Six Degrees of Freedom GPS Antenna Attitude Determination

Robert Wellington, Six Degrees of Freedom GPS

The DoD is investing in technologies and strategies to enable GPS in ways not originally intended as well as guarding against adversary impacts of GPS degradation and denial. While much of the research centers around complex and military oriented development, simple approaches may lead to gains in many areas at a greatly reduced expense.
Six Degrees of Freedom GPS has patented (US Patent: US 2011/0285590 A1) a novel algorithm titled GPS6D to provide full attitude determination of a GPS antenna utilizing carrier-to-noise (C/N0) values and the ephemeris. Although currently using GPS signals, the implementation easily extends to all GNSS constellations. While attitude determination has been done with multiple antennas, this algorithm allows the use of a single antenna. The antenna coupled with the algorithm provides the ability to determine orientation, sense multipath, and act as a redundant source of attitude determination highly applicable to small systems. Initial investigation of results in various GIANT scenarios demonstrate the potential of this application and suggest possible uses with maturation of the technology and concepts.

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1) GPS6D Technology Overview

GPS is well-known as the Global Positioning System, but GPS also provides the infrastructure to be a Global Orientation System. Indeed, the constellation of GPS satellites provides a public reference for geographic coordinate systems, particularly locally-level azimuth and elevation. In particular, if we measure various angles-of-arrival for GPS satellite signals in body coordinates, and we know the satellite configuration in geographic North-East-Down (NED) coordinates, then GPS6D will determine the rotation that transforms between body and NED coordinates.

This is simply one application of the GPS6D navigation system design that measures angles of arrival to extend GPS to a six degree-of-freedom (6DOF) navigation system for airborne platforms. The technology has wider application than GPS, but we introduce it in this context.

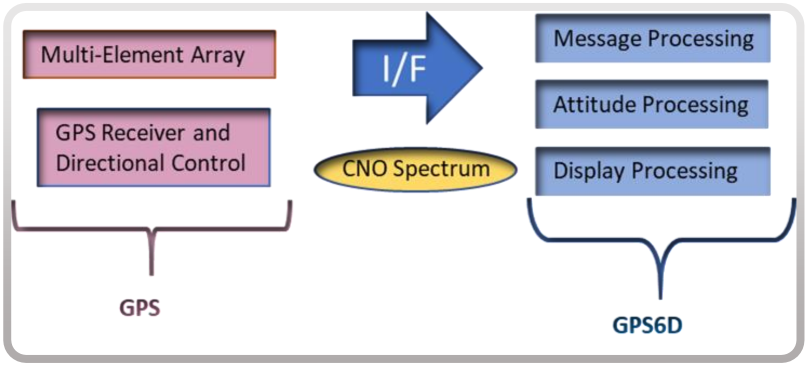

Figure 1. Distinction between GPS and GPS6D

GPS6D is set up as a post-processing system to add an attitude reference to the GPS system so that both platform position and platform orientation are provided. The context of the figure 1 includes a GPS system controlling a small directional antenna and producing a stream of CN0 values as the antenna is rotated. GPS6D picks up this stream and processes it to derive the platform attitude that accounts for the CN0 stream.

This approach is automatic in the sense that the data is gathered from the GPS receiver while the antenna is changing direction, and then GPS6D acts as a post-processor that provides the 6DOF orientation of the platform measuring the arrival angles.

Conceptually, we are relying on the antenna directionality to distinguish the alignment angles for the antenna rotational axis. The actual system is in its fourth generation and the concept has been proven in several contexts. Currently we are simulating a square array. Main practical concern is that platform must remain nominally upright for antennas to have line of sight (LOS) to the satellite. Beyond that, GPS6D will be able to compute the antenna orientation as well as other information of interest.

Figure 2 shows the different ways that GPS6D currently displays orientation. In essence, the orientation is represented by q quaternion and the quaternion orientation data is easily integrated into any software system with a USB interface. GPS6D provides a display of attitude-related information that is not available from ordinary GPS. In particular, GPS6D will produce the yaw, pitch and roll of the antenna under certain conditions, and should be able to provide an automatic heading and attitude reference dependent only on the GPS subsystem. This provides redundant attitude information for GPS6D-equipped platforms.

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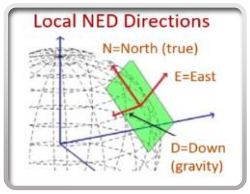
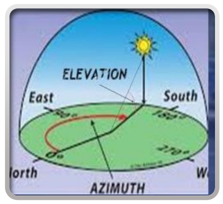
GPS6D software extends GPS position and time measurements by sensing the attitude of the antenna and direction of the transmitter (i.e. 6-DOF orientation) relative to the GPS satellite constellation.

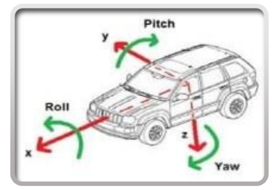
MULTIPATH INDICATOR
Detects directions of interference with GPS signal, indicates line of obstruction by nearby physical features





ORIENTATION INDICATOR
Determines the 3 navigation directions (NED)

ALIGNMENT INDICATOR
Measures the antenna angle at which GPS signals are strongest



INCLINOMETER INDICATOR
Provides 3 body angles relative to local NED

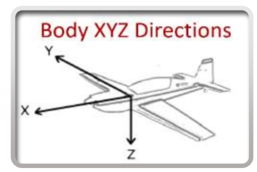


Figure 2. GPS6D Orientation (Yaw, Pitch, Roll)

LinQuest and GPS6D agreed to collaborate in bringing this technology out of the civilian world into the military. Aside from engineering subject-matter expertise, LinQuest maintains the GPS Interference and Navigation Tool (GIANT) that evaluates the impact of Position, Navigation, and Timing (PNT) system performance on operational effectiveness. GIANT has been continually upgraded, adding fidelity and functionality, to reflect the evolution of the GPS enterprise architecture, GPS augmentations, other Global Navigation Satellite Systems (GNSS), and network-centric technologies. As the Air Force's standard model for GPS, GIANT has evolved from a constructive simulation and analysis tool to support live and virtual exercises and experiments, web-based applications, and air and space operations centers. Essentially, GIANT provides data for proof-of-concept experiments that validate the concepts of this paper.

