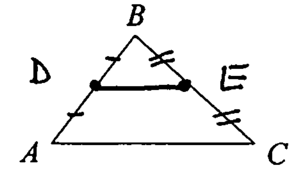
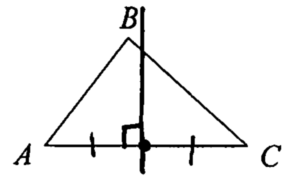
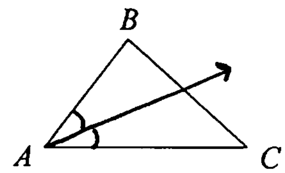
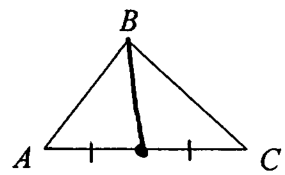
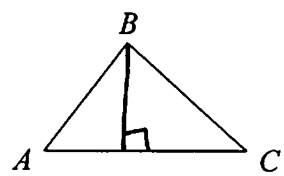
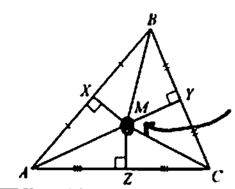
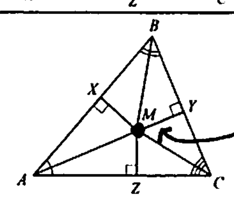
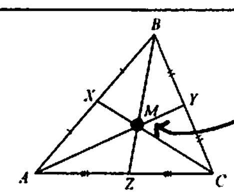
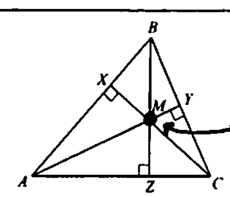
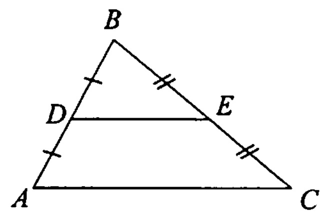
SPECIAL SEGMENTS & CENTERS of triangles

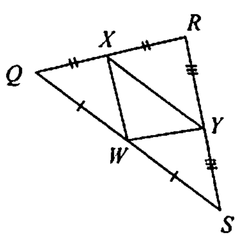
TERM	DEFINTION	PICTURE
MIDSEGMENT	A segment that joins the midpoints of two sides of a triangle.	
PERPENDICULAR BISECTOR	A line, segment, or ray that divides a segment into two equal parts and is perpendicular to the segment.	
ANGLE BISECTOR	A line, segment, or ray that divides an angle into two equal parts.	
MEDIAN	A segment that connects a vertex of a triangle to the midpoint of the opposite side.	
ALTITUDE	A segment that connects a Vertex of a triangle to the opposite side so that it is perpendicular to that side.	
CIRCUMCENTER	The point at which the three perpendicular bisectors intersect in a triangle.	
INCENTER	The point at which the three angle bisectors intersect in a triangle.	
CENTROID	The point at which the three medians intersect in a triangle.	
ORTHOCENTER	The point at which the three altitudes intersect in a triangle.	

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
TRIANGLE MIDSEGMENT	• A **triangle midsegment** is a segment connecting the midpoints of two sides of the triangles. • **Example:** $\overline{DE}$
TRIANGLE MIDSEGMENT Theorem	If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and half as long. Using the diagram above, if $\overline{DE}$ is a midsegment of ΔABC, then: 1) $\overline{AC} \parallel \overline{DE}$ 2) $DE = \frac{1}{2}AC$ $(AC = 2 \cdot DE)$

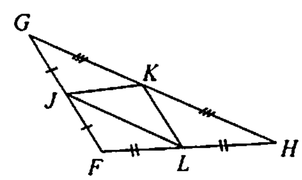


1. Identify all pairs of parallel segments.



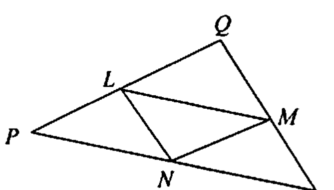
a) $\overline{XY} \parallel \overline{QS}$
b) $\overline{WY} \parallel \overline{QR}$
c) $\overline{WX} \parallel \overline{SR}$

2. Identify all pairs of parallel segments.



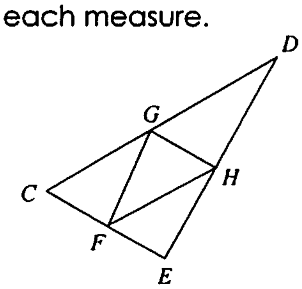
a) $\overline{JL} \parallel \overline{GH}$
b) $\overline{JK} \parallel \overline{FH}$
c) $\overline{KL} \parallel \overline{GF}$

3. If L, M, and N are the midpoints of the sides of ΔPQR, $PR = 46$, $PQ = 40$, and $LN = 17$, find each measure.



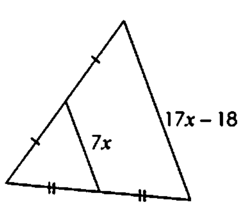
a) $LM =$ 23
b) $MN =$ 20
c) $QR =$ 34
d) $MR =$ 17

4. If F, G, and H are the midpoints of the sides of ΔCDE, $FG = 9$, $GH = 7$, and $CD = 24$, find each measure.



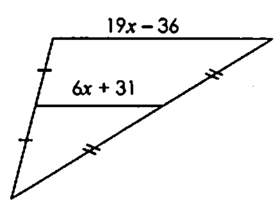
a) $CE =$ 14
b) $DE =$ 18
c) $FH =$ 12
d) Perimeter of ΔCDE: 56

5. Find the value of x.



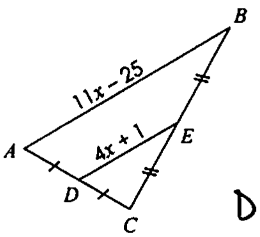
$2(7x) = 17x - 18$
$14x = 17x - 18$
$-3x = -18$
$\boxed{x = 6}$

6. Find the value of x.



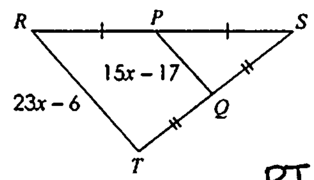
$2(6x+31) = 19x - 36$
$12x + 62 = 19x - 36$
$98 = 7x$
$\boxed{14 = x}$

7. Find DE.



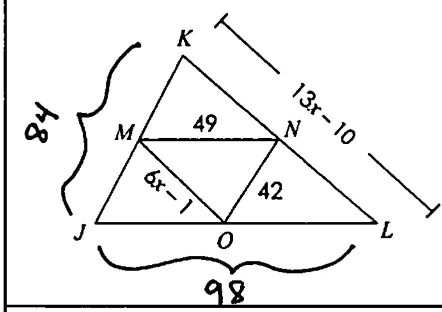
$2(4x+1) = 11x-25$
$8x+2 = 11x-25$
$27 = 3x$
$9 = x$
DE: $4(9)+1 = \boxed{37}$

8. Find RT.



$2(15x-17) = 23x-6$
$30x-34 = 23x-6$
$7x = 28$
$x = 4$
RT: $23(4)-6 = \boxed{86}$

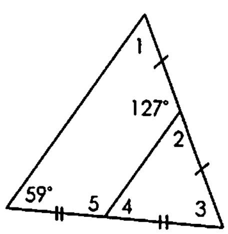
9. If $\overline{MN}$, $\overline{NO}$, and $\overline{MO}$ are midsegments, find the perimeter of ΔJKL.



