,945, i.e. $28,945 invested in this portfolio "will generate the expected tuition payments." This portfolio may generate a shortfall or surplus due to return and inflation volatilities. Note that these volatilities play no role in these calculations, but the aforementioned "riskless" risk premium does.
C. *The "prepaid tuition" portfolio*. The corresponding "obligation" is $40,000, i.e. $40,000 invested in this portfolio funds the commitment with certainty.

Let us look at the SPVs for portfolios A and B using the return and inflation volatilities. In *Examples 6-7*, conventional discount rate based "deterministic" present values are compared to the key "stochastic" measurements of SPVs.

Example 6: Present Values for Portfolios A: deterministic vs. stochastic. The key measurements of the SPV for portfolio A are presented in *Exhibit 6* (see the Appendix I for technical details).[35]

***Exhibit 6*. Portfolio A Present Value Measurements: Deterministic vs. Stochastic**

Portfolio A	SPV Mean	SPV St Dev	SPV Median	"Obligation"
Deterministic	35,871	0	35,871	35,871
Stochastic	35,890	1,127	35,872	

In this example, the volatility of the present value is driven by the inflation volatility, which is low. The volatility of the present value is low as well; the mean and the median of the present value are close; the "obligation" is a good approximation of the mean and the median. If needed, the discount rate implied by the median SPV $35,872 can be calculated now. This discount rate is 5.9997%, which is very close to portfolio A return 6%.

Example 7: Present Values for Portfolios B: deterministic vs. stochastic. The key measurements of the SPV for portfolio B are presented in *Exhibit 7* (see the Appendix I for technical details).[36]

***Exhibit 7*. Portfolio B Present Value Measurements: Deterministic vs. Stochastic**

Portfolio B	SPV Mean	SPV St Dev	SPV Median	"Obligation"
Deterministic	28,945	0	28,945	28,945
Stochastic	32,716	16,714	29,134	

In this example, the volatility of the present value is driven by the return volatility and the inflation volatility. The volatility of the present value is relatively high; the "obligation" is not a good approximation of the mean; the "obligation" is a "passable" approximation of the median. If needed, the discount rate implied by the median SPV $29,134 can be calculated now. This discount rate is 7.9378%, which is reasonably close to portfolio B median return 8%.

Similar to *Example 4*, *Examples 6* and *7* demonstrate that the calculation of an "obligation" as the commitment discounted at a fixed rate makes sense as an estimate of the SPV median. The risk premium embedded in these calculations is not "riskless"– the return and inflation volatilities are indispensable parts of the calculations. Also, these calculations do not use a discount rate. Once again, the use of a discount rate is a choice, not a necessity.

Example 8: Selected SPV percentiles for Portfolios A and B. Selected SPV percentiles for portfolios A and B are presented in *Exhibit 8* (see the Appendix I for technical details).[37]

***Exhibit 8*. Selected SPV Percentiles**

	Median	60th %ile	70th %ile	80th %ile	90th %ile

Portfolio A	35,872	36,158	36,467	36,832	37,344
Portfolio B	29,134	32,915	37,504	43,694	54,004
Portfolio C	40,000	40,000	40,000	40,000	40,000

Similar to *Exhibit 5*, *Exhibit 8* adds valuable information to the investor's choices presented in the brief: to invest $35,871 in portfolio A with 50% probability of funding, or $28,945 in portfolio B with 50% probability of funding, or $40,000 in portfolio C with 100% probability of funding. The brief's framework allows *two* probabilities of funding (50% and 100%) along with required present values. The use of SPVs allows *all* probabilities along with required present values. For instance, if the investor required 70% probability of funding, then $36,467 invested in portfolio A would be the lowest required present value. If the investor requires 90% probability of funding the commitment, then $37,344 invested in portfolio A would be the lowest required present value. This information is not available within the confines of the framework utilized in the brief.

Overall, there are certain important issues in the actuarial model that are not properly addressed in leading actuarial publications. The 2013-2017 AAA issue briefs and ASOP 27 attempt to clarify these issues and offer some guidance and numerical examples. Yet, these publications do not utilize appropriate and readily available quantitative and qualitative tools. Ambivalent guidance from the highest echelons of the actuarial profession is detrimental to the profession as well as to the development of innovative solutions retirement plan participants need.

3.4. What is *FE*?

"We have now sunk to a depth at which
restatement of the obvious is
the first duty of intelligent men."
George Orwell

Given the role the *FE* mindset plays in the DC and DB plan industries, it is important to spell out the key attributes of *FE*.

FE reflects the principles of asset pricing and corporate finance. These principles may be perfectly reasonable when used to achieve *pricing* objectives. Yet, these principles may be inappropriate and even detrimental to *funding* objectives. Overall, *FE* is an attempt to apply the principles of corporate finance and asset pricing beyond the scope of their applicability.

FE prioritizes "pricing" over funding, risk-free "transparency" over risk management, hypothetical portfolios over real life investing, deterministic present values over stochastic present values.

These priorities are based on asset allocation preferences, not on sound principles of finance. As an economic theory, *FE* conflicts with the tenets of a broadly defined financial economics.[38]

In *FE*, present values are "market values" that do not depend on investment portfolios. Market values cannot be optimized – they are what they are. Since portfolio selection is irrelevant in *FE*, the primary objectives of retirement investors – selecting portfolios that maximize the safety of retirement payments and minimize the cost of their funding – cannot be achieved in *FE*.

From a practical standpoint, *FE* has no place for some of the most important tasks of retirement plan management. From a theoretical standpoint, *FE* contains glaring violations of established academic standards as related to assumptions, terminology, and objectives. From an asset allocation standpoint, *FE* gravitates toward quasi-risk-free assets (e.g., matching bonds, annuities). From a common-sense standpoint, *FE* does not withstand even a cursory examination. A comprehensive framework that encompasses *FE* is feasible. Overall, *FE* is a "scandal."

4. NEXT STEPS

"Your assumptions are your windows on the world.
Scrub them off every once in a while,
or the light won't come in." [39]

The deterioration of the foundation of retirement investing is a serious issue. A shaky foundation may impact the key tasks of retirement plan management and deliver sub-optimal outcomes. A shaky foundation may adversely affect the magnitude and safety of retirement income as well as the cost of its funding. A shaky foundation may also adversely affect the industry's reputation.

Actuarial organizations have long been reluctant to revisit the foundations of actuarial practices. While many realize that *FE* does not represent a credible alternative, the flaws of *FE* should not disguise the shortcomings of the conventional actuarial methodologies. It is true that the "cure" of *FE* may be worse than the "disease," but this "disease" exists nonetheless.

To correct this situation, the actuarial and financial economics communities should jointly take several steps. These steps include clarifying the basics, clearing up the language, eliminating the confusion that is abundant in current publications, and creating a solid foundation for further developments. This section sketches out one way to do so.

The first step is to define the key concepts. A retirement commitment is defined as a series of future payments. There are two types of retirement commitments: spending commitments (out-flows) and contribution commitments (in-flows). Retirement commitments are contingent long-term cash flows of uncertain magnitude and timing.

The primary objective of a retirement investor is to fund spending commitments. To achieve this and other objectives, measurements of retirement commitments must be consistent with the objectives. Vital measurements of retirement commitments called "present values" come from the principle of time value of money. Different objectives may require different present values. Series of investment portfolios that extend through the commitments' time horizon (commonly called "glide paths") play a critical role in the calculations of present values.