2) GPS6D Measurement Process

2.1) Measuring Alignment Angles with Rotating Antenna

GPS is not a new invention; yet it has capabilities that have yet to be exploited by industry. We are claiming that the orientation of the receiver antenna can be determined from the CN0 output stream from the GPS under certain assumptions and reasonable constraints like line-of-sight (LOS) visibility of the satellites. In this section we will give one example of how to measure certain alignment angles that are related to angles of arrival. This paper describes how the GPS CNO patterns can be decoded to determine the attitude of the antenna, and this section focuses on how alignment angles are extracted in one application to reach this interesting conclusion.

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To distill this example to its essence, we simplify the mission profile to consist of an airborne platform with an upward pointed antenna mounted on top. We further suppose the antenna can somehow be rotated degrees around the longitudinal X axis through the platform nose. To be specific, the rotation passes through the zenith at angles in the range -45 to 45 directed in the body YZ-plane. To be simplistic, we gave the platform trajectory a constant heading and used platform roll angles to sweep the range of potential alignment angles between the satellites and antenna. The details are shown in Figure 3. Note that the antenna need not point directly at the satellite when best aligned by rolling the platform. The alignment angle is whatever roll angle brings the antenna closest to the satellite direction of arrival.

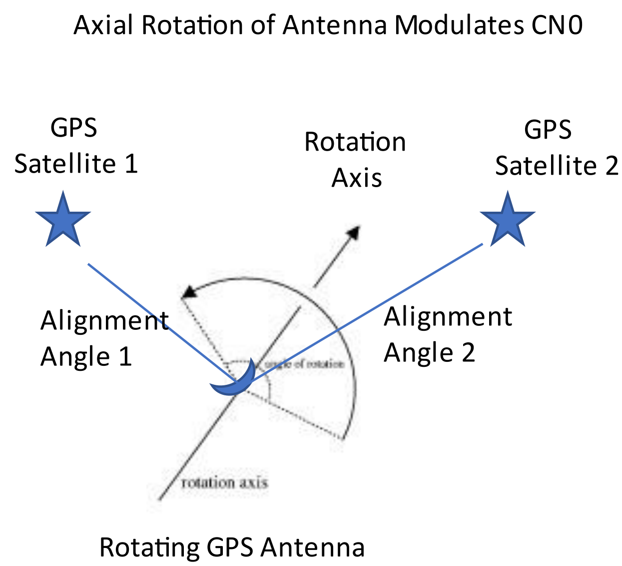

Figure 3. Axial Rotation of Antenna

The GIANT GPS simulator produced CNO readings for 10 GPS satellites in view while rotating the antenna and gathering CNO samples. It is not really a surprise to find that the CN0 varies to reflect the antenna position relative to the satellite. Of course, this conclusion assumes LOS between satellite and antenna, and multipath will cause some satellites to be less useful.

We can estimate the attitude of the GPS antenna by measuring the alignment angle between each satellite and the antenna boresight angle (bank angle) relative to vertical. Figure 4 shows the high SNR signal that is produced as the antenna is rotated toward and away from the signal source on the left. In this simulation, the signals had a Emitter present, but this did not prevent the precise location of nulls toward the right in the modulation patterns when banked away from the GPS satellite. As the antenna turns 90 degrees away from the satellite, this particular antenna creates a deep null in the CN0 pattern. Fitting these patterns to a template, we are able to determine that alignment angles for the satellites.

Contrast Figure 4 with figure 5. In figure 5 it is clear that the satellite is off to the left of zenith, and again this fact is not obscured by the Emitter. These measurements are derived from the CN0 data by estimating a phase associated with rolling the antenna relative to the GPS satellites around the local X axis. Collectively, these measured angles determine the attitude of the antenna axis in GPS coordinates, as will be shown in later sections, and GPS6D algorithms produce the attitude quaternion that best fits the measured angles.

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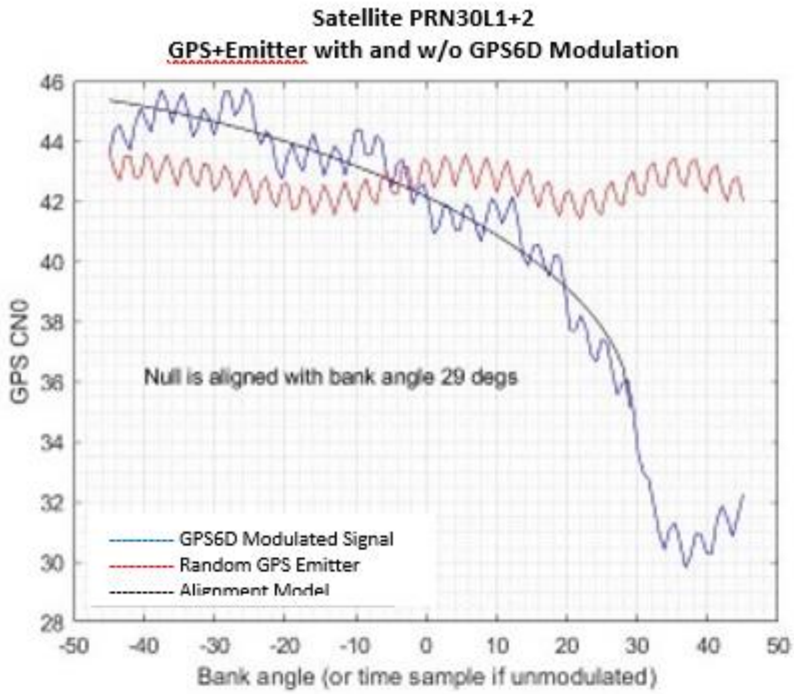