$2(6x-1) = 13x-10$
$12x-2 = 13x-10$
$8 = x$
KL: $13(8)-10$
$= 94$

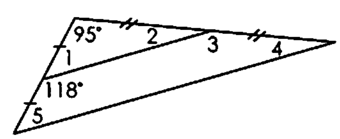
Perimeter =
$94+84+98$
$\boxed{= 276}$

10. Find the measure of each missing angle.



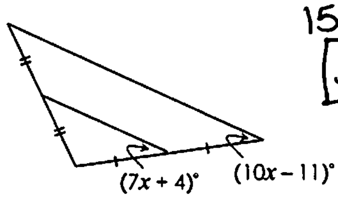
$m\angle 1 =$ 53°
$m\angle 2 =$ 53°
$m\angle 3 =$ 68°
$m\angle 4 =$ 59°
$m\angle 5 =$ 121°

11. Find the measure of each missing angle.



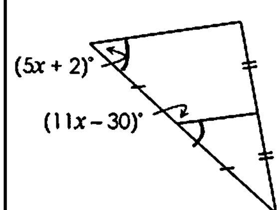
$m\angle 1 =$ 62°
$m\angle 2 =$ 23°
$m\angle 3 =$ 157°
$m\angle 4 =$ 23°
$m\angle 5 =$ 62°

12. Find the value of x.



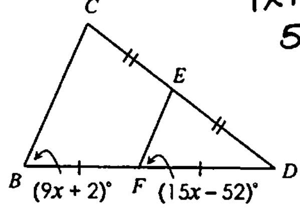
$7x+4 = 10x-11$
$15 = 3x$
$\boxed{5 = x}$

13. Find the value of x.



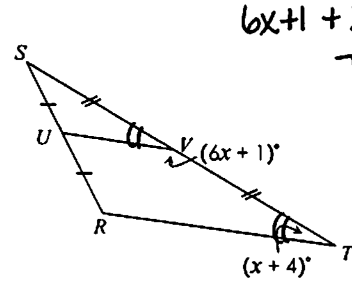
$5x+2+11x-30 = 180$
$16x-28 = 180$
$16x = 208$
$\boxed{x = 13}$

14. Find $m\angle CBF$.



$9x+2 = 15x-52$
$54 = 6x$
$9 = x$
$m\angle CBF = 9(9)+2$
$\boxed{= 83°}$

15. Find $m\angle RTV$.



$6x+1+x+4 = 180$
$7x+5 = 180$
$7x = 175$
$x = 25$
$m\angle RTV = 25+4$
$\boxed{= 29°}$

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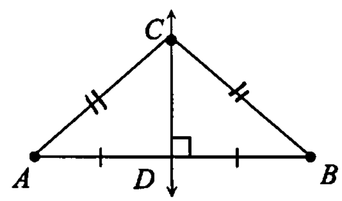
Main Ideas/Questions	Notes/Examples
PERPENDICULAR BISECTOR Theorems	

Perpendicular Bisector Theorem

If a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

If $\overleftrightarrow{CD} \perp \overline{AB}$ **and** $AD = BD$**,**

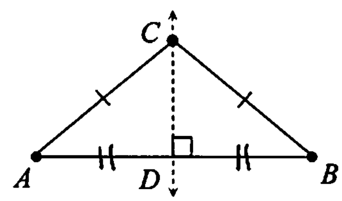
then $\overline{AC} \cong \overline{CB}$.



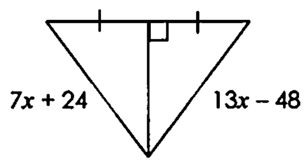
Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

If $CA = CB$, **then a line exists through** C **such that** $\overleftrightarrow{CD} \perp \overline{AB}$ **and** $\overline{AD} \cong \overline{DB}$.



1. Find the value of x.

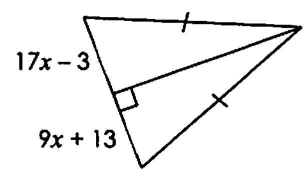


$7x+24 = 13x-48$

$72 = 6x$

$\boxed{x=12}$

2. Find the value of x.

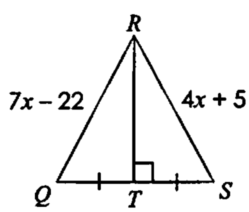


$17x-3 = 9x+13$

$8x = 16$

$\boxed{x=2}$

3. Find RS.



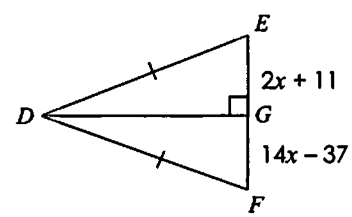
$7x-22 = 4x+5$

$3x = 27$

$x = 9$

$RS: 4(9)+5 = \boxed{41}$

4. Find EG.



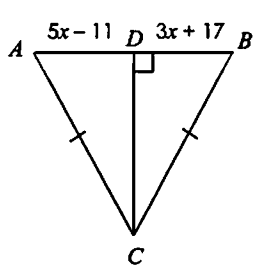
$2x+11 = 14x-37$

$48 = 12x$

$x = 4$

$EG: 2(4)+11 = \boxed{19}$

5. Find AB.



$5x-11 = 3x+17$

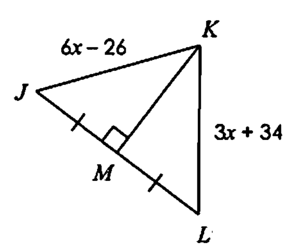
$2x = 28$

$x = 14$

$AD = 5(14)-11 = 59$

$AB = 2(59) = \boxed{118}$

6. Find JK.



$6x-26 = 3x+34$

$3x = 60$

$x = 20$

$JK: 6(20)-26$

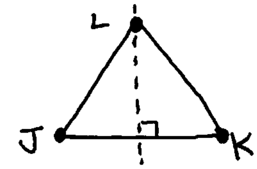
$\boxed{=94}$

7. If $\overline{JK}$ is formed by $J(-7, -8)$ and $K(1, 4)$, determine if $L(-9, 2)$ lies on the perpendicular bisector of $\overline{JK}$.

$\overline{JL}$: $d = \sqrt{(-9+7)^2+(2+8)^2}$
$= \sqrt{4+100} = \sqrt{104}$

$\overline{KL}$: $d = \sqrt{(-9-1)^2+(2-4)^2}$
$= \sqrt{100+4} = \sqrt{104}$

$\boxed{\text{Yes!}}$



(JL must = KL)

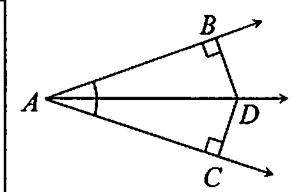
ANGLE BISECTOR
Theorems

Angle Bisector Theorem

If a point is on a bisector of an angle, then the point is equidistant from the sides of the angle.

If $\overrightarrow{AD}$ **bisects** $\angle BAC$, $\overrightarrow{AB} \perp \overline{BD}$, **and** $\overrightarrow{AC} \perp \overline{CD}$,

then $\overline{BD} \cong \overline{DC}$.

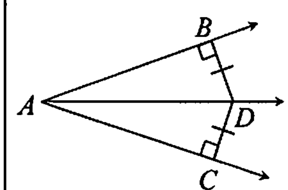


Converse of the Angle Bisector Theorem

If a point is on the interior of an angle and equidistant from the sides of the angle, then the point is on the angle bisector.

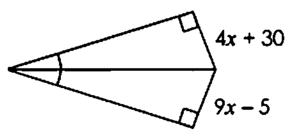
If $BD = CD$, $\overrightarrow{AB} \perp \overline{BD}$, **and** $\overrightarrow{AC} \perp \overline{CD}$,

then $\overrightarrow{AD}$ bisects $\angle BAC$ ($\angle BAD \cong \angle CAD$)



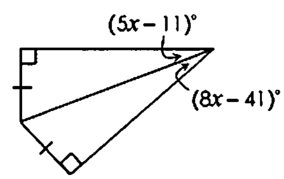
8. Find the value of x.

$9x-5 = 4x+30$
$5x = 35$
$\boxed{x=7}$



9. Find the value of x.

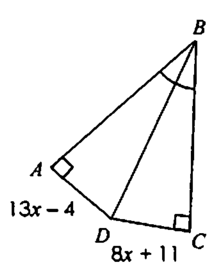
$5x-11 = 8x-41$
$30 = 3x$
$\boxed{10=x}$



10. Find AD.

$13x-4 = 8x+11$
$5x = 15$
$x=3$

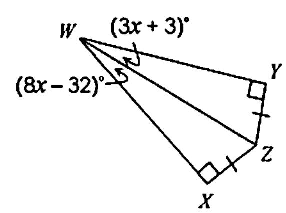
AD: $13(3)-4$
$\boxed{=35}$



11. Find $m\angle XWZ$.

$8x-32 = 3x+3$
$5x = 35$
$x=7$

$m\angle XWZ$: $8(7)-32$
$\boxed{=24}$

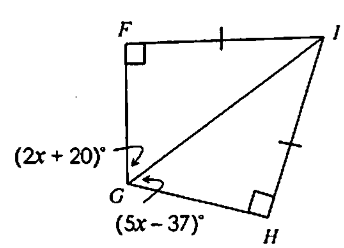


12. Find $m\angle FGH$.

$5x-37 = 2x+20$
$3x = 57$
$x=19$

$m\angle FGI$: $2(19)+20$
$= 58°$

$m\angle FGH$: $2(58)$ $\boxed{=116°}$



Name:	Date:
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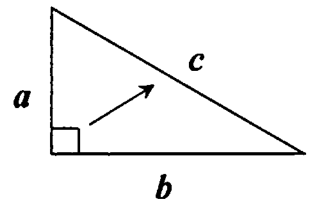
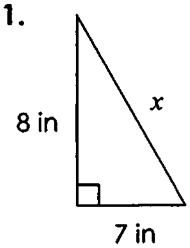
Main Ideas/Questions	Notes/Examples
Parts of a **RIGHT TRIANGLE**	 • Sides ***a*** and ***b*** are called legs • Side ***c*** is called the hypotenuse
What is the **PYTHAGOREAN THEOREM?**	The **Pythagorean Theorem** is used to find a **missing side length** on a right triangle! **Formula:** $a^2+b^2=c^2$

EXAMPLES

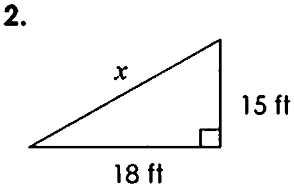
Find the missing side of each triangle. Round to the nearest tenth.