The following points may serve as a conceptual foundation for the development of present values of retirement commitments funded by risky assets.

- To fund a commitment means being able to make every payment when it comes due.
- A present value of a commitment represents the asset value at the present required to fund the commitment.
- A present value is calculated via a discounting procedure. Any discounting procedure assumes investing in a glide path. Different glide paths may generate different present values.
- A present value of a payment is equal to the payment divided by the compounded return factor. A present value of a commitment is equal to the sum of the present values of all payments.
- If the investor endeavors to fund his commitments via investing in risky assets, present values of these commitments are generally stochastic and should be modeled as random variables.
- There may be valid reasons to require deterministic present values. These reasons may include solvency, valuation, financial reporting, and compliance considerations. The requirement for present values to be deterministic is an assumption that should be disclosed and justified.
- One way to calculate a deterministic present value is a) to select deterministic measurements of the commitments; b) to select deterministic measurements of portfolio returns as discount rates; c) to calculate conventional present values. Another way is to estimate the present value distribution and utilize a measurement of this distribution (e.g., the mean or a percentile) as the deterministic present value. The former uses discount rates, the latter does not. *The use of discount rates for present value calculations is a choice, not a necessity*.
- The terminology used in the industry should be revised. Retirement commitments and their present values are commonly called "liabilities." This dual use of the term "liabilities" should be discontinued. The term "liability" has specific meanings defined by the actuarial, accounting, and other governing bodies. This term should not be used outside of the boundaries determined by these governing bodies. Present values of retirement commitments need a separate general term. This author proposes "*Required Assets*" as a candidate for this term.

- The primary risk is defined as the failure of the investor's primary objective (to fund spending commitments). In other words, *the primary risk is defined as the shortfall event* – contribution commitments and investment returns are insufficient to fund spending commitments.
- Further steps include a detailed description of funding problems and the identification of appropriate risk measurements. For example, DB plan participants may want to maximize the safety of their benefits given the level of contributions and the risk "budget"; taxpayers/shareholders may want to minimize the cost of pension funding given the benefits and the risk "budget." Another example, DC plan participants may want to maximize their retirement payouts.

These points should be thoroughly vetted and debated. The actuarial and financial economics communities should jointly sponsor a wide-ranging debate in this area. An open debate that promotes diverse views should be highly beneficial to the stakeholders of retirement plans.

Conclusion

"One's got to change the system,
or one changes nothing."
George Orwell

Academic standards evolve over time. What appeared to be a reasonable framework at some point may turn into a "scandal" when/if a better framework becomes feasible. Ending this "scandal" should be a priority.

The pricing assumption in *FE* may have been an acceptable shortcut at some point. Today, under the banner of financial economics, the proponents of *FE* promote an inferior framework while a superior framework is viable. The negative impact of the *FE* "scandal" is tangible and substantial. *FE* is likely to be detrimental to the best interests of retirement plan participants.

Should retirement investors utilize the princples of financial economics? Absolutely. DB and DC, corporate and public, ongoing and closed, frozen, terminated, etc., – all retirement plans should utilize the sound princples of finance. But should retirement investors utilize *FE*? They should be exceedingly skeptical. Investors should utilize sound economic theories, and *FE* is at best an asset allocation viewpoint, not a sound economic theory.

Why do many economists and asset managers support *FE*? It is natural for any professional group to pursue more applications of their expertise. Yet it is essential for any group to appreciate the limits of the applicability of their expertise and the fundamental difference between consensus and groupthink. This appreciation appears lacking among the proponents of *FE*.

On the surface, this lack of appreciation looks like a clear case of the law of the instrument (a.k.a. Kaplan-Maslow law of the hammer). This concept is popularly formulated as "if all you have is a hammer, everything looks like a nail." In case of *FE*, this metaphor can be rephrased as "if all you have is asset pricing tools, every cash flow looks like a tradable asset." This author is skeptical that anything of value lies beneath the surface, but open to suggestions.

This paper is a call to jump-start an open debate about the foundation of retirement investing. This call is timely for several reasons. Some leading publications as well as education and examination materials in this area utilize "rules of thumb and folklore" when more disciplined approaches are viable. Certain solutions currently offered to retirement investors are likely to generate sub-optimal outcomes. The challenge is to develop an investor-specific "genuine" approach to retirement plan management that is based on the best interests of the stakeholders of retirement plans.

It remains to be seen if the leadership of the actuarial and financial economics communities are up to this challenge.

APPENDIX I: Numerical Examples, Technical Details

This appendix presents the details of the calculations for all numerical examples in this paper. The following basic definitions, assumptions, and facts about portfolio returns are applicable to all numerical examples.

Arithmetic mean A for random variable R is defined as follows:

$$A = E(R) \tag{A1}$$

Standard deviation S for random variable R is defined as follows:

$$S = \sqrt{E\left(\left(R - A\right)^2\right)} \tag{A2}$$

Geometric mean G for random variable R such that $1 + R > 0$ is defined as follows:

$$G = \exp\left(E\left(\ln\left(1 + R\right)\right)\right) - 1 \tag{A3}$$

It is assumed that all expected values in definitions (A1) – (A3) exist and are finite.

There are n asset classes under consideration. For asset class k, R_k is the investment return that is modeled as a random variable. For investment return R_k, A_k is the arithmetic mean and S_k is the standard deviation. For asset classes i and j, c_{ij} is the correlation coefficient between the investment returns for these asset classes.

A portfolio is defined as series $\{X_k\}_{k=1}^{n}$, where X_k is the proportion of total assets invested in asset class k, $0 \le X_k \le 1$ for all k, and $\sum_{k=1}^{n} X_k = 1$.

Portfolio arithmetic mean A is calculated as follows:

$$A = \sum_{k=1}^{n} X_k A_k \tag{A4}$$

Portfolio standard deviation S is calculated as follows:

$$S = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j S_i S_j c_{ij} \tag{A5}$$

Return factor is defined as $1+R$, where R is portfolio return. *Return factors are assumed to have lognormal distributions* in all numerical examples in this paper, i.e., $\ln(1+R)$ is assumed to have normal distribution $N(\mu,\sigma)$. This assumption is called *the lognormal assumption* in this paper.[40]

Let Y be lognormal random variable. Then $\ln(Y)$ has normal distribution $N(\mu,\sigma)$, and the relationships between $E(Y)$, $Var(Y)$, μ, and σ are well-known.[41] Given μ and σ, $E(Y)$ and $Var(Y)$ are calculated as follows:

$$E(Y) = \exp\left(\mu + \frac{1}{2}\sigma^2\right) \tag{A6}$$

$$Var(Y) = \exp\left(2\mu + 2\sigma^2\right) - \exp\left(2\mu + \sigma^2\right) \tag{A7}$$

Given $E(Y)$ and $Var(Y)$, μ and σ are calculated from the following equations:

$$\exp(\mu) = \frac{E(Y)}{\sqrt{1 + \frac{Var(Y)}{E(Y)^2}}} \tag{A8}$$

$$\exp(\sigma^2) = 1 + \frac{Var(Y)}{E(Y)^2} \tag{A9}$$

Under the lognormal assumption, the median of $1+R$ is equal to $\exp(\mu)$. From (A3), $\exp(\mu)=1+G$, where G is the geometric mean, so the geometric mean is equal to the median. The relationship between A, G, and S is as follows:

$$1+G=\frac{1+A}{\sqrt{1+\frac{S^2}{(1+A)^2}}} \tag{A10}$$

or, equivalently,

$$1+A=(1+G)\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{4S^2}{(1+G)^2}}}$$

The return *p*th percentile T_p is calculated as follows:

$$T_p=\exp\left(\mu+\sigma\Phi_{0,1}^{-1}(p)\right)-1 \tag{A11}$$

where $\Phi_{0,1}^{-1}(p)$ is the inverse of the standard normal distribution.