Figure 4. CN0 Indicates Leftward Alignment

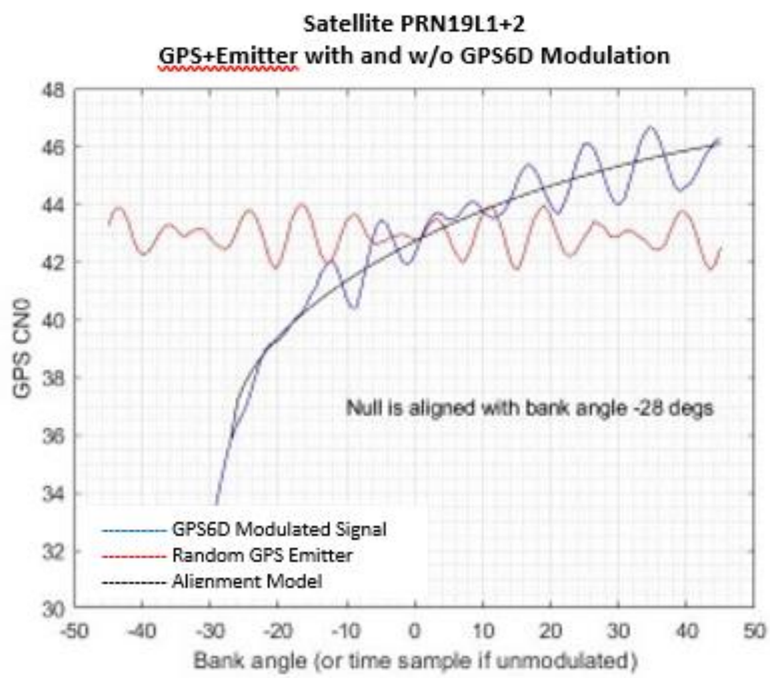

Figure 5. CN0 Indicates Rightward Alignment2.2 Emitter Processing

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The goal of the signal processing step is to measure the alignment angles in the emitter environment. This is the major innovative effort in each new application of the orientation technology. The second part of the problem is the transformation of the satellite alignment angles into the antenna orientation, and this is a feature of the software.

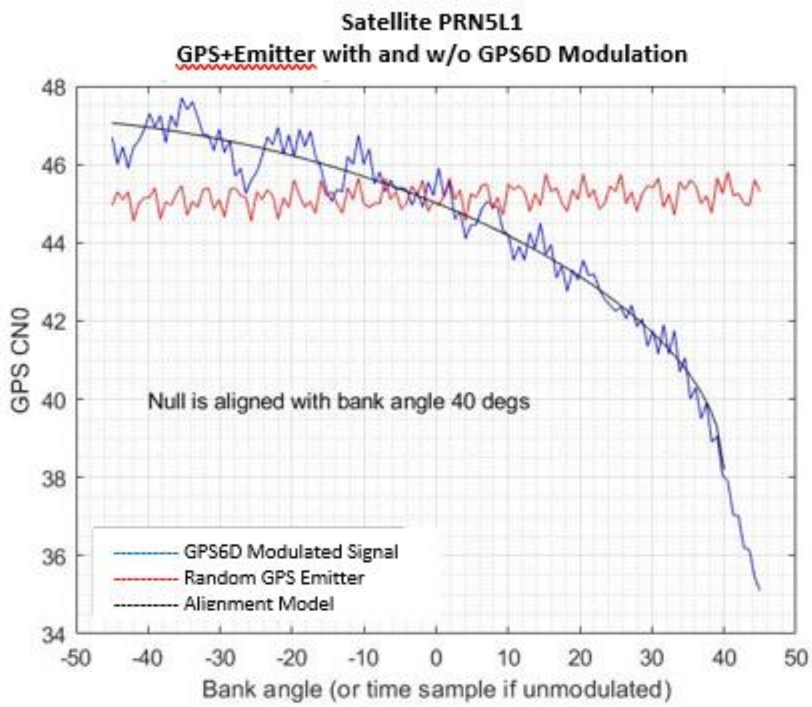

Figure 6. Emitter does not affect alignment angle

We have included the effect of an emitter on the scenario in the simulation. The added challenge is to find the correct alignment angles in the presence of the Emitter. The Emitter places peaks and valleys throughout the signal and makes it difficult to spot features in the received signal. However, the Emitter pattern is not similar to the GPS6D modulation that comes from rotating the antenna. The template for the GPS6D modulation does not match the Emitter signal. This is easily seen by comparing the modulated curves in Figures 4 and 6, which show different satellites with similar modulation. Similarly, Figure 5 and 7 should be compared.

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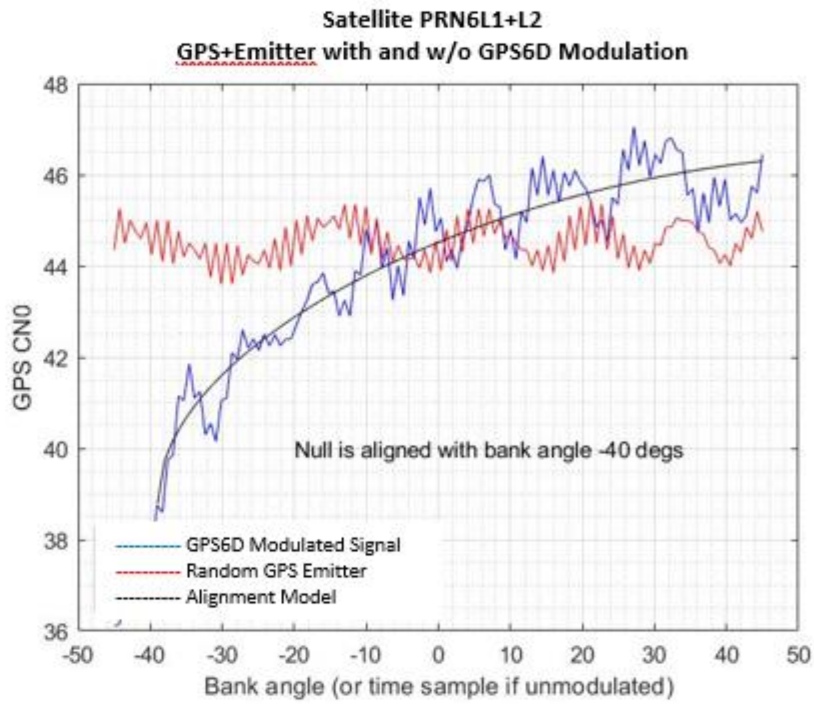

Figure 7 Emitter does not obscure alignment

We summarize the main preliminary conclusions from our work with Emitters and this particular antenna simulated by GIANT.

1) This particular antenna pattern appears to track a root cosine rather well, with a little more flattening around the peak, while the antenna beam has a 204-degree width, beyond which the signal drops off (or out).
2) The effective position of the L2 antenna seems to be the same as the L1 antenna. Data processing provides 10 L1 alignment angles and 10 L2 alignment angles. They are coherently modulated and can be added.
3) If the satellite is low in the sky, look for nulls in the modulation pattern. Otherwise, attempt to correlate with the middle of the modulation pattern (which is symmetric about zero).

The correlation peak is well defined in these cases where the pattern is cupped like a cosine near zero. When the pattern dives in a null, the correlation peak is generally very broad. However, the null remains visible in this case.

We were able to extract the information from the signals even in the presence of the Emitters. This information is recorded in terms of the alignment angle for each satellite, which is used to generate the GPS6D orientation.