1.



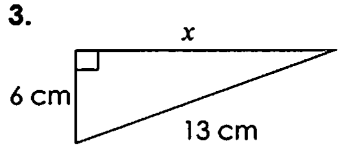
$7^2+8^2=x^2$
$49+64=x^2$
$113=x^2$
$\boxed{x=10.6\text{ in}}$

2.



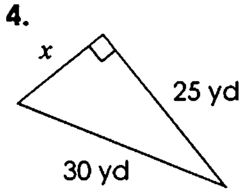
$18^2+15^2=x^2$
$324+225=x^2$
$549=x^2$
$\boxed{x=23.4\text{ ft}}$

3.



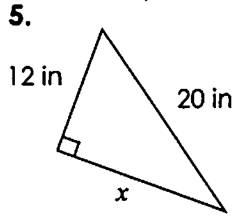
$6^2+x^2=13^2$
$36+x^2=169$
$x^2=133$
$\boxed{x=11.5\text{cm}}$

4.



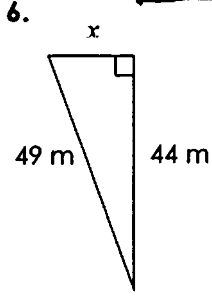
$x^2+25^2=30^2$
$x^2+625=900$
$x^2=275$
$\boxed{x=16.6\text{ yd}}$

5.



$12^2+x^2=20^2$
$144+x^2=400$
$x^2=256$
$\boxed{x=16\text{in}}$

6.



$x^2+44^2=49^2$
$x^2+1936=2401$
$x^2=465$
$\boxed{x=21.6\text{ m}}$

Name:	Date:
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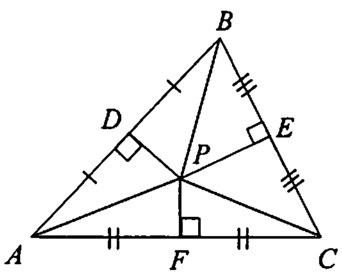
Main Ideas/Questions	Notes/Examples
CIRCUMCENTER	• The perpendicular bisectors of the sides of a triangle intersect at a point called the **circumcenter.**
	• The circumcenter is always equidistant from the vertices of the triangle.



Use the diagram to the left to answer the following questions:

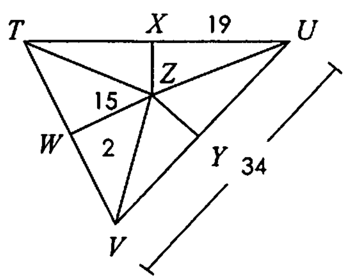
1) List the perpendicular bisectors: $\overline{PF}$, $\overline{PE}$, $\overline{PD}$

2) Name the circumcenter: P

3) List all congruent segments: $\overline{AD} \cong \overline{DB}$, $\overline{BE} \cong \overline{EC}$, $\overline{AF} \cong \overline{FC}$, $\overline{AP} \cong \overline{BP} \cong \overline{CP}$

Directions: Find each measure using the information given.

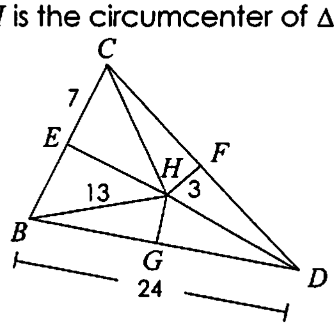
1. Z is the circumcenter of ΔTUV.



D) $x^2 + 15^2 = 21^2$
$x^2 + 225 = 441$
$x^2 = 216$
$x = 14.7$

a) TU	38
b) VY	17
c) UZ	21
d) WV	14.7
e) TV	29.4

2. H is the circumcenter of ΔBCD.

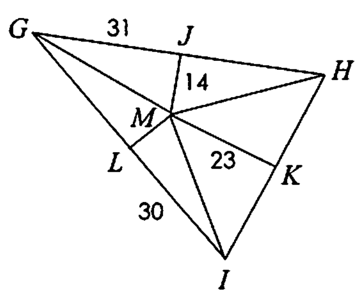


C) $7^2 + x^2 = 13^2$
$49 + x^2 = 169$
$x^2 = 120$
$x = 11$

D) $3^2 + x^2 = 13^2$
$9 + x^2 = 169$
$x^2 = 160$
$x = 12.6$

a) GD	12
b) BC	14
c) EH	11
d) FD	12.6
e) CD	25.2

3. M is the circumcenter of ΔGHI.

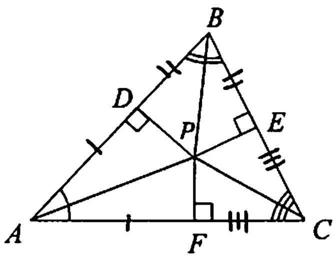


B) $31^2 + 14^2 = x^2$
$1157 = x^2$
$x = 34$

C) $23^2 + x^2 = 34^2$
$529 + x^2 = 1156$
$x^2 = 627$
$x = 25$

a) GI	60
b) MH	34
c) IK	25
d) HI	50
e) MG	34

INCENTER



- The angle bisectors of the angles of a triangle intersect at a point called the **incenter.**
- The incenter is always equidistant from the sides of the triangle.

Use the diagram to the left to answer the following questions:

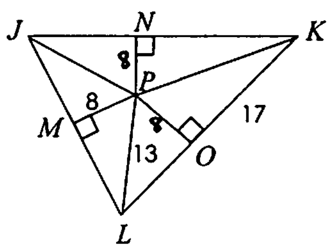
1) List the angle bisectors: $\overline{PB}$, $\overline{PC}$, $\overline{PA}$

2) Name the incenter: P

3) List all congruent segments: $\overline{PD} \cong \overline{PE} \cong \overline{PF}$, $\overline{AD} \cong \overline{AF}$, $\overline{BD} \cong \overline{BE}$, $\overline{CF} \cong \overline{CE}$

Directions: Find each measure using the information given.

4. P is the incenter of ΔJKL.

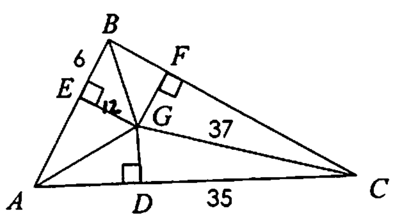


c) $8^2 + 17^2 = x^2$
$353 = x^2$
$x = 18.8$

D) $8^2 + x^2 = 13^2$
$64 + x^2 = 169$
$x^2 = 105$
$x = 10.2$

a) NP	8
b) NK	17
c) PK	18.8
d) LO	10.2

5. G is the incenter of ΔABC.

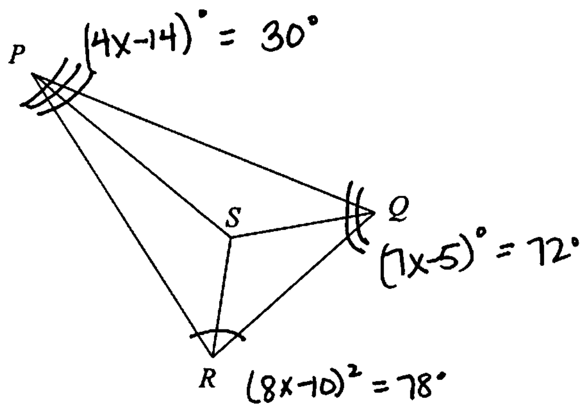


A) $x^2 + 35^2 = 37^2$
$x^2 + 1225 = 1369$
$x^2 = 144$
$x = 12$

B) $6^2 + 12^2 = x^2$
$180 = x^2$
$x = 13.4$

a) GD	12
b) BG	13.4
c) FC	35
d) BF	6

6. If S is the incenter of ΔPQR, $m\angle PRQ = (8x - 10)°$, $m\angle RPQ = (4x - 14)°$, and $m\angle PQR = (7x - 5)°$, find each measure.