Examples 1 – 5 utilize the following capital market assumptions:

Return/Risk

	Geometric Mean (%)	Arithmetic Mean (%)	Standard Deviation (%)
Stocks	7.00	8.16	16.00
Bonds	4.00	4.12	5.00

Correlation Matrix

	Stocks	Bonds
Stocks	1	0.20
Bonds	0.20	1

Portfolio returns in different years are assumed independent.

Example 1: Estimate Future Values Given Present Value.

Given \$100 at the present, the goal of is to estimate measurements of future value $100(1+R)$. The following three portfolios are considered: 100% stocks/0% bonds, 50% stocks/50% bonds, and 0%

stocks/100% bonds. The calculations for the 50/50 portfolio are below; the calculations for the other portfolios are similar:

the arithmetic mean is calculated in (A4): $A = 0.0614$;
the standard deviation is calculated in (A5): $S = 0.0885$;
the geometric mean is calculated in (A10): $G = 0.0578$.

Under the lognormal assumption, $\ln(1+R)$ has normal distribution $N(\mu, \sigma)$. For this distribution, parameters μ and σ are calculated as follows:

μ is calculated in (A8): $\mu = 0.0561$;
σ is calculated in (A9): $\sigma = 0.0832$.

The 30th percentile is calculated in (A11): $T_p = 0.0126$.

Given *PV*=$100, the *FV* measurements are calculated directly form the corresponding portfolio return measurements:

FV Mean $= 100(1+0.0614) = 106.14$
FV Median $= 100(1+0.0578) = 105.78$
FV Standard Deviation $= 100 \cdot 0.0885 = 8.85$
FV 30th percentile $= 100(1+0.0126) = 101.26$

These results are summarized in *Exhibit 1*.

***Exhibit 3*. Future Value Estimates**

	100% Stocks 0% Bonds	50% Stocks 50% Bonds	0% Stocks 100% Bonds
FV Mean	108.16	106.14	104.12
FV Median	107.00	105.78	104.00
FV St Deviation	16.00	8.85	5.00
FV 30th %ile	99.06	101.26	101.42

Example 2: Estimate Present Values Given Future Value.

The goal of is to estimate the asset value required to fund \$100 at the end of the period, i.e. present value $\frac{100}{1+R}$. As in *Example 1*, the following three: 100% stocks/0% bonds, 50% stocks/50% bonds, and 0% stocks/100% bonds. The calculations for the 50/50 portfolio are below; the calculations for the other portfolios are similar:

Under the lognormal assumption, $\ln(1+R)$ is normal $N(\mu,\sigma)$, therefore $\ln\left(\frac{1}{1+R}\right)$ is normal $N(-\mu,\sigma)$. As calculated in *Example 1*, $\mu = 0.0561$ and $\sigma = 0.0832$.

PV Mean is calculated in (A6):

$$PV\text{ Mean } = 100\exp\left(-0.0561 + \frac{1}{2}0.0832^2\right) = 94.87$$

$$PV\text{ Median } = 100\exp(-0.0561) = 94.54$$

PV Standard Deviation is calculated in (A7):

$$PV\text{ St Dev } = \sqrt{\exp\left(-2\cdot 0.0561 + 0.0832^2\right)\left(\exp\left(0.0832^2\right) - 1\right)} = 7.91$$

PV 70th percentiles is calculated in (A11):

$$T_{0.7} = \exp\left(-0.0561 + 0.0832\cdot\Phi_{0,1}^{-1}(0.7)\right) - 1 = 98.76$$

These results are summarized in *Exhibit 2*.

***Exhibit 4*. Present Value Estimates**

	100% Stocks 0% Bonds	50% Stocks 50% Bonds	0% Stocks 100% Bonds
PV Mean	94.47	94.87	96.26
PV Median	93.46	94.54	96.15
PV St Deviation	13.98	7.91	4.62
PV 70th %ile	100.95	98.76	98.60

Example 3: Probability of Fixed Return over N Years.

The basic measurements of the 57.8% stocks and 42.2% bonds portfolio return are as follows:

the arithmetic mean is calculated in (A4): $A = 0.0646$;
the standard deviation is calculated in (A5): $S = 0.0988$.

The geometric mean is calculated in (A10): $G = 0.0600$.

Under the lognormal assumption, $\ln(1+R)$ has normal distribution $N(\mu,\sigma)$. For this distribution, parameters μ and σ are calculated as follows:

μ is calculated in (A8): $\mu = 0.0583$;
σ is calculated in (A9): $\sigma = 0.0926$;

The probability of portfolio return to be between a and b ($a \le b$) for N years is equal to

$$\left(\Pr(a < R < b)\right)^N = \left(\Phi_{\mu,\sigma}\left(\ln(1+b)\right) - \Phi_{\mu,\sigma}\left(\ln(1+a)\right)\right)^N \quad \text{(A12)}$$

where $\Phi_{\mu,\sigma}$ is the normal cumulative distribution function with parameters μ and σ. For example, the probability of portfolio return to be between 5.9% and 6.1% for 10 years is equal to

$$\left(\Pr(0.059 < R < 0.061)\right)^{10} = \left(\Phi_{\mu,\sigma}\left(\ln(1.061)\right) - \Phi_{\mu,\sigma}\left(\ln(1.059)\right)\right)^{10} =$$
$$= (0.50406 - 0.49593)^{10} = 1.25 \cdot 10^{-21}$$

Some numerical applications of formula (A12) are summarized in *Exhibit 3*.

***Exhibit 3*. Probability of Return in Ranges over *N* Years**

Range	$N = 1$	$N = 10$	$N = 80$
Between 5.9% and 6.1%	$8.13 \cdot 10^{-3}$	$1.25 \cdot 10^{-21}$	$6.15 \cdot 10^{-168}$
Between 29.9% and 30.1%	$5.85 \cdot 10^{-4}$	$4.69 \cdot 10^{-33}$	$2.35 \cdot 10^{-259}$

Example 4: Present values of $1,000,000 in ten years.

In this example, the results are rounded to the nearest thousand (to be consistent with the 2013 brief).

As in *Example 3*, the given portfolio consists of 57.8% stocks and 42.2% bonds. The goal of is to estimate the asset value required to fund \$1,000,000 at the end of 10 years, i.e. present value $\frac{1,000,000}{(1+R_1)\ldots(1+R_{10})}$, where $R_1,\ldots,R_{10}$ are independent identically distributed portfolio returns.