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The Emitter (in the case Figure 7) does not have enough power to obscure the pattern null. What is distinctive about the null is that the derivative appears to be undefined at the null, and this is the case with the nulls of a root cosine antenna pattern. This measurement was made by aligning the null with the zero at 90 degrees for a cosine. In fact, the nulls appear to be 104 degrees off the antenna boresight at zero degrees. The uncertainty appears to be 1 or 2 degrees, although the Emitter does create a floor that can blur the measurement a bit. We see an example in Figure 6 where placing the null at bank angle -40 degrees means the satellite 5 is aligned with -40 + 104 = 64 degrees. A null is also apparent for satellite 6 in Figure 2. The signal fits well with the null at -40 degrees, which corresponds to -40+ 104 = 64 degrees.

2.3) Gathering Alignment Angles from Scan of Horizontal Array

There is of course no requirement that direction finding be accomplished using moving antennas. Arrays can be electrically steered, and in this example, we contemplate a 2D array sweeping around the body vertical Z direction. Since the body may not actually be level, the body Z axis may be tilted compared to geographic Down. In other words, the platform may have pitch and roll angles that change. When we measure azimuth in the body frame, have to designate the X axis as zero angle, and increase the angle as we rotate toward the Y axis. To avoid confusion with the geographic azimuth, we shall call this body azimuth a “pseudo-azimuth”. The difficulty is whether the body orientation is actually level and aligned with North. The same concern arises with elevation, and we shall speak of “pseudo-elevation” angles to make clear that we are referring to body axes. In this example, the pseudo-azimuth measurements will serve as our alignment angle measurements.

Figure 8 considers a square GPS array of about a wavelength in diameter mounted on top of an airborne platform that is roughly level with the horizon. Using beam forming we can steer the GPS beam to different pseudo-azimuths and pseudo-elevation angles as measured in the body frame. In particular we can scan the apparent azimuth at a fixed elevation angle (relative to the antenna) using the array steering vector to change azimuth. By rotating the beam around the body Z axis using the array steering vector, we get an array gain.

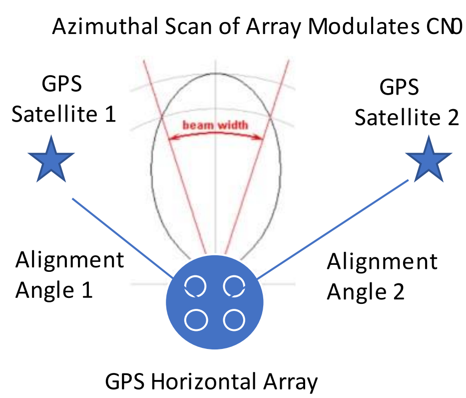

Figure 8. Self-Orienting GPS Array

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The array gain comes from coherently adding the signals from the four antenna elements using complex phases and the wave vector for the signal. For example, if the signal is located at pseudo-az = 100 degrees, then the array gain may look like Figure 9. The vertical red line is located at the actual pseudo-az of the satellite. The green line indicates the actual azimuth considering the platform yaw of 75 degrees, pitch -5 degrees, and roll = 5 degrees. Figure 9 illustrates the array gain for a satellite located at azimuth 23 degrees and elevation 20 degrees. However, the platform is oriented toward azimuth -75 degrees and it has a roll right of 5 degrees and a pitch down of -5 degrees. This causes the pseudo-azimuth and elevation angles to be 100 degrees and 24 degrees respectively.

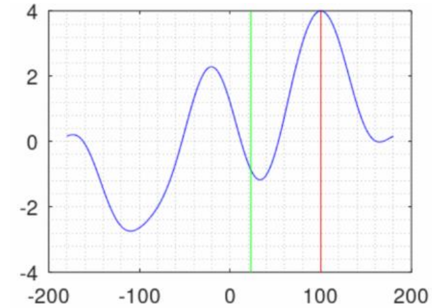

Figure 9. Gain from Steering GPS Array

As can be seen, the pseudo-azimuth angle can be determined as the peak of the gain pattern. In this manner we can measure alignment angles for satellites using a small GPS array. We will give more array processing details in the application section. We have not yet been able to use the GIANT simulator to validate these results, but we expect to do that in the near future. We also want to note that we had to work a little harder to find which peak corresponds to the alignment in Figure 10, because the peaks have the same height.

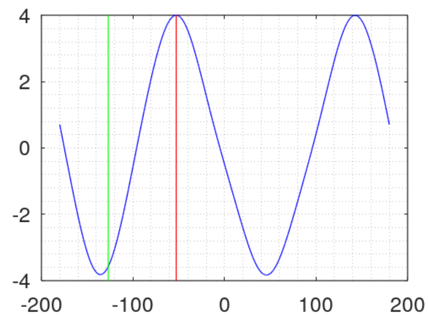

Figure 10.Satellite with Pseudo-Alignment -53 deg

The take-away point is that the pseudo-azimuth measurements are the alignment angles for the Z axis. Since the satellite NED coordinates are public, the alignment angles determine the platform orientation. Once the platform orientation has been estimated, we are able to better derive the alignment angles. Comparing the measured and predicted alignment angles allows us to update the estimate of the platform quaternion, allowing an accurate solution through an iterative process. The next section talks about how quaternion is estimated from the alignment measurements.

3) GPS6D Attitude Processing

The second major conceptual part of the process of determining the antenna alignment relative to the GPS constellation is to pose a mathematical problem that can be solved in terms of the measurements available by monitoring the modulated GPS signal. It is not exactly obvious how one uses the measured alignment angles to compute orientation. Nevertheless, there is a mathematical equivalence between the alignment angles measured by the platform, and the actual platform orientation. Knowing this equivalence exists allows us to focus our energy on deriving the alignment angles. The implementation of the algorithm is tedious. The table in

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Figure 11 defines the information including the alignment measurement data that drives the GPS6D engine to compute the antenna orientation in GPS coordinates.

Name	Description
Lat, Lon, Alt from GPS solution	Latitude and Longitude Position for antenna
Measured Alignment Angles for approximately 10 Satellites	Rotation angle around Antenna Axis that best aligns antenna boresight with the satellite line–of–sight.
Az, El for 10 Satellites	Azimuth and Elevation angles from Ephemeris. These are taken to be local line of sight angles toward the satellites. If the satellites were visible it would be easy to tell directions by GPS knowing the correct Az and El for the satellite.