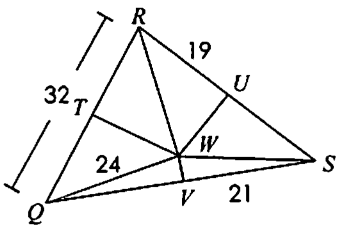
$8x - 10 + 4x - 14 + 7x - 5 = 180$
$19x - 29 = 180$
$19x = 209$
$x = 11$

a) $m\angle PRQ$	78°
b) $m\angle RPQ$	30°
c) $m\angle PQR$	72°
d) $m\angle RPS$	15°
e) $m\angle PQS$	36°
f) $m\angle PRS$	39°
g) $m\angle PSR$	126°

more practice with

CIRCUMCENTER & INCENTER

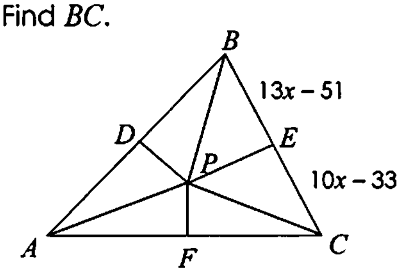
Directions: If W is the **circumcenter** of $\triangle QRS$, find each measure.



1. RS 38	**2.** TQ 16
3. WS 24	**4.** QV 21
5. TW $x^2+16^2=24^2$ $x^2+256=576$ $x^2=320$ $\boxed{x=17.9}$	**6.** WV $x^2+21^2=24^2$ $x^2+441=576$ $x^2=135$ $\boxed{x=11.6}$

Directions: If P is the **circumcenter** of $\triangle ABC$, find each measure.

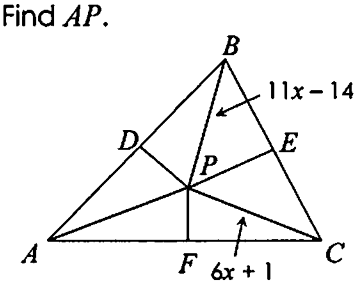
7. Find BC.



$13x-51=10x-33$
$3x=18$
$x=6$

$EC=10(6)-33=27$
$BC=2(27)=\boxed{54}$

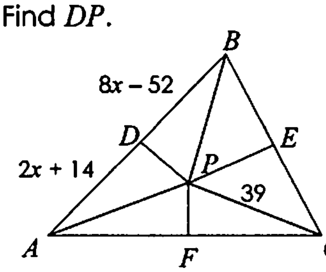
8. Find AP.



$11x-14=6x+1$
$5x=15$
$x=3$

$FC=6(3)+1=19$
$\boxed{AP=19}$

9. Find DP.

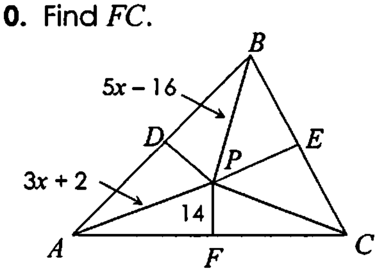


$8x-52=2x+14$
$6x=66$
$x=11$

$AD=2(11)+14$
$=36$

DP: $36^2+x^2=39^2$
$1296+x^2=1521$
$x^2=225$
$\boxed{x=15}$

10. Find FC.

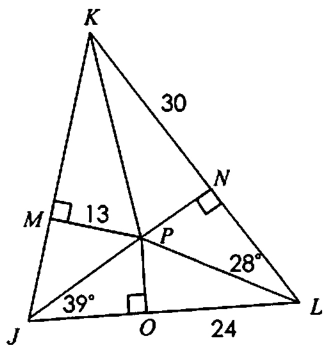


$5x-16=3x+2$
$2x=18$
$x=9$

$AP=3(9)+2$
$=29$

FC: $14^2+x^2=29^2$
$196+x^2=841$
$x^2=645$
$\boxed{x=25.4}$

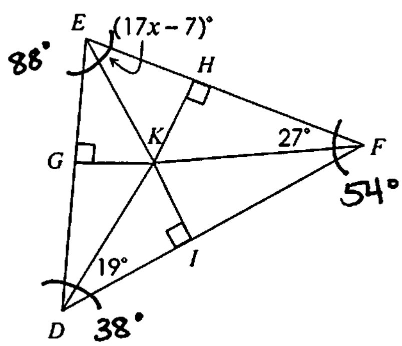
Directions: If P is the **incenter** of $\triangle JKL$, find each measure.



11. $m\angle MJP$ $39°$	**12.** $m\angle JKL$ $46°$
13. $m\angle NKP$ $23°$	**14.** $m\angle KLJ$ $56°$
15. NP 13	**16.** KM 30
17. KP $13^2+30^2=x^2$ $1069=x^2$ $\boxed{x=32.7}$	**18.** PL $13^2+24^2=x^2$ $745=x^2$ $\boxed{x=27.3}$

Directions: If K is the **incenter** of $\triangle DEF$, find each measure.

19. Find x.

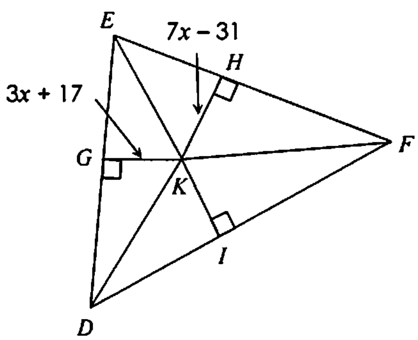


$17x-7=44$

$17x=51$

$\boxed{x=3}$

20. Find KI.



$7x-31 = 3x+17$

$4x=48$

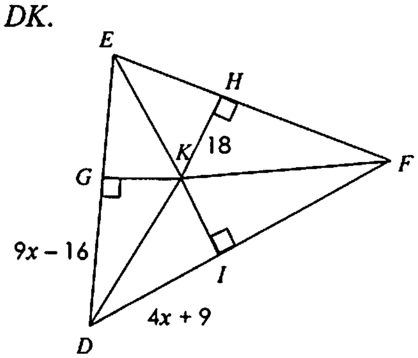
$x=12$

$GK = 3(12)+17$

$=53$

$\boxed{KI=53}$

21. Find DK.



$9x-16 = 4x+9$

$5x=25$

$x=5$

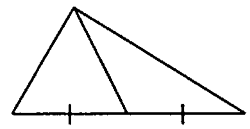
$DK: \ 18^2+29^2=x^2$

$1165=x^2$

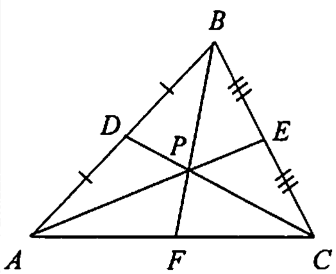
$\boxed{x=34.1}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
What is a **MEDIAN?**	A **median** is a segment joining a vertex to the midpoint of the opposite side.



CENTROID



- The three medians of a triangle intersect at a point called the **centroid.**

Use the diagram to the left to answer the following questions:

1) List the medians: $\overline{AE}$, $\overline{BF}$, $\overline{CD}$

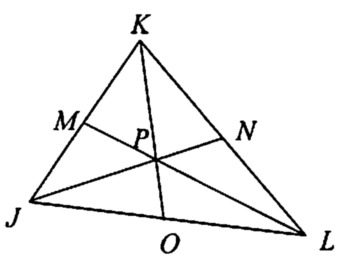
2) Name the centroid: P

3) What special properties exist for each median?

$AP = 2 \cdot PE$; $PE = \frac{1}{2} \cdot AP$

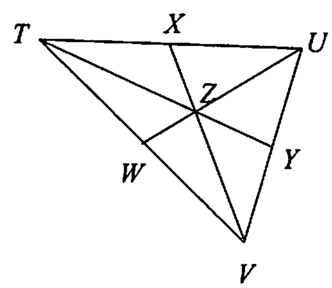
$AP = \frac{2}{3} \cdot AE$; $PE = \frac{1}{3} \cdot AE$

1. If P is the centroid of ΔJKL, $JK = 22$, $KN = 13$, and $OL = 18$, find each measure.



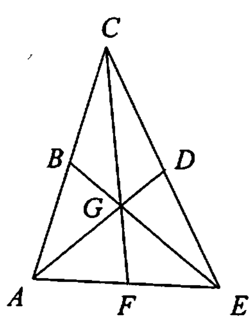
a) KM = 11
b) NL = 13
c) KL = 26
d) JO = 18
e) JL = 36

2. If Z is the centroid of ΔTUV, $TZ = 60$, $XZ = 28$, and $WZ = 25$, find each measure.



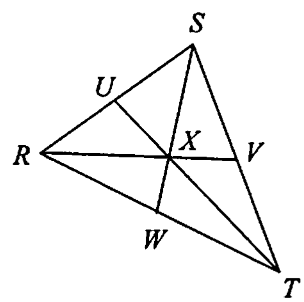
a) ZV = 56
b) ZY = 30
c) ZU = 50
d) TY = 90
e) XV = 84