As in *Example 3*, the basic measurements of the 57.8/42.2 portfolio return are $\mu = 0.0583$ and $\sigma = 0.0926$. Under the lognormal assumption, $\ln\left(\frac{1}{1+R}\right)$ is normal $N(-\mu,\sigma)$, therefore $\ln\left(\frac{1}{(1+R_1)\ldots(1+R_{10})}\right)$ is normal $N\left(-10\mu,\sigma\sqrt{10}\right)$.

PV Mean is calculated in (A6):

$$1,000,000\exp\left(-0.0583\cdot 10+\frac{1}{2}0.0926^2\cdot 10\right)=582,873\approx 583,000$$

PV Standard Deviation is calculated in (A7):

$$1,000,000\sqrt{\exp\left(-2\cdot 0.0583\cdot 10+0.0926^2\cdot 10\right)\left(\exp\left(0.0926^2\cdot 10\right)-1\right)}=174,472\approx 174,000$$

PV Median $=1,000,000\exp(-0.0583\cdot 10)=558,394\approx 558,000$

These results are summarized in *Exhibit 4*.

***Exhibit 4*. PVs of \$1,000,000 in Ten Years**

	SPV Mean	SPV St Dev	SPV Median	Solvency Value	Budget Value
Portfolio A	744,000	0	744,000	744,000	
Portfolio B	583,000	174,000	558,000		558,000

<u>Example 5</u>: Selected SPV percentiles for Portfolios A and B.

In this example, the results are rounded to the nearest thousand to be consistent with the 2013 brief.

Under the lognormal assumption, $\ln\left(\frac{1}{(1+R_1)\dots(1+R_{10})}\right)$ is normal $N\left(-10\mu,\sigma\sqrt{10}\right)$. Similar to (A11), percentiles of $\frac{1,000,000}{(1+R_1)\dots(1+R_{10})}$ are equal to $1,000,000\left(\exp\left(-10\mu+\sigma\sqrt{10}\cdot\Phi^{-1}(p)\right)-1\right)$.

For example, the 70th percentile of this SPV is equal to

$$1,000,000\left(\exp\left(0.0583\cdot 10+0.0926\cdot\sqrt{10}\cdot\Phi^{-1}(0.7)\right)-1\right)=651,110\approx 651,000$$

These results are summarized in *Exhibit 5*.

***Exhibit 5*. Selected SPV Percentiles**

	Median	60th %ile	70th %ile	80th %ile	90th %ile
Portfolio A	744,000	744,000	744,000	744,000	744,000
Portfolio B	558,000	601,000	651,000	715,000	813,000

In the numerical examples considered so far, the financial commitments contain just one certain payment. In contrast, the numerical examples in the 2017 issue brief deal with multiple uncertain payments. Specifically, the financial commitment considered in the higher education funding problem consists of four uncertain payments of $10,000 adjusted for inflation payable 10, 11, 12, and 13 years from now. The corresponding SPV is defined as follows:

$$RA_0=\sum_{i=10}^{13}Y_1\cdot\ldots\cdot Y_k=Y_1\cdot\ldots\cdot Y_9\cdot\left(Y_{10}+Y_{10}Y_{11}+Y_{10}Y_{11}Y_{12}+Y_{10}Y_{11}Y_{12}Y_{13}\right) \qquad \text{(A13)}$$

where $Y,Y_1,\ldots,Y_{13}$ are independent identically distributed random variables. If RA_{10} is defined as

$$RA_{10}=Y_{10}+Y_{10}Y_{11}+Y_{10}Y_{11}Y_{12}+Y_{10}Y_{11}Y_{12}Y_{13}$$

then from (A13) we have

$$RA_0=Y_1\cdot\ldots\cdot Y_9\cdot RA_{10} \qquad \text{(A14)}$$

The moments of RA_{10} are calculated as follows.

$$E\left(RA_0^k\right)=\left(E\left(Y^k\right)\right)^9 E\left(RA_{10}^k\right) \qquad \text{(A15)}$$

While $E\left(Y^k\right)$ in (A15) are calculated directly from the CMAs, $E\left(RA_{10}^k\right)$ require additional work. The following proposition provides related formulas for the first and second moments.[42]

<u>Proposition.</u> Let $Y, Y_1, \ldots, Y_n$ be independent identically distributed random variables and $RA = \sum_{k=1}^{n} Y_1 \ldots Y_k$. If $m_1 = E(Y) < \infty$ and $m_2 = E\left(Y^2\right) < \infty$, then

$$E(RA) = m_1 \frac{m_1^n - 1}{m_1 - 1} \quad \text{(A16)}$$

$$Var(RA) = \frac{m_2 + m_1}{m_2 - m_1} m_2 \frac{m_2^n - 1}{m_2 - 1} - \frac{2m_2}{m_2 - m_1} m_1 \frac{m_1^n - 1}{m_1 - 1} - \left(m_1 \frac{m_1^n - 1}{m_1 - 1} \right)^2 \quad \text{(A17)}$$

Examples 6 – 10 utilize the following capital market assumptions (to be consistent with the 2017 issue brief):

Return/Risk

	Geometric Mean (%)	Arithmetic Mean (%)	Standard Deviation (%)
Asset Class A	6.000	6.000	0.000
Asset Class B	8.000	9.154	16.000
Inflation	5.000	5.005	1.000

Correlation Matrix

	Asset Class A	Asset Class B	Inflation
Asset Class A	1	0.00	0.00
Asset Class B	0.00	1	0.00
Inflation	0.00	0.00	1

Return and inflation factors are assumed lognormal; returns and inflation in different years are assumed independent.

<u>Example 6</u>: *Present Values for Portfolio A: deterministic vs. stochastic*.

Portfolio A is 100% in asset class A. Portfolio A return has geometric mean 6% and standard deviation 0%.

Let R_k and I_k be portfolio returns and inflation in year *k*, where $k = 1, \ldots, 13$. We consider inflation adjusted payments of $1; the results will be scaled up to $10,000 later.

Return factors $1+R_k$ are assumed lognormal, so $\ln(1+R_k)$ is normal $N(\mu_1,\sigma_1)$. Then μ_1 and σ_1 are calculated from (A8) and (A9) as follows:

$$\mu_1 = \ln(1.06) = 0.0583$$

$$\sigma_1 = \sqrt{\ln\left(1+\frac{0^2}{1.06^2}\right)} = 0$$

Inflation factors $1+I_k$ are assumed lognormal, so $\ln(1+I_k)$ is normal $N(\mu_2,\sigma_2)$. Then μ_2 and σ_2 from (A8) and (A9) are calculated as follows:

$$\mu_2 = \ln(1.05) = 0.0488$$

$$\sigma_2 = \sqrt{\ln\left(1+\frac{0.01^2}{1.05005^2}\right)} = 0.0095$$

Let us define $Y_k = \frac{1+I_k}{1+R_k}$ for $k=1,\ldots,13$. Then Y_k is lognormal, so $\ln(Y_k)$ is normal $N(\mu,\sigma)$. Then μ and σ are calculated as follows:

$$\mu = \mu_2 - \mu_1 = 0.0488 - 0.0583 = -0.0095$$

$$\sigma = \sigma_2 = 0.0095$$

SPV RA_0 is defined in (A13) as $RA_0 = \sum_{i=10}^{13} Y_1 \cdot \ldots \cdot Y_k$. The moments of RA_0 are calculated in (A15).

