The "Annuity Puzzle" And Model Portfolios

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SUMMARY

The paper presents a model that demonstrates the advantages of partial annuitization in the most straightforward manner. The paper also briefly discusses "the annuity puzzle", a decades old problem.

The "Annuity Puzzle"

Annuity manufacturers as an industry have long strived to market annuities to retirement investors. Many economists and industry practitioners believe that annuitization would be beneficial to these investors. Yet, most retirement investors do not annuitize despite the industry's continued efforts.

The industry employs a specific term – the "annuity puzzle" – to refer to the disconnect between the apparent theoretical advantages of annuities and their unpopularity among retirement investors. This disconnect has been observed for several decades (Franco Modigliani discussed it in his 1985 Nobel Prize acceptance speech). Yet, the "annuity puzzle" remains unresolved even though a variety of quantitative and qualitative arguments have been advanced to rationalize it.

There are two major schools of thought that deal with the "annuity puzzle." Behavioral finance offers insights into the financial propensities of investors, identifies obstacles to efficient decision-making, and recommends better ways to communicate. Financial economics delivers the substance of these communications in terms of optimal contribution and portfolio selection strategies that include annuities.

And here lies the key problem. The conventional economic approach to annuitization is the expected utility theory (EUT) based rational expectation model. The limitations of EUT have long been known, and some economists have enunciated their reservations about EUT. For instance, A. Roy (the author of the Roy's safety-first criterion) noted in 1952:

> *"In calling in a utility function to our aid, an appearance of generality is achieved at the cost of a loss of practical significance and applicability in our results. A man who seeks advice about his actions will not be grateful for the suggestion that he maximise expected utility."*

Still, EUT has remained popular among many economists and industry practitioners. As first demonstrated over half a century ago, expected utility-maximizing retirement investors should fully annuitize their assets. This and similar results have led many to believe that the quantitative arguments in favor of full annuitization are strong.

This author disputes this belief. EUT based recommendations for full annuitization demonstrate the shortcomings of EUT, not the advantages of full annuitization. Full annuitization is unpopular for good reasons – it makes little sense to most retirement investors in voluntary markets.

Outside of EUT, the quantitative arguments in favor of full annuitization are somewhere between unimpressive and non-existent. The answers to some of the most important questions regarding annuities in retirement portfolios are hard to find. To name a couple, does partial annuitization add

risk capacity to the conventional part of a retirement portfolio? Yes, it does (but try to find out how to quantify this capacity). Should bequest motives discourage annuitization? No, they should not. To the contrary, bequest motives may serve as an incentive for partial annuitization (but try to find out why). The next section takes initial steps to answer these questions.

The shortcomings of EUT based models are not limited to their debatable recommendations. Many in the industry appear to find the idea that such models may justify full annuitization is too tempting to abandon easily. Wouldn't it be great if the right product "bells-and-whistles" along with clever "nudges" were sufficient to jump-start high demand for annuities? Hence, many in the industry embrace a product-centric view that implies that the solution to the "annuity puzzle" should be found in better communications and product design.

This author has little optimism that product-centric views can solve the "annuity puzzle." Outcome-centric views are much more likely to lead to a solution. Annuities are likely to become popular when the industry offers clear quantitative arguments that portfolios with annuities generate superior outcomes compared to their conventional counterparts.

Unfortunately, outcome-centric views do not always get the recognition they deserve. Not too many retirement plans provide guidance with respect to outcomes and their riskiness. Many service providers – asset managers, record keepers, consultants, etc. – still operate in the risk/return space that is not the most hospitable place for annuities. Annuity manufacturers have not been terribly helpful – their support for outcome-centric views has long been called for.

A practical way to give retirement plan participants access to annuities would be to include annuities in the model portfolios that asset managers and consultants/advisors develop for the plan participants. Many DC plans do not (see compelling reasons to) offer in-plan annuities. Without in-plan annuities, it makes little sense to include annuities in the model portfolios.

It would also be valuable if the portfolio selection tools that growing numbers of advisors rely on incorporated annuities. More annuity-friendly regulatory environment would be helpful as well. Even though there has been notable progress in these areas, annuitization remains unpopular.

The primary objective of this paper is to present clear arguments that annuities improve outcomes. The model presented in the paper demonstrates the advantages of partial annuitization in the most straightforward manner. The obviousness of these advantages should facilitate the resolution of the "annuity puzzle."