3. If G is the centroid of ΔACE, $AG = 8$, $GF = 7$, and $BG = 5$, find each measure.



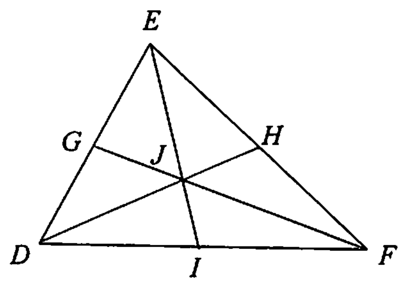
a) GD = 4
b) AD = 12
c) CG = 14
d) GE = 10
e) BE = 15

4. If X is the centroid of ΔRST, $TU = 27$, $SW = 18$, and $RV = 21$, find each measure.



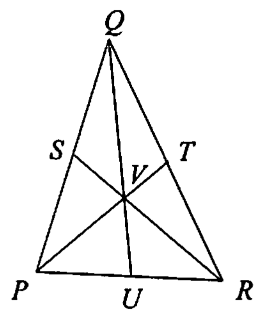
a) TX = 18
b) XU = 9
c) SX = 12
d) XW = 6
e) RX = 14

5. If J is the centroid of ΔDEF, $DH = 51$, $GF = 60$, and $EI = 57$, find each measure.



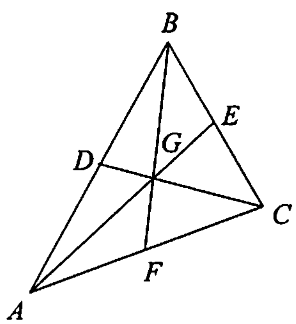
a) $DJ =$ 34
b) $JH =$ 17
c) $GJ =$ 20
d) $JF =$ 40
e) $EJ =$ 38
f) $JI =$ 19

6. If V is the centroid of ΔPQR, $SR = 21$, $VU = 8$, and $PT = 15$, find each measure.



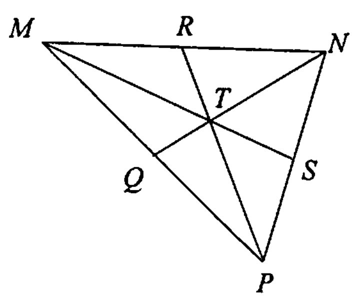
a) $SV =$ 7
b) $VR =$ 14
c) $QV =$ 16
d) $QU =$ 24
e) $PV =$ 10
f) $VT =$ 5

7. If G is the centroid of ΔABC, $BF = 72$, $AC = 64$, and $GE = 27$, find each measure.



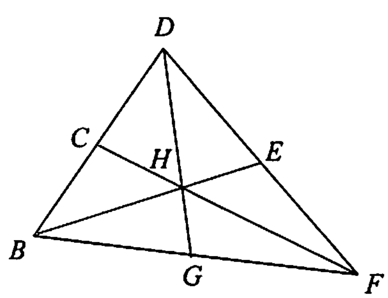
a) $AF =$ 32
b) $FC =$ 32
c) $BG =$ 48
d) $GF =$ 24
e) $AG =$ 54
f) $AE =$ 81

8. If T is the centroid of ΔMNP, $TN = 16$, $MQ = 23$, and $RP = 18$, find each measure.



a) $QT =$ 8
b) $QN =$ 24
c) $QP =$ 23
d) $MP =$ 46
e) $RT =$ 6
f) $TP =$ 12

9. If H is the centroid of ΔBDF, $DF = 50$, $CF = 42$, and $BH = 22$, find each measure.



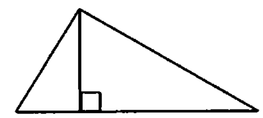
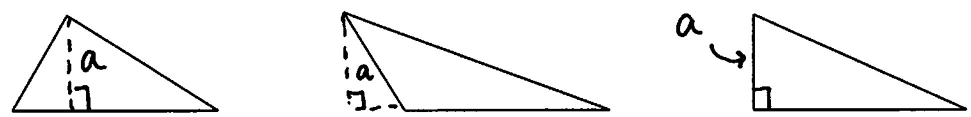
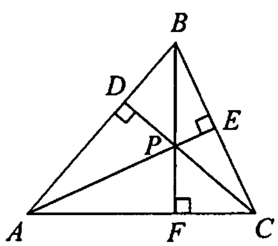
a) $HE =$ 11
b) $EF =$ 25
c) $HF =$ 28
d) $DE =$ 25
e) $CH =$ 14
f) $BE =$ 33

Name:

Date:

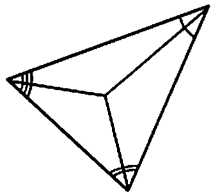
Topic:

Class:

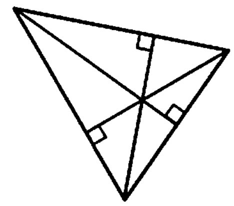
Main Ideas/Questions	Notes/Examples
What is an **ALTITUDE?**	An **altitude** is a segment joining a vertex to the opposite side so that it is perpendicular to that side.  ***Altitudes can be inside, outside, or a side of the triangle*** 
ORTHOCENTER 	• The three altitudes of a triangle intersect at a point called the **orthocenter.** **Use the diagram to the left to answer the following questions:** **1)** List the altitudes: $\overline{AE}$, $\overline{BF}$, $\overline{CD}$ **2)** Name the orthocenter: P
CENTERS OF TRIANGLE Review	**Fill in the blanks.** 1. The **circumcenter** is created by the intersection of the three perpendicular bisectors of each side. 2. The **incenter** is created by the intersection of the three angle bisectors of each angle. 3. The **centroid** is created by the intersection of the three medians in the triangle. 4. The **orthocenter** is created by the intersection of the three altitudes in the triangle. 5. A circumcenter is equidistant from the **vertices** of a triangle. 6. An incenter is equidistant from the **sides** of a triangle.

:: NAME THAT TRIANGLE CENTER! ::

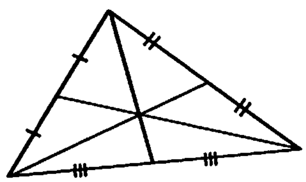
Directions: Based on the markings, classify each center as a **circumcenter**, **incenter**, **centroid**, or **orthocenter**.



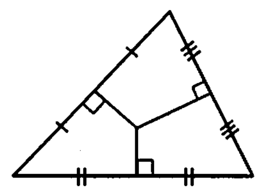
1. incenter



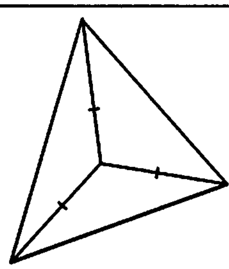
2. orthocenter



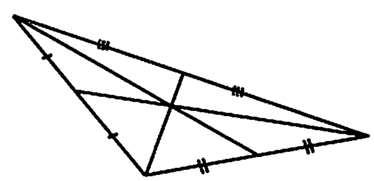
3. centroid



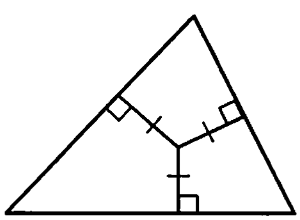
4. circumcenter



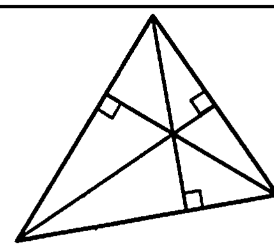
5. circumcenter



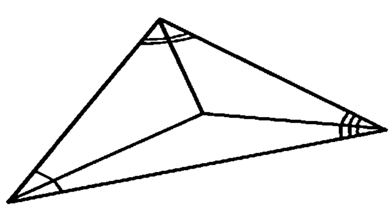
6. centroid



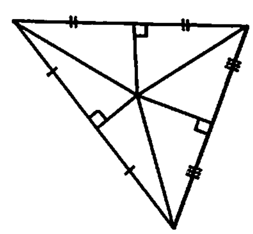
7. incenter



8. orthocenter



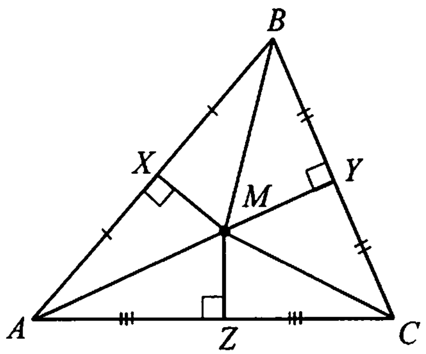
9. incenter



10. circumcenter

Centers of Triangles

Circumcenter



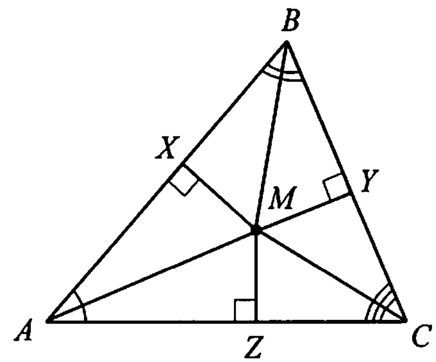
Created by: perpendicular bisectors

Important Facts:

The circumcenter is equidistant from each vertex of the triangle.