$$E\left(RA_0^k\right) = \left(E\left(Y^k\right)\right)^9 E\left(RA_{10}^k\right) = m_k^9 E\left(RA_{10}^k\right) \quad \text{(A18)}$$

The moments $m_1 = E(Y)$ and $m_2 = E\left(Y^2\right)$ are calculated as in (A6) and (A7):

$$m_1 = \exp\left(-0.0095 + \frac{1}{2}0.0095^2\right) = 0.9906 \quad \text{(A19)}$$

$$m_2 = \exp\left(-2\cdot 0.0095 + 2\cdot 0.0095^2\right) = 0.9814 \quad \text{(A20)}$$

The first and second moments of RA_{10} are calculated from (A16), (A17), (A19), and (A20):

$$E\left(RA_{10}\right)=m_1\frac{m_1^4-1}{m_1-1}=3.9070 \tag{A21}$$

$$E\left(RA_{10}^2\right)=\frac{m_2+m_1}{m_2-m_1}m_2\frac{m_2^4-1}{m_2-1}-\frac{2m_2}{m_2-m_1}m_1\frac{m_1^4-1}{m_1-1}=15.2671 \tag{A22}$$

The first and second moments of RA_0 are calculated from (A18), (A19), (A20), (A21), and (A22):

$$E\left(RA_0\right)=m_1^9E\left(RA_{10}\right)=3.5890 \tag{A23}$$

$$E\left(RA_0^2\right)=m_2^9E\left(RA_{10}^2\right)=12.8934 \tag{A24}$$

The standard deviation of RA_0 is calculated from (A23) and (A24):

$$StDev\left(RA_0\right)=\sqrt{E\left(RA_0^2\right)-\left(E\left(RA_0\right)\right)^2}=0.1127 \tag{A25}$$

It should be noted that the distribution of RA_0 is not known (even though the moments of RA_0 can be calculated). Still, we can estimate the median and other percentiles of RA_0, but may need additional approximations to do so. One way to do it is to utilize the lognormal approximation of the distribution of RA_0, i.e. to calculate the parameters of lognormal random variable Y that has the same first and second moments as RA_0.[43] In this case, $\ln\left(Y\right)$ is normal $N\left(\mu_3,\sigma_3\right)$, and parameters μ_3 and σ_3 are calculated from (A8) and (A9) as follows:

$$\mu_3=\ln\left(\frac{E\left(RA_0\right)}{\sqrt{1+\frac{Var\left(RA_0\right)}{E\left(RA_0\right)^2}}}\right)=1.2774$$

$$\sigma_3=\sqrt{\ln\left(1+\frac{Var\left(RA_0\right)}{E\left(RA_0\right)^2}\right)}=0.0314$$

The median of RA_0 is estimated as follows:

$$\exp(\mu_3) = 3.5872 \tag{A26}$$

It is possible to calculate the first *three* moments of RA_0 and calculate the parameters of random variable Y that has the same three moments as RA_0. The median of Y is close to 3.5872 in (A26).[44]

These results scaled up from $1 to $10,000 as well as the deterministic calculations from the 2017 issue brief are presented in *Exhibit 6*.

***Exhibit 6*. Portfolio A Present Value Measurements: Deterministic vs. Stochastic**

Portfolio A	SPV Mean	SPV St Dev	SPV Median	"Obligation"
Deterministic	35,871	0	35,871	35,871
Stochastic	35,890	1,127	35,872	

Example 7: Present Values for Portfolio B: deterministic vs. stochastic.

Portfolio B is 100% in asset class B. Portfolio B return has geometric mean 8% and standard deviation 16%.

Let R_k and I_k be portfolio returns and inflation in year *k*, where $k = 1,\ldots,13$. We consider inflation adjusted payments of $1; the results will be scaled up to $10,000 later.

Return factors $1+R_k$ are assumed lognormal, so $\ln(1+R_k)$ is normal $N(\mu_1,\sigma_1)$. Then μ_1 and σ_1 are calculated from (A8) and (A9) as follows:

$$\mu_1 = \ln(1.08) = 0.0770$$

$$\sigma_1 = \sqrt{\ln\left(1+\frac{0.16^2}{1.09154^2}\right)} = 0.1458$$

Inflation factors $1+I_k$ are assumed lognormal, so $\ln(1+I_k)$ is normal $N(\mu_2,\sigma_2)$. Then μ_2 and σ_2 are calculated from (A8) and (A9) as follows:

$$\mu_2 = \ln(1.05) = 0.0488$$

$$\sigma_2 = \sqrt{\ln\left(1+\frac{0.01^2}{1.05005^2}\right)} = 0.0095$$

Let us define $Y_k = \frac{1+I_k}{1+R_k}$ for $k = 1,\ldots,13$. Then Y_k is lognormal, so $\ln(Y_k)$ is normal $N(\mu,\sigma)$. Then μ and σ are calculated as follows (note that the inflation and portfolio B return are assumed uncorrelated):

$$\mu = \mu_2 - \mu_1 = 0.0488 - 0.0770 = -0.0282$$
$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} = 0.1461$$

SPV RA_0 is defined in (A13) as $RA_0 = \sum_{i=10}^{13} Y_1 \cdot \ldots \cdot Y_k$. The moments of RA_0 are calculated in (A15).

$$E\left(RA_0^k\right) = \left(E\left(Y^k\right)\right)^9 E\left(RA_{10}^k\right) = m_k^9 E\left(RA_{10}^k\right) \quad \text{(A27)}$$

The moments $m_1 = E(Y)$ and $m_2 = E\left(Y^2\right)$ are calculated in (A6) and (A7):

$$m_1 = \exp\left(-0.0282 + \frac{1}{2} \cdot 0.1461^2\right) = 0.9827 \quad \text{(A28)}$$
$$m_2 = \exp\left(-0.0282 \cdot 2 + 2 \cdot 0.1461^2\right) = 0.9864 \quad \text{(A29)}$$

The first and second moments of RA_{10} are calculated from (A16), (A17), (A28), and (A29):

$$E\left(RA_{10}\right) = m_1 \frac{m_1^4 - 1}{m_1 - 1} = 3.8295 \quad \text{(A30)}$$
$$E\left(RA_{10}^2\right) = \frac{m_2 + m_1}{m_2 - m_1} m_2 \frac{m_2^4 - 1}{m_2 - 1} - \frac{2m_2}{m_2 - m_1} m_1 \frac{m_1^4 - 1}{m_1 - 1} = 15.2602 \quad \text{(A31)}$$

The first and second moments of RA_0 are calculated from (A27), (A28), (A29), (A30), and (A31):

$$E\left(RA_0\right) = m_1^9 E\left(RA_{10}\right) = 3.2716 \quad \text{(A32)}$$
$$E\left(RA_0^2\right) = m_2^9 E\left(RA_{10}^2\right) = 13.4969 \quad \text{(A33)}$$

The standard deviation of RA_0 is calculated from (A32) and (A33):

$$StDev\left(RA_0\right) = \sqrt{E\left(RA_0^2\right) - \left(E\left(RA_0\right)\right)^2} = 1.6714 \quad \text{(A34)}$$

To estimate the median of RA_0, we calculate the parameters of lognormal random variable Y that has the same first two moments as RA_0. In this case, $\ln(Y)$ is normal $N(\mu_3,\sigma_3)$, and parameters μ_3 and σ_3 are calculated from (A8) and (A9) as follows:

$$\mu_3 = \ln\left(\frac{E(RA_0)}{\sqrt{1+\frac{Var(RA_0)}{E(RA_0)^2}}}\right) = 1.0693$$

$$\sigma_3 = \sqrt{\ln\left(1+\frac{Var(RA_0)}{E(RA_0)^2}\right)} = 0.4816$$

The median of RA_0 is estimated as follows:

$$\exp(\mu_3) = 2.9134 \tag{A35}$$

As in *Example 6*, it is possible to calculate the first *three* moments of RA_0 and calculate the parameters of random variable Y that has the same three moments as RA_0. The median of Y is 2.9126, which is close to 2.9134 in (A35).[45]

These results scaled up from $1 to $10,000 as well as the deterministic calculations from the 2017 issue brief are presented in *Exhibit 7*.