Figure 11. Inputs for GPS6D Attitude Processing

The alignment angle display (see Figure 12) of GPS6D shows how the measured alignment angles stack up against the alignment angles that can be predicted from the estimated platform attitude. If the angles do not match, the satellite signal is suspect, or the attitude estimation is off due to bad measurements.

To read this display start on the left and imagine looking down the rotation axis for measuring alignment angles. In this case, you can imagine the wings of a plane at +-90 degrees looking forward while the antenna rotates left to right. The red pointer indicates the current angle of the antenna. The red, orange and yellow circles represent the satellites twice over. The orange circles indicate the alignment angles at which we measured the satellites. Furthermore, the yellow circles on the inner radius are the actual measurements and the and the orange circles (outer radius) are the simulation’s best estimate of the correct input measurement. If there were no errors, then the orange and yellow circles would be at the same alignment angles. The close alignment between orange and yellow circles means there is little error in the measurements and estimate.

The list on the right of Figure 12 provides the numerical data associated with the satellites and alignment measurements. The row high-lighted in red corresponds to the two red circles on the alignment dial. We note that the implementation uses a Quality of Service (QoS) flag to indicate when there are measurement problems. The rms errors between the satellite angles in Figure 12 are of the same magnitude as errors in the orientation of the platform, with the exception that geometric constraints must be satisfied. This is consistent with the attitude quaternion produced by the algorithm. It tells us that in this case our alignment measures have an rms error of about 6 degrees

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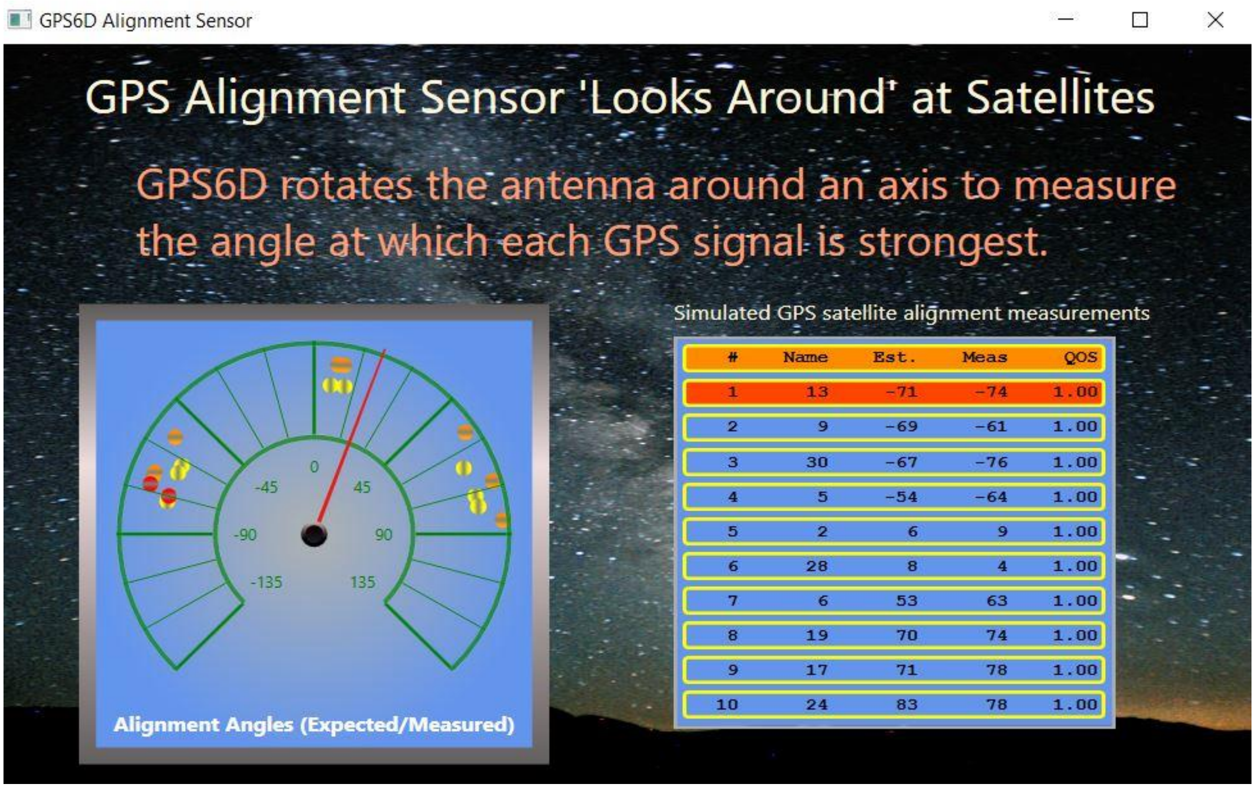

Figure 12. GPS6D Alignment Display

The GPS6D inclinometer display is shown in Figure 13. It reinterprets the attitude quaternion in terms of Tait-Bryan Yaw, Pitch and Roll angles relative to the NED reference frame. Envision the platform oriented due North with the Z axis down. Then we reach the actual platform attitude by performing a composite of first a Yaw, then a Pitch, and finally a Roll transformation. The Yaw is effectively the azimuthal heading of the platform. The pitch is the normal pitch up of a platform, and the roll indicates whether bank angle 0 really is vertical up or rolled to some nonzero bank angle. Note that the azimuth would be reversed 180 degrees if the sign of the alignment angles were changed. This display shows some tiny deviations in angles from Yaw/Pitch/Roll = 0/0/0. This is an Eastward heading, flying level with Z up.

Regarding the use of GPS6D, the biggest automation challenge is the extraction of the alignment angles. The matched filter works well in the middle of the antenna pattern but is surprisingly useless near a null. However, the large derivative at the null makes it possible to pick out an approximate null.