AM = BM = CM

Incenter



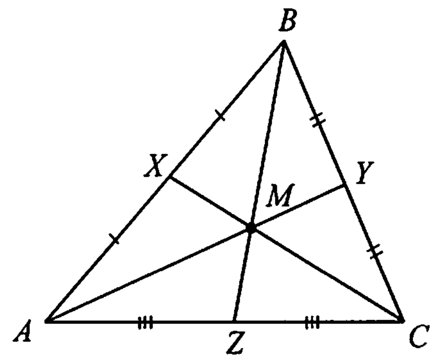
Created by: angle bisectors

Important Facts:

The incenter is equidistant from each side of the triangle.

XM = YM = ZM

Centroid



Created by: medians

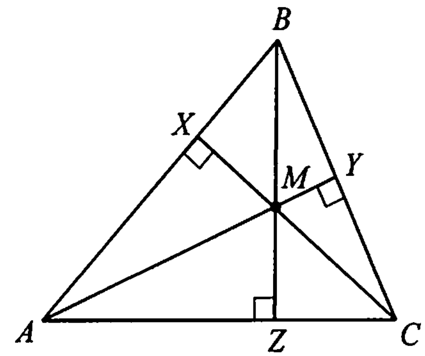
Important Facts:

A median is created by a vertex connected to the midpoint of the opposite side.

$AM = \frac{2}{3} \cdot AY$ $\quad AM = 2 \cdot MY$

$MY = \frac{1}{3} \cdot AY$ $\quad MY = \frac{1}{2} AM$

Orthocenter



Created by: altitudes

Important Facts:

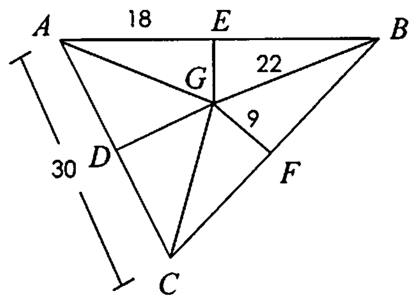
An altitude is created by a vertex connected to the opposite side so that it is perpendicular to that side.

Name: ______________________ Date: ______________ Per: ______

CENTERS OF TRIANGLES

Review!

1. If G is the circumcenter of $\triangle ABC$, find each measure.

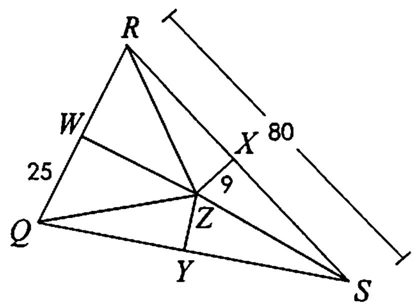


b) $x^2+9^2=22^2$
$x^2+81=484$
$x^2=403$
$x=20$

c) $x^2+18^2=22^2$
$x^2=160$
$x=12.6$

a) AD = 15
b) FC = 20
c) EB = 18
d) AG = 22
e) EG = 12.6

2. If Z is the **circumcenter** of $\triangle QRS$, find each measure.

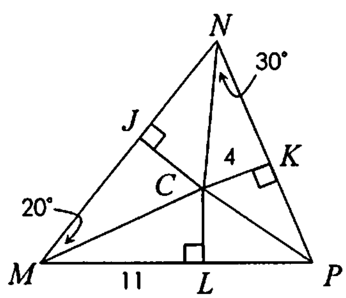


b) $9^2+40^2=x^2$
$1681=x^2$
$41=x$

e) $x^2+25^2=41^2$
$x^2+625=1681$
$x^2=1056$
$x=32.5$

a) QR = 50
b) RZ = 41
c) XS = 40
d) ZS = 41
e) WZ = 32.5

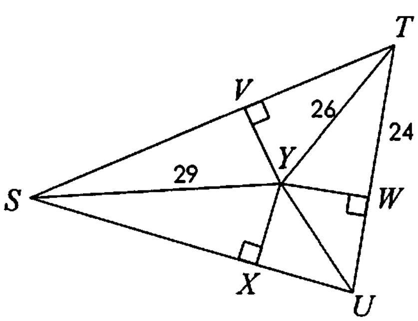
3. If C is the **incenter** of $\triangle MNP$, find each measure.



e) $4^2+11^2=x^2$
$137=x^2$
$x=11.7$

a) $m\angle CML$ = 20°
b) $m\angle MNP$ = 60°
c) $m\angle NPC$ = 40°
d) JC = 4
e) MC = 11.7

4. If Y is the **incenter** of $\triangle STU$, find each measure.

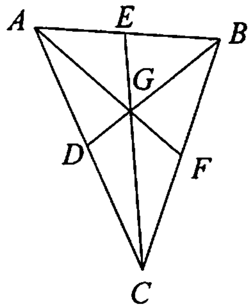


b) $x^2+24^2=26^2$
$x^2+576=676$
$x^2=100$
$x=10$

c) $x^2+10^2=29^2$
$x^2=741$
$x=27.2$

a) VT = 24
b) YW = 10
c) SX = 27.2
d) YX = 10
e) SV = 27.2

5. If G is the **centroid** of $\triangle ACE$, $AG = 26$, $BC = 44$, and $DG = 12$, find each measure.



a) GF = 13

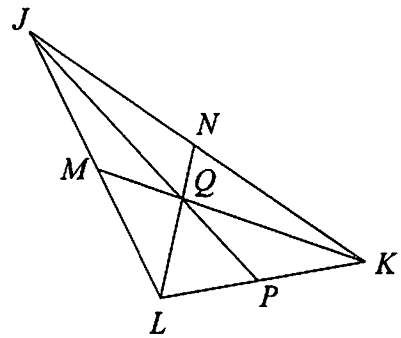
b) AF = 39

c) FC = 22

d) GB = 24

e) DB = 36

6. If Q is the **centroid** of $\triangle JKL$, $LN = 72$, $JP = 93$, and $MK = 78$, find each measure.



a) LQ = 48

b) QN = 24

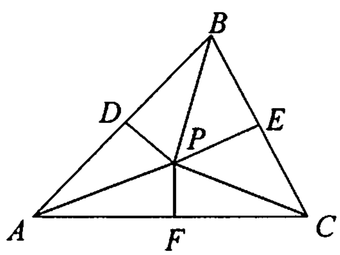
c) QP = 31

d) JQ = 62

e) QK = 52

For questions 7 and 8, *P* is the circumcenter of $\triangle ABC$.

7. If $BE = 8x - 11$, and $EC = 13x - 31$, find BC.



$13x - 31 = 8x - 11$

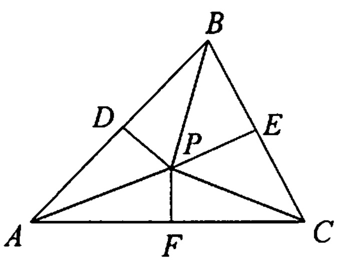
$5x = 20$

$x = 4$

$BE = 8(4) - 11 = 21$

$\boxed{BC = 42}$

8. If $BP = 9x - 29$, $AP = 5x - 1$, and $PF = 15$, find FC.



$9x - 29 = 5x - 1$

$4x = 28$

$x = 7$

AP: $5(7) - 1 = 34$

FC: $x^2 + 15^2 = 34^2$

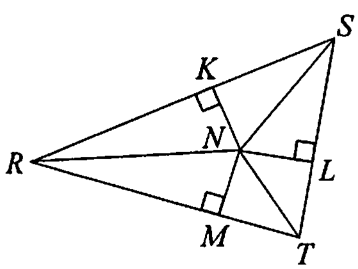
$x^2 + 225 = 1156$

$x^2 = 931$

$\boxed{x = 30.5}$

For questions 9 and 10, *N* is the incenter of $\triangle RST$.