***Exhibit 7*. Portfolio B Present Value Measurements: Deterministic vs. Stochastic**

Portfolio B	SPV Mean	SPV St Dev	SPV Median	“Obligation”
Deterministic	28,945	0	28,945	28,945
Stochastic	32,716	16,714	29,134	

<u>Example 8</u>*: Selected SPV percentiles for Portfolios A and B.*

Under the lognormal assumption, $\ln(Y)$ is normal $N(\mu_3,\sigma_3)$. Similar to (A11), percentiles of Y are equal to $\exp\left(\mu_3+\sigma_3\Phi^{-1}(p)\right)$. For example, the 70th percentile of the SPV for portfolio A is calculated as follows:

$$\exp\left(1.2774 + 0.0314 \cdot \Phi^{-1}(0.7)\right) = 3.6467$$

The 70^{th} percentile of the SPV for portfolio B is calculated as follows:

$$\exp\left(1.0693 + 0.4816 \cdot \Phi^{-1}(0.7)\right) = 3.7504$$

These and similar results scaled up from $1 to $10,000 are presented in *Exhibit 8*.

***Exhibit 8*. Selected SPV Percentiles**

	Median	60th %ile	70th %ile	80th %ile	90th %ile
Portfolio A	35,872	36,158	36,467	36,832	37,344
Portfolio B	29,134	32,915	37,504	43,694	54,004
Portfolio C	40,000	40,000	40,000	40,000	40,000

APPENDIX II: "Liabilities" in Maginn (2007)

"I always knew what the right path was.
Without exception, I knew.
But I never took it. You know why?
It was too damn hard."
Frank Slade, Lieutenant Coronel (Retired)[46]

This appendix is reproduced with minor changes from Mindlin (2011).

The tendency to utilize the term "liability" in an undisciplined manner is widespread and largely overlooked. Numerous publications give different definitions of "liabilities," but even more publications give no definition at all. This tendency is as regrettable as it is out of sync with established academic standards.

Mcginn (2007), a textbook published by CFA Institute, presents the basics of asset-only and asset-liability methodologies in its Chapter 5 "Asset Allocation" (called the Chapter in this section). The comments in this appendix are limited to the use of the term "liability" in the Chapter; this appendix does not attempt to review other aspects of the Chapter.

The Chapter presents the principles of asset-liability management and discusses the objectives of asset allocation and the role of the policy portfolio for investors with financial commitments to fund. When it comes to the concept of "liability," unfortunately, the Chapter is very confusing with respect to the following basic questions: Why should the concept of "liability" be used in the development of optimal portfolios? What is the “liability”? Is the “liability” a cash flow or a present value of a cash flow?

The textbook sends clear messages regarding the concept of "liabilities" prior to the Chapter. In the first definition of the “pension surplus,” the need to “present value” those “future liabilities” is perfectly clear (page 66).

“*The pension surplus equals pension plan assets at market value minus* ***the present value of pension plan liabilities***.”[47]

The presence of “the present value” in this definition is absolutely necessary. “Pension liabilities” are future payments made at different points in time, and these payments must be “present valued” in order to be subtracted from today’s assets.

A statement on page 69 reiterates the need for the “present value” in this definition.

“*For a DB plan, one key ALM concept is the pension surplus, defined as pension assets at market value minus* ***the present value of pension liabilities***.”

At the beginning, the Chapter is clear about the meaning of the term "liabilities." On page 236, “future liabilities” are obviously a cash flow:

“*… insurers, defined benefit (DB) pension plans, and certain other institutional investors face* ***streams of significant future liabilitie****s. Controlling the risk related to* ***funding future liabilities*** *is a key investment objective for such investor.”*

The emphasis on funding continues on the same page. These “future liabilities” are instrumental in the development of optimal asset allocation:

“*In the context of determining a strategic asset allocation, the asset–liability management (ALM) approach involves explicitly modeling liabilities and adopting the optimal asset allocation in relationship to* ***funding liabilities****.”*

A statement on page 237 reiterates once again that "pension liabilities" require "present valuing."

“*DB pension plan may want to maximize the future risk-adjusted value of pension surplus (the value of pension assets minus* ***the present value of pension liabilities****)*.”

The meaning of the term "funding" is further clarified on page 286, where "a liability" is a future payment:

"*... **funding a liability** means being able to pay the liability **when it comes due***."

The emphasis on funding as a multi-period endeavor in these statements is of paramount importance. According to these definitions, the investor's job is to make sure there is enough money to make all payments when they come due in the future.

The whole thing starts unraveling on page 237. The "present value" mysteriously disappears in the following statement:

"*ALM strategies run from those which seek to minimize risk with respect to net worth or surplus (**assets minus liabilities**) to those which deliberately bear surplus risk in exchange for higher expected surplus*."

Readers beware, those "liabilities" cannot be subtracted from the assets – it is against the basic principle of the time value of money. As defined in the textbook, "liabilities" are benefit payments made at different points in time, while the asset value is at the present. To be able to subtract "liabilities" from "assets," one must calculate a present value of these "liabilities."

Unfortunately, as presented in the Chapter, the term "liabilities" represents entirely different objects. This is not just a minor slip of a tongue – the Chapter contains multiple occurrences of the term "liabilities" utilized to represent either a cash flow or a present value. Even worse, in some cases, the reader has to guess the meaning of the term. Below are just a few examples of these multiple meanings with additional comments.

- <u>"Liabilities" is a present value</u> (page 237): "*ALM strategies run from those which seek to minimize risk with respect to net worth or surplus (assets minus liabilities) to those which deliberately bear surplus risk in exchange for higher expected surplus, analogous to the trade-off of absolute risk for absolute mean return in an asset-only approach.*" The objectives of "funding future liabilities" and "minimizing the surplus risk" are fundamentally different. They are based on different assumptions and may lead to different optimal policy portfolios.
- <u>"Liabilities" is a cash flow</u> (page 237): "*A cash flow matching approach structures investments in bonds to match (offset) future **liabilities** or quasi-liabilities. When feasible, cash flow matching minimizes risk relative to funding **liabilities**.*"
- <u>"Liabilities" is either a present value or a cash flow</u> (page 238): "*In general, the ALM approach tends to be favored when … the market value of liabilities or quasi-liabilities are interest rate sensitive*." This statement is controversial for several reasons. First, pension benefits are non-

tradable and non-transferable, so "the market value of liabilities" in a conventional sense does not exist. Second, if the term "liabilities" in this sentence means a cash flow, then their benefit payments are not directly interest rate sensitive. In this case, this statement implies that "the ALM approach" should not "be favored" by retirement plans – a silly proposition. Third, if "the liabilities" in this sentence mean a present value, then the sensitivity to interest rates must be justified somehow, which the Chapter does not do.