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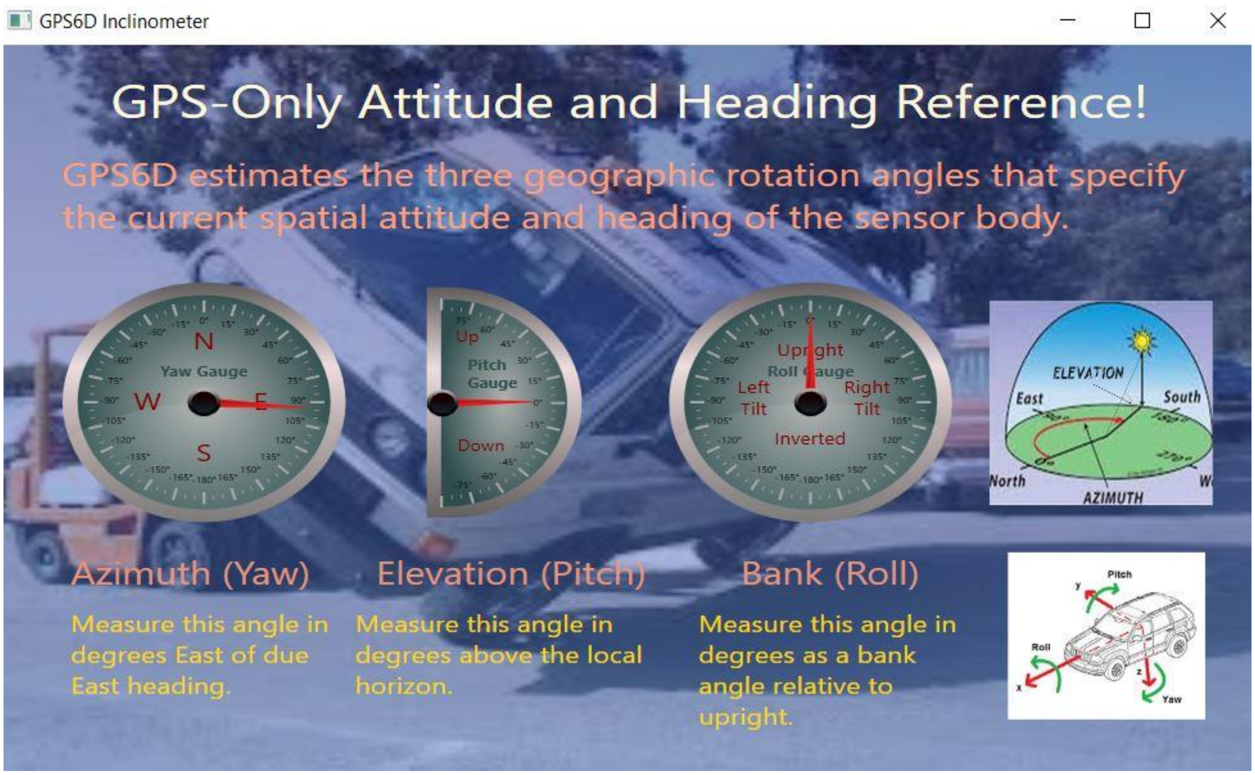

Figure 13. GPS6D Attitude and Heading Reference

4) Methods: Orientation, Perspective, Projection and Alignment Angles

Perspective: We judge 3D-orientations from our point of view using changes in apparent angles and directions. While this is not obvious it becomes intuitively clear if we consider a camera taking a picture like in Figure 14. If we project a 3D view onto an image, we can later determine the point of view of the camera relative to objects in the picture. Although the camera uses point projection through the origin to produce an image, we can also reconstruct the camera's physical orientation from a parallel projection image if we know the physical location of features in the image. Now we remind ourselves that with polar coordinates all the locations in the figure can be described by a radial and an angular measurement. Furthermore, it is clear that the radial dimension is relative to the size of the picture, which does not directly determine the orientation of the camera (although it might determine the position or range of the camera in some manner). However, the polar angles carry orientation information directly. They are the alignment angles for the LOS axis of the camera, and they depend entirely on how the camera is oriented.

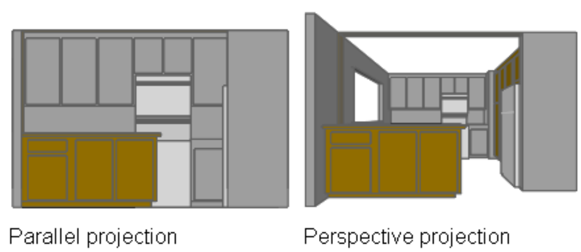

Figure 14. A Projection Determines Camera Pose

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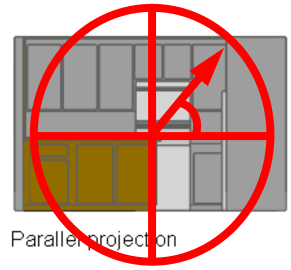
Alignment Angles: Figure 15 is intended to help one imagine looking down an axis $\vec{A}$ and observing points, $\vec{S_i}$ for i =1..n, positioned around (but generally off) the line-of-sight (LOS) for the axis. Imagine a large dial like a clock face that you can rotate around $\vec{A}$ until an arrow points toward the projected object. Pick an origin for zero rotation and measure the polar angle α_i that points toward the projected object $\vec{S_i}$, and this is the alignment angle for this object and orientation. Alignment angles $(\alpha_1, \alpha_2, .., \alpha_n)$ are features that depend on projection parallel to the axis $\vec{A}$. Under reasonable assumptions, the alignment angles uniquely determine the orientation of the projection axis A and the reference angle.



Orientation of the platform (physical body) coordinate XYZ axes relative to the local geographic NED frame is given by a quaternion q rotating the NED axes to XYZ axes. The Z-axis generally remains down when GPS is involved (to keep satellites visible), but the platform may not be level involving some pitch and roll as well as azimuthal heading. Fixing the satellites Si, we can let the axis orientation q vary and obtain various alignment angles. Figure 15 shows a subtle part of the geometric alignment model G. This geometric model G is a mapping of the quaternions (or some subset remaining upright) into multiple alignment angles depending on the number of satellites. The first part of the map transforms the quaternion into an orientation to measure alignment angles. This is a map of the 3-sphere onto the projective space or rotations, and this transformation is exactly 2 to 1 sending antipodal points on the sphere to the same rotation.