9. If $MN = 9x - 1$, $NL = 16x - 15$, find KN.



$16x - 15 = 9x - 1$

$7x = 14$

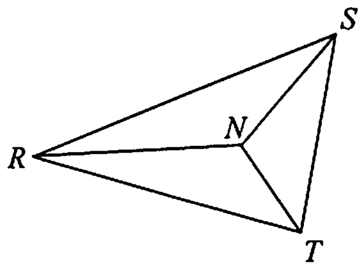
$x = 2$

$MN = 9(2) - 1$

$= 17$

$\boxed{KN = 17}$

10. If $m\angle RST = (3x + 17)°$, $m\angle STR = (8x - 32)°$, and $m\angle TRS = (2x)°$, find $m\angle RSN$.



$3x + 17 + 8x - 32 + 2x = 180$

$13x - 15 = 180$

$13x = 195$

$x = 15$

$m\angle RST =$

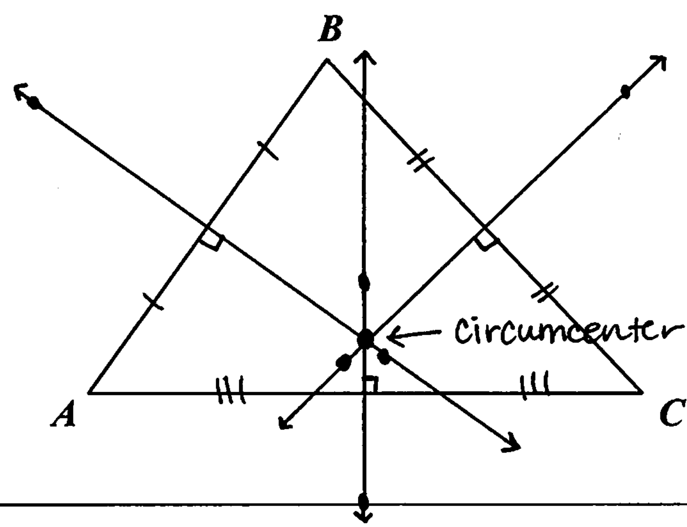
$3(15) + 17 = 62°$

$\boxed{m\angle RSN = 31°}$

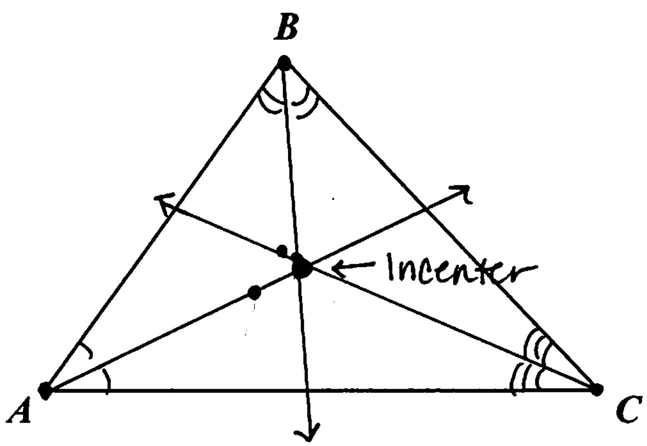
∷ CONSTRUCTING CENTERS OF TRIANGLES ∷

Directions!

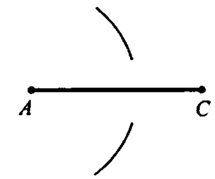
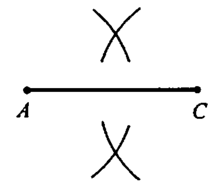
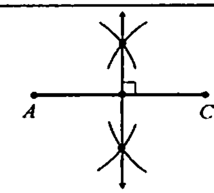
CIRCUMCENTER (Created by **Perpendicular Bisectors**)



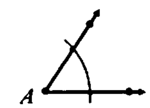
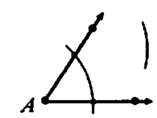
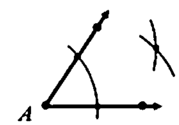
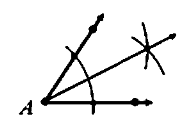
INCENTER (Created by **Angle Bisectors**)



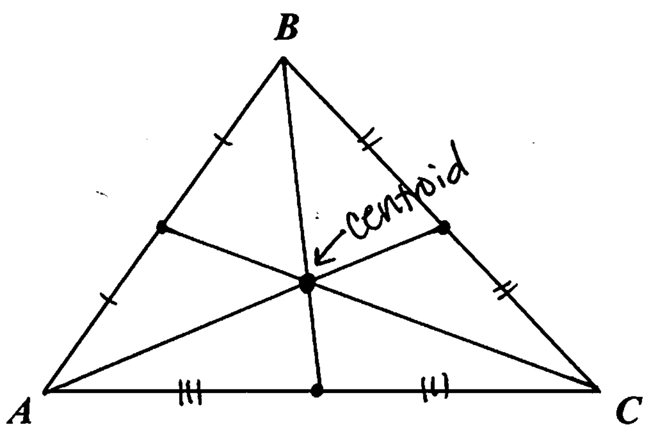
TO CONTSTRUCT A PERPENDICULAR BISECTOR:

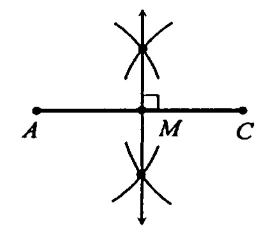
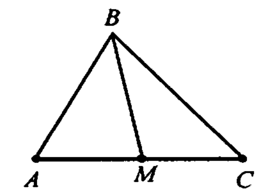
Step 1: Place the compass on point A. Adjust the radius so it's more than one half the length of the segment. Draw two arcs.	
Step 2: Keeping the same radius, place the compass on point C. Draw two more arcs intersecting the previous arcs.	
Step 3: Using the straightedge, connect the intersection points of the arcs.	

TO CONSTRUCT AN ANGLE BISECTOR:

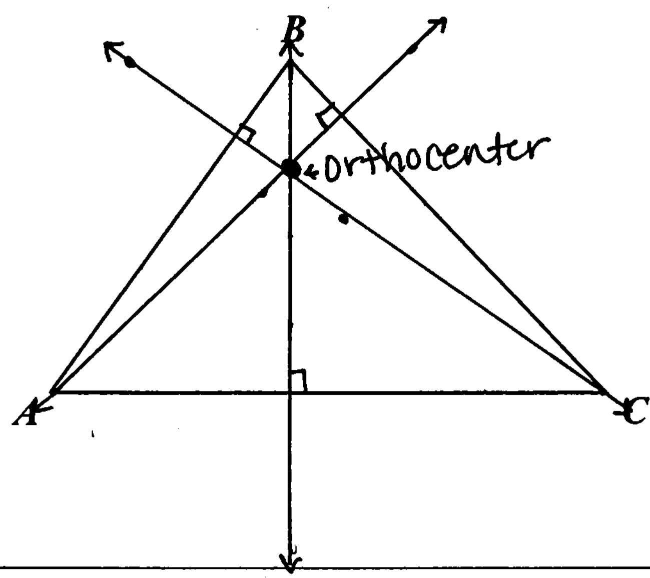
Step 1: Place the compass on vertex A. Draw two arcs as shown.	
Step 2: Place the compass on the intersection of the first arc and AB. Drawn another arc on the interior of the angle.	
Step 3: Without changing the radius, place the compass on the intersection of the second arc and AC. Draw another arc on the interior of the angle, intersection the arc drawn in Step 2.	
Step 4: Using a straight edge, connect the vertex and the intersection point of the two interior arcs.	

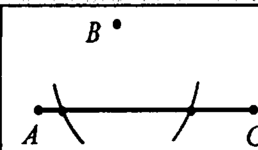
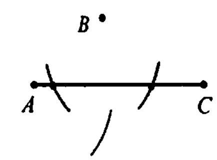
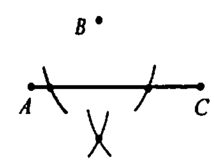
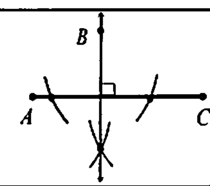
CENTROID (Created by **Medians**)



TO CONSTRUCT MEDIAN:	
Step 1: Find the midpoint of AC by constructing the perpendicular bisector of AC. Use the instructions on the previous page.	
Step 2: Connect vertex B to the midpoint found in Step 2.	

ORTHOCENTER (Created by **Altitudes**)



TO CONSTRUCT AN ALTITUDE:	
Step 1: Place the compass on point B. Using any radius, draw arcs intersecting AC at two points.	
Step 2: Place the compass at the intersection of the first arc and AC. Adjust the radius so it's more than half the distance between the two intersection points. Draw an arc below the line.	
Step 3: Keeping the same radius, place the compass on the intersection of the second arc and AC. Draw another arc intersecting the arc drawn in Step 2.	
Step 4: Using a straight edge, connect point B and the intersection of the arcs drawn below the line.	