- <u>"Liabilities" either a present value or a cash flow</u> (page 245): "*Shortfall risk in relation to liabilities is a key focus of ALM approaches to asset allocation.*" What a great point! The ability of a particular asset value to fund a stream of future payments is surely one of the main challenges in the area of asset allocation. But this point is immediately followed by the following incredible statement: "*An asset-only approach can also easily incorporate shortfall risk in a variety of ways.*" Really? Dear authors, do you honestly believe that "an asset-only approach" can calculate, for example, the shortfall risk – the probability that the commitment will not be funded? Apparently, the term "shortfall risk" in the statement regarding "a key focus of ALM approaches" is actually the "surplus shortfall risk." In this case, the statement is both incorrect and in conflict with the abovementioned statement on page 236: "*Controlling the risk related to **funding future liabilities** is a key investment objective for such investor."* Readers, beware: "funding future liabilities" and surplus risk management are fundamentally different endeavors.
- <u>"Liabilities" is a present value</u> (page 255): "Net worth" is again defined as "***assets minus liabilities***." We are back in the "present value" mode.
- <u>"Liabilities" is a cash flow</u> (page 286): "*In many cases, however, an asset portfolio is meant **to fund a specified liability schedule** (funding a liability means being able to pay the liability **when it comes due**).*" Again, this is a very important clarification.
- <u>"Liabilities" is a present value</u> (page 287): "*A single variable that summarizes the interaction of assets and liabilities is net worth (**the difference between the market value of assets and liabilities**), also called surplus.*" We are back in the "present value" mode once again.
- <u>"Liabilities" is a cash flow</u> (page 288): "*The funding ratio, calculated by dividing the value of pension assets by **the present value of pension liabilities**, measures the relative size of pension assets compared with pension liabilities.*" The "present value" reappears.

As the Chapter continues, it gets even worse. It is troubling enough that a stream of "liabilities" gets "present valued" and becomes just a "liability" for no particular reason. Now this present value of a non-tradable cash flow conveniently turns into an asset (held short) for no particular reason as well. The implied assumptions are a) this benefit stream can be replicated using a portfolio of tradable bonds; b) the only legitimate present value of the commitment is the price of this hypothetical bond portfolio. The Chapter makes no attempt to justify these assumptions.

Here are a few examples of this convenient transformation.

- Page 290: "*Assume that the **return on the pension liability** tracks the return on U.S. government long-term bonds.*" Suddenly, the pension liability *is assumed* to be a return-generating asset.
- Page 291: "*The pension liability behaves as a long-term bond, by assumption.*" Interestingly, a similar statement is presented as a "fact" in Bader-Gold [2003].[48] Much to the authors' credit, they do not go that far and reiterate that the similarity of "the pension liability" and "a long-term bond" *is assumed*.
- Page 308: "*Pension liabilities behave similarly to bonds as concerns interest rate sensitivity*." Finally, the similarity of "pension liabilities" and bonds is stated unequivocally as a fact. The proponents of LDI have reached the Promised Land.

Yet, it would be unwise to put the blame for the problems in the Chapter on its authors. Resolving the problems with the dual use of the term "liabilities" was neither easy nor the authors' goal. The Chapter is essentially a survey of the existing publications on the subject. Let us not blame the messenger for the trouble with the message.

APPENDIX III: "Human Capital" in Ibbotson (2007)

"Okay, sweetie, I know you think
you're explaining yourself,
but you're really not."
Penny Hofstadter[49]

Let us walk through the basics of the "human capital" theory as presented in Ibbotson (2007), a monograph published by CFA Institute Research Foundation.

Initially, "human capital" is defined as *a series of payments*:

> *"The education and skills … provide us with a capacity to earn income or wages for the remainder of our lives. This earning power we call "human capital…"*

Immediately, "human capital" is redefined as "*the present value"* of the series:

> *"… and we define it as the present value of the anticipated earnings over one's remaining lifetime."*

These seemingly unremarkable two sentences give us four (!) essentially equivalent terms – "a capacity to earn income," "earning power," "human capital," and "anticipated earnings." At least four more – "income flow," "future labor income," "net incomes," and "random income" – are added later in the text. The second sentence also offers two conflicting definitions of "human capital" – as a series of payments and "the present value" of the series. This is the first but not the last conflict in this area.

The next step is to integrate human capital and financial capital:

> "*… our total wealth is made up of two parts: our human capital and our financial capital. Human capital should be treated like any other asset class; it has its own risk and return properties and its own correlations with other financial asset classes."*

The goal here is to turn human capital into a conventional asset class and use it in a one-period portfolio-centric model. The authors realize that simply defining human capital as "the present value of an investor's future labor income" is insufficient. So, they turn "the present value" into "the *economic* present value" in their next definition of human capital:

> *"... Human capital is defined as the economic present value of an investor's future labor income."*

Dear reader, please see the epigraph to this appendix. After several definitions, human capital is still undefined – the authors still have to define the term "economic present value" (later in the monograph, this term is also defined as "the expected discounted value" of the "income flow"). To calculate this present value, the means of random payments of the "income flow" are discounted using the discount rate equal to the "relevant" risk-free rate plus a "subjective" parameter.

The journey to define "human capital" continues in Appendix A, which presents a detailed description of the model. "Human capital" was already defined as a series of payments and immediately re-defined as "the present value." Appendix A offers another definition of human capital as "the present value." Note the evolution of "the present value" to "the economic present value" and then to "the financial economic present value":

> *"We define human capital as "the financial, economic present value of net incomes that depend on a number of subjective or market pricing factors."*

Dear reader, please see the epigraph to this appendix again. On the same page, the authors offer yet another definition of human capital – "the expected discounted value of the income flow" (*EDVIF*). In this definition, there are the means of random income in the numerator and a deterministic discount rate in the denominator. The authors are perfectly open about their goal to convert "the random income into a scalar."

Curiously, this version of human capital is abbreviated *DVHC* (the understandable *EDVIF* or previously introduced *HC* would make more sense). This author conjectures that *DVHC* stands for "discounted value of human capital." This conjecture is noteworthy because human capital was initially a series of payments, then turned into "the present value," and it is turning into a series of payments again (otherwise, there would be no need for further discounting). Moreover, the authors explicitly confirm that they view human capital as a series of payments when they express their feelings (!) toward *DVHC*:

> *"Therefore, because we are discounting with an explicit risk premium, we feel justified in also using the term "financial economic value of human capital" to describe DVHC."*

Again, if human capital were still "the present value," there would be no need to consider its "financial economic" value. After several definitions, human capital's round trip from a series of payments to "the present value" and back is complete. Most of these arguments regarding the term "human capital" are little more than exercises in semantics. Human capital remains a series of contingent payments of uncertain magnitude and timing, semantics exercises notwithstanding.