Alignment Model: $G: H \rightarrow R^n$ by $G(q) = \vec{\alpha} = (\alpha_1, \alpha_2, .., \alpha_n) \epsilon R^n$

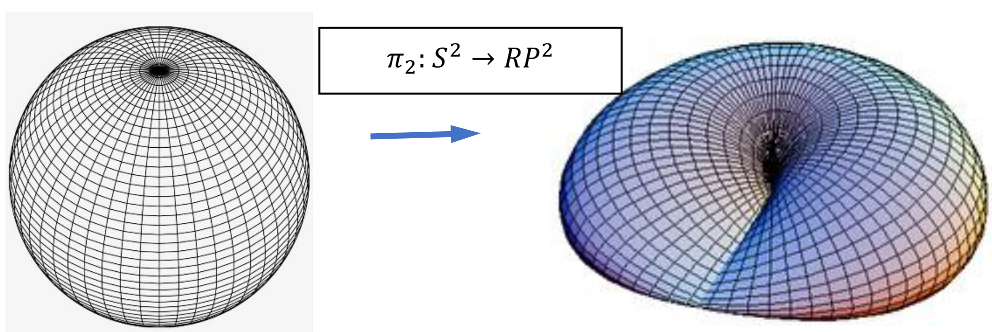


Figure 15. Projecting Quaternions onto Rotations

Claim: Under reasonable assumptions, for n > 3, the model G is a smooth embedding of S3 into R^n where each set of actual alignment angles is the image of two antipodal points +q and -q in H. Therefore, G can be factored through the fundamental projection $\pi: S^3 \rightarrow RP^3$ so that $G: H \approx S^3 \rightarrow RP^3 \rightarrow R^n$.

The projection shown in Figure 15 is a 2D representation of a 3D problem. It is similar to examining the equator of the 3-sphere, which is an ordinary 2-sphere. On the rotation side, we

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see the figure does not directly embed in 3-space because it somehow twists through itself. However, in 4 dimensions, there is room to turn, and the rotations from 3 space can be embedded one-to-one. The important point is that a one-to-one map can be inverted, and our problem is basically to invert the geometric model G. What we learn from Figure 15 is that there is an intrinsic curvature to this problem that tells us to favor quadratic solution techniques. It is similar to using Newton's method to find the minimum of $f(x) = x^{10}$. Convergence will be slow if curvature is ignored.

The **inverse problem** is to determine the quaternion q that produces the alignment angles $\vec{\alpha}$. This is a generalized parameter estimation problem with unique features due to the unusual structure of the rotation group SO(3) $\approx RP^3$, which is equivalent to real projective 3-space. The quaternion representation H is a double covering of the rotation group by the space S^3. The fundamental group of the RP3 rotations includes a nontrivial loop created by the identification of North and South poles for example. The projection identifies antipodal points on the sphere, so $\pi: S^3 \to RP^3$ is 2 to 1 on quaternions identifying (-1,0,0,0) and (1,0,0,0) as the same rotation, which projects to the identity rotation. Surprisingly, the quaternion (-1,0,0,0) represents any 360-degree rotation but it is not exactly the identity in our 3D space of rotations and translations. Instead, a 720-degree rotation is needed to create the same quaternions, a point that is illustrated by Dirac's string trick, or the Belt or Plate Tricks (see Wikipedia). This is because there is a loop in RP3 (and RP2) that cannot be contracted to zero, and a 360 rotation goes once around the loop because (-1,0,0,0) gets identified with (1,0,0,0). In general, curvature requires a focus on the Hessian to invert the model and use quadratics to estimate the orientation given the alignment angles.

5) GPS6D Applications

5.1 Application to Self-Orienting Array on Airborne Platform

The problem to be solved is to find the orientation of a square horizontal array used for GPS denied/degraded applications. The approach is to use the array geometry to measure the pseudo-azimuth (or alignment) angles and then estimate the true orientation of the array relative to the GPS constellation.

Make a general assumption that the CN0 measurements produced by the GPS receiver are proportional to the complex amplitude of the received signal strength. Informally, the CN0 needs to peak when the antenna is aimed in the direction of the satellite generating the GPS signal. Neglect multipath reception and make a general assumption that this is an airborne platform with the square array placed conformally on top of the platform.

Specify how the array is mounted relative to the platform attitude. For simplicity, assume the array elements are omnidirectional in the upward hemisphere. Suppose the platform has roll, pitch and yaw axes named X, Y and Z. X is out the nose, Y is out the right wing, and Z is down for the platform. Don't worry about the details of the 4 antenna elements beyond their locations at the four corners $\vec{r}(i)$, i = 1… 4, of a square with a diagonal length L and X,Y coordinates:

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$$\vec{r}(1) = (L/(2\sqrt{2}) * (1, 1, 0); \vec{r}(2) = (L/2\sqrt{2}) * (-1, 1, 0);$$

$$\vec{r}(3) = (L/(2\sqrt{2}) * (-1, -1, 0); \vec{r}(4) = (L/(2\sqrt{2}) * (1, -1, 0);$$

Suppose the complex signal arrives as a plane wave of the form S(t) = R(t)*exp(j*theta(t)) where we ignore polarization for R(t) and recognize that theta(t) is changing as the GPS frequency. It is convenient to describe the arriving signal as having a pseudo-azimuth phi0 relative to X and a vertical polar angle theta0, so the vector direction in X, Y and Z is then:

$$S0 = (sin(\theta 0) * cos(\varphi 0), sin(\theta 0) * sin(\varphi 0), cos(\theta 0))$$

Which gives a wave vector: $\vec{k} = (2\pi\ \lambda) * S0$. Projection of the wavevector onto the array vectors gives the relative phase delays at the corners of the square:

$$\vec{k} \cdot \vec{r}(1) = (\pi/\sqrt{2}) * sin(\theta 0) * (cos(\varphi 0) + sin(\varphi 0))$$

$$\vec{k} \cdot \vec{r}(2) = (\pi/\sqrt{2}) * sin(\theta 0) * (-cos(\varphi 0) + sin(\varphi 0))$$

$$\vec{k} \cdot \vec{r}(3) = (\pi/\sqrt{2}) * sin(\theta 0) * (-cos(\varphi 0) - sin(\varphi 0))$$

$$\vec{k} \cdot \vec{r}(4) = (\pi/\sqrt{2}) * sin(\theta 0) * (cos(\varphi 0) - sin(\varphi 0))$$

To coherently sum the complex signals S(i), i=1…4, remove the relative phase shifts to get a matching phase for all 4 elements. The phase exactly matches in the direction of the satellite $S0$. Use the steering vector:

$$\vec{V}(\vec{k}) = [\ exp(-j\vec{k} \cdot \vec{r}(1)), exp(-j\vec{k} \cdot \vec{r}(2)), exp(-j\vec{k} \cdot \vec{r}(3)), exp\ (-j\vec{k} \cdot \vec{r}(4))\ ]$$

Add coherently to form the complex sum:

$$F = \sum \vec{V}(\vec{k})(i) S(i)\ i \approx 4 * S(t)$$

Expect a complex amplitude gain of up to 4 when steering in the correct direction.