Or so this author hopes.

The Model

This section presents the results of a simplified portfolio selection model that assumes that in-plan annuities are available. The model incorporates annuities in the context of a funding problem. In a typical funding problem, the investor's primary objective is to fund a financial commitment (e.g. a particular level of retirement income). In this model, annuities "compete" with conventional assets (e.g. stocks, bonds) for allocations in outcome optimizing portfolios. Annuities are assumed to be "risk-free" and valued at the "book value" (see the appendix for more details). In contrast, conventional assets are "risky" and generate uncertain outcomes.

The stochastic analysis of outcomes is commonly performed using Monte-Carlo simulations. However, Monte-Carlo simulations are not well-suited for outcomes-based portfolio optimization. Simulations generally *illustrate* pre-selected portfolios rather than *generate* optimal portfolios. The results of Monte-Carlo simulations are usually not perfectly robust and replicable as they depend on certain technicalities (e.g. sample size, seeds, software platform) and may vary from vendor to vendor. Overall, Monte-Carlo simulations possess several structural problems.

The model presented in this paper is based on a *simulation-free stochastic analysis of outcomes*. Simulation-free analysis of outcomes allows direct solutions to a variety of portfolio (glide path) optimization problems. Simulation-free estimates allow instant comparisons of the upside and downside properties of outcomes for various conventional portfolios and annuitization levels. Such estimates may demonstrate the ability of annuities to improve outcomes as well as add risk capacity in the conventional part of the portfolio.

To illustrate these points, this section presents a simplified numerical example. John is 65, just retired, and has 1,000,000 in his retirement account. John has made a commitment to withdraw $40,000 adjusted for inflation annually for the next 30 years.

To fund his commitment, we assume that John will select one of the model portfolios offered by his retirement plan. Let us try to envision John's choices if annuities and outcome estimates were available. For every model portfolio with no annuitization, John would see comparable portfolios with partial annuitization. To see the pros and cons of these portfolios, John would need measurements of the outcomes these portfolios generate. The capital market and annuity pricing assumptions required for the comparison are presented in the appendix.

John's assets would generate uncertain outcomes (with or without partial annuitization). What kinds of outcome measurements John would find useful? John is interested in higher asset values in his retirement account, so John would be interested in seeing higher means and percentiles of uncertain asset values (among other things). Also, the use of risky assets implies the possibility of

shortfall, i.e. there may be insufficient assets to make all payments. John should be interested in seeing lower shortfall risk measurements that include shortfall probability, size, and volatility.

In this example, we consider the following measurements of John's asset values in each year: means, standard deviations, 1st and 99th percentiles, every fifth percentile from 5th to 95th (in total, 21 percentiles each year). Overall, we consider 630 percentiles = 21 percentiles each year times 30 years. Shortfall probability, size, and volatility are considered as well.

Let us define annuitization in this example. We say that John's portfolio has *X*% annuitization if the portfolio has a single premium immediate annuity (SPIA) that pays *X*% of John's commitment. For example, we say that a portfolio has 15% annuitization if the portfolio contains a SPIA that will pay $6,000 (= $40,000 times 15%) adjusted for inflation annually for the next 30 years.

For simplicity, let us assume that John's plan offers just two investment choices – stock and bond index funds. If John is considering a conventional 40/60 portfolio (40% stocks/60% bonds; this portfolio is called the conventional portfolio), would partial annuitization improve outcomes? Let us compare the outcomes generated by the conventional portfolio to their counterparts for the 50/50 portfolio with 15% annuitization (called the annuitized portfolio).

Here is what John would see. All abovementioned 630 asset value percentiles are higher for the annuitized portfolio than their counterparts for the conventional portfolio. In other words, the annuitized portfolio generates *higher asset values at all selected risk levels in all years*. The annuitized portfolio generates *lower shortfall probability, size, and volatility*. The expected terminal asset value (the mean asset value at the end of the 30-year period) is 11% higher for the annuitized portfolio than its counterpart for the conventional portfolio. In other words, the bequest generated by the annuitized portfolio is expected to be 11% higher than its counterpart generated by the conventional portfolio.

If John is considering more conservative portfolios, the benefits of partial annuitization are even more pronounced. If John is considering a conventional 30/70 (30% stocks/70% bonds) portfolio with no annuitization (called again the conventional portfolio), he may want to compare the outcomes generated by this portfolio and the 40/60 portfolio with 15% annuitization (called the annuitized portfolio).