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Endnotes

[1] In this paper, the term "leading organizations" means major organizations that represent finance professionals. These organizations include, but are not limited to, CFA Institute, the Society of Actuaries (SOA), the American Academy of Actuaries (AAA), large asset managers, record keepers, insurance companies, and consultants. The term "leading publications" means publications sponsored/endorsed/promoted by the leading organizations.

[2] The phrase "rules of thumb and folklore" is taken from Bernstein (2007): "Before Harry Markowitz's 1952 essay on portfolio selection, there was no genuine theory of portfolio construction – there were just rules of thumb and folklore."

[3] Specifically, Samuelson (1970) discusses some works of Leonhard Euler that utilized certain debatable assumptions. These assumptions would be later found unnecessary in the works of K. Weierstrass, C. Jacobi, and others.

[4] Shortly before Samuelson (1970), Samuelson (1969) demonstrates that, under certain conditions, the optimal glide paths are stationary. Clearly dissatisfied with this result, Samuelson called his result "a defiance of folk wisdom and casual introspection" (see Samuelson (1989)) and "a failure" (quoted from the transcript of the Paul Samuelson's interview to Robert Merton conducted for the History of Finance project sponsored by the American Finance Association in 2005). Samuelson had endeavored to design a theoretical framework that would rationalize evolving glide paths essentially for the rest of his life. See Mindlin (2013A) for more details.

[5] William of Occam (sometimes spelled as Ockham or Ockam), *Summa Logicae* (1323).

[6] This is the first of the Rules of Reasoning in Philosophy in the third section of Isaac Newton's *Principia Mathematica* (1687).

[7] From "On the Method of Theoretical Physics," lecture delivered at Oxford, June 10, 1933. Often quoted as "*Everything should be made as simple as possible, but not simpler.*" Also attributed to Albert Einstein is "*The grand aim of all science [is] to cover the greatest number of empirical facts by logical deduction from the smallest possible number of hypotheses or axioms*." Lincoln Barnett The Universe and Dr Einstein (1950 ed.).

[8] For the purposes of this paper, the "financial economics" community includes the actual and aspiring members of CFA Institute, financial service providers (e.g., asset managers, banks, insurance companies, investment consultants, financial advisors), as well as academics working in the area of financial economics. The actuarial community includes the actual and aspiring members of the major actuarial organizations – the American Academy of Actuaries (AAA) and the Society of Actuaries (SOA), professionals working at actuarial departments of various organizations, as well as academics working in the area of actuarial science. Obviously, these communities and groups may overlap.

[9] An example of such "surcharges" is presented in Mindlin (2015A).

[10] Albert Einstein: Philosopher-Scientist, editor Paul Arthur Schilpp, New York (1949).

[11] For example, the term "liability" in Maginn (2007) stands for *both* a series of payments and a present value. Similarly, the term "human capital" in Ibbotson (2007) stands for *both* a series of payments and a present value.

[12] Donald L. Kohn, Vice-Chairman of the Federal Reserve System, The Economic Outlook, May 20, 2008, https://www.federalreserve.gov/newsevents/speech/kohn20080520a.htm.

[13] The author would like to encourage the reader to offer a counterexample to this statement.

[14] Guide (2006): "Under financial economics, the driving force for investing pension assets is corporate finance, not portfolio selection. Thus, whether equities or bonds provide higher expected returns is irrelevant. Pension investing seen through the financial economic lens has nothing to do with the size or even the existence of an equity risk premium. The equity risk premium is a critical feature of portfolio theory and analytics but is all but irrelevant to corporate finance."

[15] Richard Feynman, Cargo Cult Science, Caltech commencement address, 1974.

[16] It should be noted that the "pricing" of retirement income commitments is also problematic. To see why it is so, let us assume that a DC plan participant has made a commitment to fund a specific level of retirement income. It is common in many publications to price this commitment as an annuity, even though full annuitization is currently quite unpopular. Most DC plan participants utilize risky assets to fund their commitments and accept a certain level of shortfall risk. "Pricing" a risky cash flow as riskless conflicts with the basic tenets of investing.

[17] This statement is very helpful. You see, the term "human capital" "often conveys a number of different ideas," and sometimes the term conveys just one idea. Sometimes the term "human capital" "conveys a number of conflicting ideas," and sometimes the ideas it conveys are in perfect agreement. Truly clarifying.

[18] The relationship between future and present values is discussed later in the paper.

[19] "The Biggest Money Mistakes We Make – Decade by Decade," The Wall Street Journal, October 26, 2016.

[20] Multi-period future value "will-do" mean-variance optimization would be one example of such optimization problem, but these technicalities are outside of the scope of this paper.

[21] See Mindlin (2009) and Mindlin (2013 A) for more details.

[22] For example, see Mindlin (2015A).

[23] One can only wonder at the scientific rigor of publications that are usually "complete and theoretically sound."

[24] One examples of these issues is "to" vs. "through" glide paths, see Mindlin (2017). Another example is discussed in Mindlin (2016B).

[25] To this author, the major issues of optimal glide path design directly related to the primary objectives of retirement investors include, but are not limited to, Nash equilibrium strategies, forward- vs. backward-looking objectives, forward vs. backward inductions, "will-do" vs. "expected-to-do" glide paths, and risk paths.

[26] Mindlin (2009), Mindlin (2015B), and Mindlin (2016A) demonstrate that evolving glide paths optimize the outcomes of funding problems.

[27] "Seinfeld," Season 3, Episode 9.

[28] See Trowbridge (1989).

[29] Kellison (2009) is the third edition of this textbook.

[30] See Bowers (1997).

[31] See Panjer (1998).

[32] See Guide (2006).

[33] This author's attitude toward this "guide" is reflected in Mindlin (2007).

[34] Even though these multiple payments create some technical complications, still all necessary formulas for this example can be found in Kellison (2009).

[35] In *Example 6*, the mean and standard deviation are calculated exactly, and the median is estimated.

[36] In *Example 7*, the mean and standard deviation are calculated exactly, and the median is estimated.

[37] In *Example 8*, all percentile values are estimates.

[38] See Mindlin (2010).

[39] This quote is often attributed to Isaac Asimov and sometimes to Alan Alda, an American actor, director, and writer.

[40] Lognormal models (approximations) of portfolio return factors are extensively used in finance. For example, see Kellison (2009), section 12.3.

[41] For example, see Kellison (2009), formulas (12.15) and (12.16)).

[42] See Kellison (2009), formulas (12.11) and (12.14).

[43] Another way is to utilize the two moments matching inverse gamma approximation.

[44] The median of the two moments matching inverse gamma distribution is 3.5866, which is also close to 3.5872.

[45] In this example, the two moments matching lognormal distribution should generate a better approximation than its inverse gamma counterpart does. To see why it should be so, the 3rd moments of RA_0, the lognormal, and the inverse gamma approximations are equal to 70.25, 70.21, and 75.35 correspondingly. The 3rd moment of the lognormal approximation is much closer to the 3rd moments of RA_0 than its inverse gamma counterpart.

[46] Scent of a Woman (1992), produced by City Light Films, distributed by Universal Pictures.

[47] In this and all other quotations in this appendix, emphasis is added by the author.

[48] See Bader-Gold [2003], page 4. Speaking of Principle 4 (which is arguably the most important point of the paper), the authors assert that "This principle follows from the fact that a company's pension liabilities are similar to debt."

[49] "The Big Bang Theory," Season 1, episode 5.

Important Information