Suppose it is possible through this mechanism to identify the pseudo-azimuth angles for all (or most) of the satellites S(i), I = 1 ... N where N is 10 or more visible GPS satellites. These angles are also called alignment angles because they are the angles at which the satellites aligned with the XY pseudo-azimuth (ignoring elevation). In practice, it may help to initially delete satellites at high elevation angles since the alignment does not resolve well.

We have not yet run this problem though GIANT and we are awaiting good array data for further test. There seems to be a straightforward iterative method for GPS6D to use the GPS alignment angles obtained from the CN0 stream from the array. This will eliminate the need to physically rotate the antenna, enabling self-orienting GPS navigation systems.

5.2) Ground-Based Application: Multipath Measurement

The GPS system can be used to measure both position and orientation (6 DOF) if multipath is not present. This is the case for airborne applications, but not the case for antennas on the

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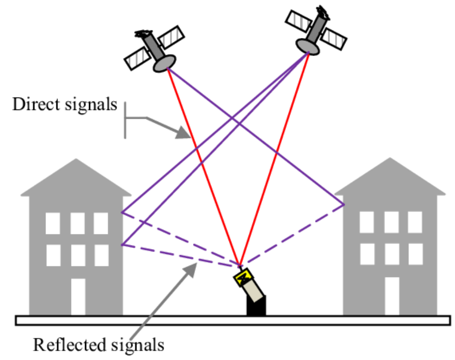
ground. The problem can be turned around again to attack Multipath on the ground. If multipath is present, the local alignment angles will demonstrate that there is an inconsistency between the angles and a known geometric orientation. For example, we may find a problematic satellite in the display of Figure 12. Figure 13 shows how this can happen even when the satellite is LOS to the antenna.



If we know our orientation on the ground, we can compute the LOS vectors to the satellites, and even the alignment angles with respect to a chosen axis. If the measured angle is off from the predicted alignment angle, then we know that satellite is being received by a reflected signal. Likely, we will find an obstruction in the LOS path or a nearby reflecting surface.

Using a model for Rician fading we see that the LOS polarization vector is altered by the reflections in the environment. When one component (often, but not necessarily, a line of sight component) dominates, a Rician distribution provides a model for Rician fading. Model this propagation channel by considering a sinusoidal GPS carrier. This signal received over a Rician multipath channel can be expressed as

$$s(t) = \cos \omega_c t; \quad v(t) = C\,\vec{E}_0 \cos \omega_c t + \sum_{n=1}^{N} \rho_n\, \vec{E}_n \cos(\omega_c t + \phi_n)$$

Here C is the amplitude of the line-of-sight component; ρ_n is the amplitude of the n-th reflected wave; ϕ_n is the phase of the n-th reflected wave; n = 1 ... N identify the reflected and scattered waves; $\vec{E}$ is the unit polarization vector; and $\vec{E}_n$ is a reflected polarization vector.

The effect of the multipath on the alignment angle is a random sum. A tool for detecting and measuring multipath enables diagnostics and a path to improved performance of systems susceptible to multipath. Such measurements will determine which satellites are affected by multipath. Research in this direction is required to determine practical application.

5.3) Application to Self-Orienting Rotating Platforms

One way to create a stable orientation is to impart a rotation rate to a platform, such as a satellite spinning in space. This differs from a non-rotating platform because inertial rotation will impose constraints on dwell-time that may be difficult to satisfy for GPS. Suppose inertial rotation about a fixed axis $\vec{A}$ and record the periodic times when some feature aligns with a beacon. The beacon could be a GPS satellite, but we can also imagine a laser beacon if rotation rate is a problem. Another option is a pseudo-Doppler measurement that can be made in some systems as the platform rotates. The key here is the ability to measure a timing pulse when the rotating sensor aligns with the beacon. The source of an optical signal could even be on the platform, and simply reflected from the beacons, like radar. For simplicity, assume some sensor aligns with the beacon at periodic times, and assume the ability to measure these times and alignment angles. This makes it possible to estimate both the orientation as well as the

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rotation rates for the platform. These measurements unlock both the orientation of the sensor as well as the angular rates.

Conceptually, knowledge of position as a function of time t should determine derivative, but the presence of noise prevents numerical differentiation of the quaternion. Thus we will talk directly about estimating the quaternion derivative dq/dt from angular velocity components (pqr) measured in body coordinates. Let $\vec{H}$ be the angular momentum of a rigid body, so $\vec{H} = I\vec{\omega}_I$ where $\vec{\omega}_I$ = [p, q, r]' is the angular rotation vector of the body about the center of mass, p is rotation about the x-axis, q is rotation about the y-axis, r is rotation about the z-axis, $\vec{\omega}_I$ is defined in an Inertial Frame, and the matrix I is the Moment of Inertia Matrix.

This derivation applies to Tait-Bryan angles relative to the right-handed coordinate system defined by the NED axes fixed to the Earth. Yaw ψ is the first rotation about the z axis, pitch Θ is the second rotation about the y axis that formed following yaw, and roll ϕ is the third rotation about the x axis that was formed following pitch.

For a rotating body, the rate of change of the spatial orientation can be calculated based on detected rate of change of alignment angle. A linear relationship exists between the derivatives of the Euler angles (dψ/dt, dΘ/dt, dϕ/dt) and the components (p,q,r) of the angular velocity. Calculation shows there is a 3x3 matrix M1 such that

$$\frac{d}{dt}(\psi, \theta, \varphi) = M1 * [p; q; r]$$

$$M1 = [1 \;\; sin\varphi \; tan\theta \;\; cos\varphi tan\theta; \; 0 \;\; cos\varphi \;\; -sin\varphi; \; 0 \;\; sin\varphi sec\theta \;\; cos\varphi sec\theta]$$

And there is a 4x3 matrix D such that the quaternion derivative is

$$\frac{dq}{dt} = D * [p; q; r]$$

$$D = 05.* [-x, -y, -z; w, -z, y; \; z, w, -x; \; -y, x, w]$$

The angular velocity in the body frame is calculated from the quaternion derivative by inverting the above equation.

6.) Conclusions

This paper is a step along the path that 6DOFGPS is walking with LinQuest to extend GPS positioning to GPS orientation. We have discussed several ways that orientation can be obtained using only the GPS system. We hope that a new approach may lead to gains in several areas at a greatly reduced expense.

Clearly, we seek opportunities to apply and test this new GPS6D technology with GPS antenna systems, or non-GPS systems, that have directional capabilities. We seek further discussions with interested parties who can provide hardware or expertise to make progress.

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