Again, all 630 asset value percentiles are higher for the annuitized portfolio. The annuitized portfolio generates lower shortfall probability, size, and volatility. The expected terminal asset value is 17% higher for the annuitized portfolio than its counterpart for the conventional portfolio.

Furthermore, John may want to consider a conventional 48/52 (48%stocks/52% bonds) portfolio with the annuitization level of 25%. Yet again, all 630 asset value percentiles are higher for the annuitized portfolio. The annuitized portfolio generates lower shortfall probability, size, and volatility. The expected terminal asset value is 29% higher for the annuitized portfolio than its counterpart for the conventional portfolio.

Overall, the evidence in favor of partial annuitization in these examples is perfectly clear. Partial annuitization may generate better outcomes – higher asset values, higher upside, lower downside – for a multitude of retirement investors.

A Few Observations

This section presents a few additional observations about the approach presented in this paper.

- The model presented in this paper is designed to be as simple as possible while capturing the relationship between annuities and conventional asset classes. More realistic examples may include, but not be limited to, additional asset classes, glide paths, efficient frontiers, multiple purchases of annuities, and the flexibility of post-retirement spending. These and related issues will be addressed in future publications.
- Adding a reasonably priced annuity and a reasonable level of annuitization to a conventional portfolio may lower the values of shortfall probability, size, and volatility. Then the conventional part of the newly annuitized portfolio can be more aggressive without increasing the riskiness of the entire portfolio. Thus, partial annuitization may create additional risk capacity for the conventional part of the portfolio when risk is defined as the shortfall event (there are insufficient assets to make all payments),
- There are different ways to measure and optimize the impact of partial annuitization. In this paper, we compare the means of expected terminal values while the 630 percentiles and the shortfall measurements serve as constraints, but there are many others.
- The paper highlights the role of an annuity as a "buy-and-hold" asset that has unique pricing characteristics due to the "mortality credits."
- In this approach, no utility function assumption is needed. The outcomes of the funding problem are optimized directly.
- One of the key factors that impacts the ability of partial annuitization to improve outcomes is the relationship between annuity pricing and CMAs.

This author just came across an article "What Happens When Income is the New Outcome?" published by a reputable asset manager recently. Here are a couple of answers to this question (among others). First, most target date funds would have to change the shape of their glide paths. Second, annuities would become much more popular.

Conclusion

This author believes that the core of the "annuity puzzle" is the inadequacy of currently available quantitative arguments in favor of annuities. Without adequate quantitative arguments, qualitative arguments are unlikely to be convincing. Without convincing arguments, service providers and policymakers are unlikely to champion annuities. Without help from service providers and policymakers, most retirement investors are unlikely to annuitize.

The following simple message should be at the core of quantitative arguments in favor of annuities. Under the appropriate conditions, *partial annuitization generates higher asset values*. Across the board. In all years.

The intuition in favor of partial annuitization is simple. As far as outcomes are concerned, partial annuitization produces better "downside," and a more aggressive conventional part of the portfolio produces a better "upside." As a result, we have better outcomes across the board.

Obviously, this paper has just scratched the surface. A lot of work is yet to be done. The biggest challenge for the thought leadership of the industry is to embrace outcome-centric views in earnest. It remains to be seen if they are up to this challenge.

APPENDIX: Capital Market and Pricing Assumptions

The following capital market assumptions (CMAs) are utilized in the paper.

	Geometric Mean (%)	Arithmetic Mean (%)	Standard Deviation (%)
Stocks	7.00	8.03	15.00
Bonds	4.00	4.08	4.00
Inflation	2.50	2.505	1.00

Correlation Matrix

	Stocks	Bonds	Inflation
Stocks	1	0.20	0.00
Bonds	0.20	1	-0.10
Inflation	0.00	-0.10	1

Portfolio returns in different years are assumed independent. Portfolio returns are assumed to be lognormally distributed.

It is assumed that a single premium immediate annuity (SPIA) that makes 30 end-of-year inflation-adjusted payments is available in the plan. This SPIA is valued each year as the present value of the remaining payments (the "book" or "carrying" value). It is assumed that the nominal discount rate for these calculations is 3.75%.

Additional technical details are available upon request.