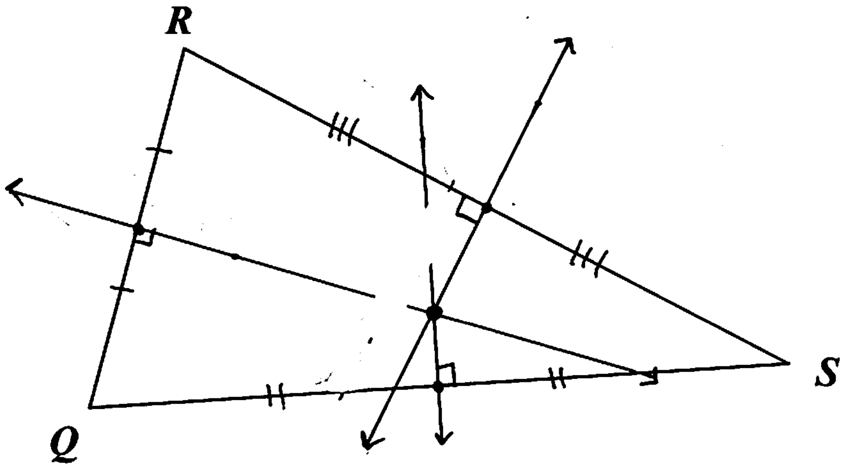
:: CONSTRUCTING CENTERS OF TRIANGLES ::

Practice!

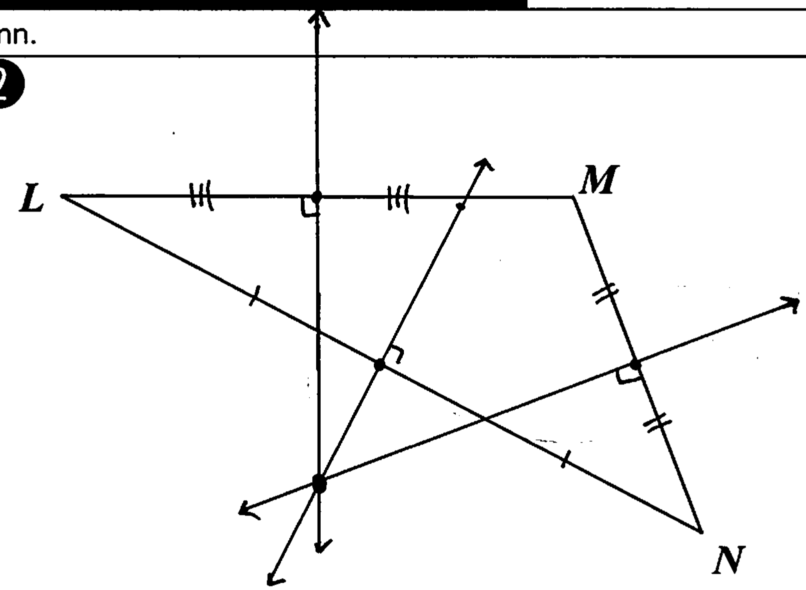
Directions: Construct the center of the triangle indicated on the left column.

CIRCUMCENTER

1

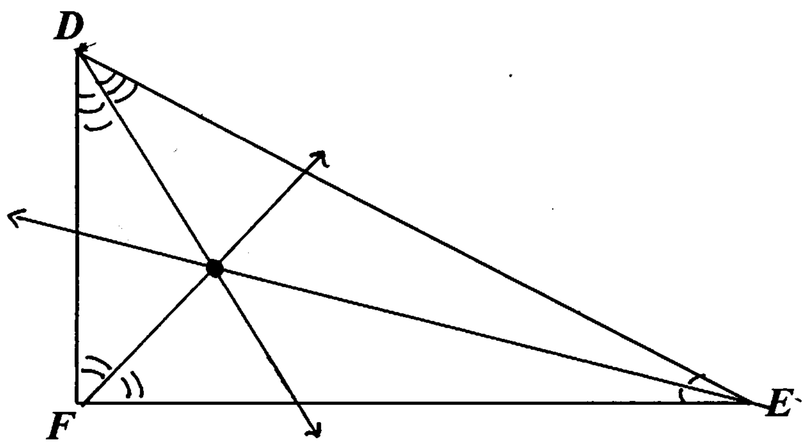


2

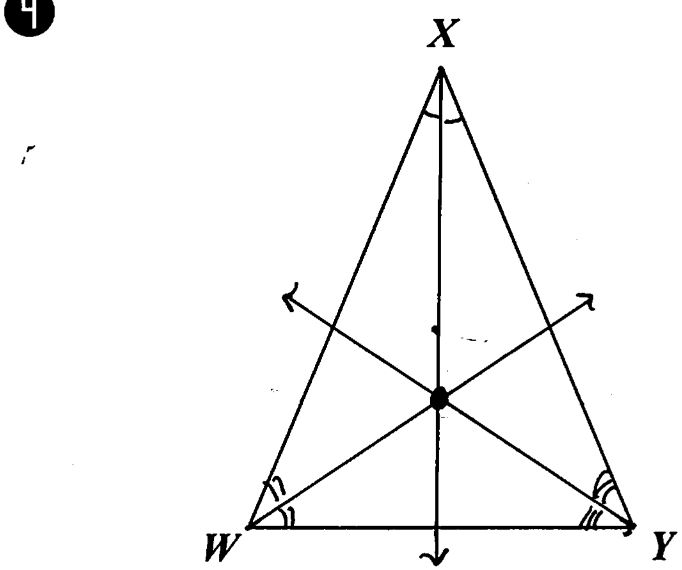


INCENTER

3

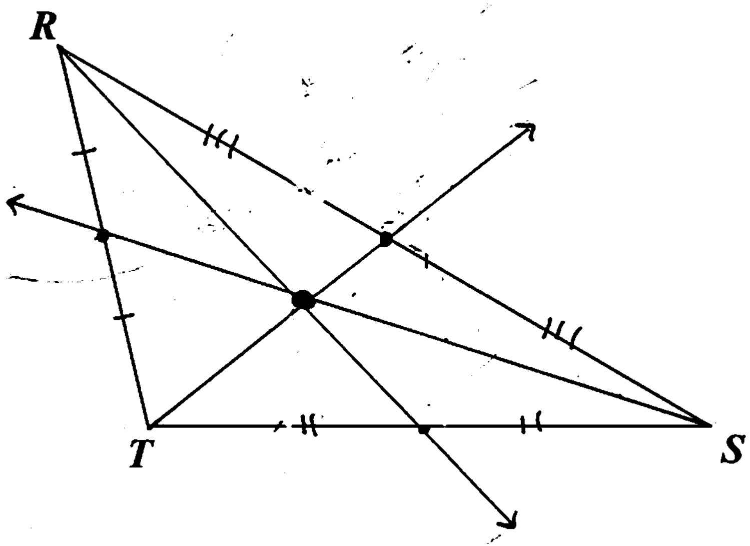


4

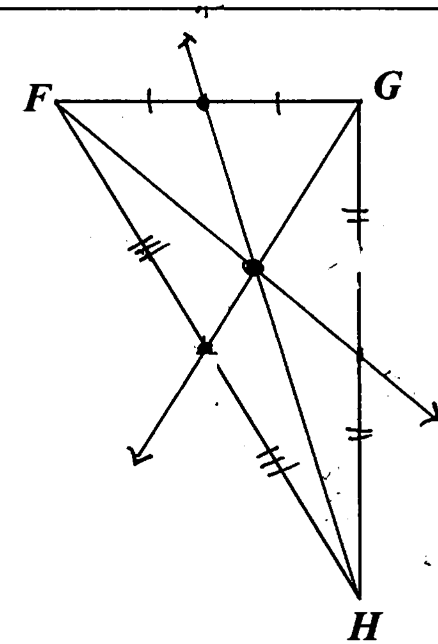


CENTROID

5

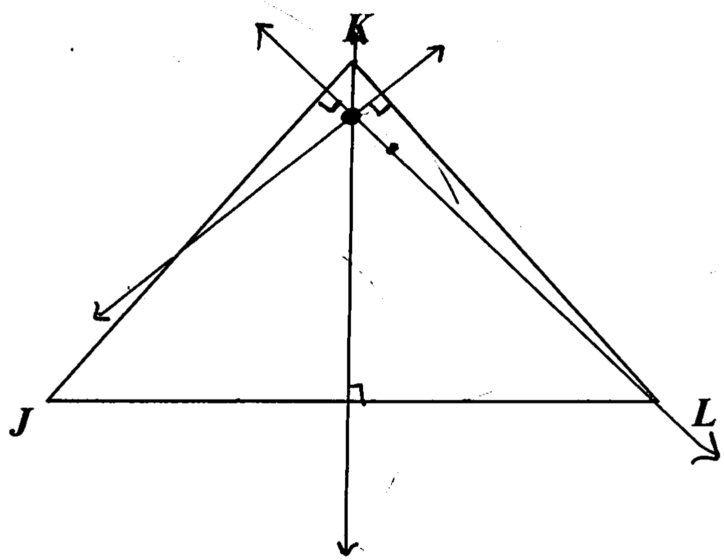


6

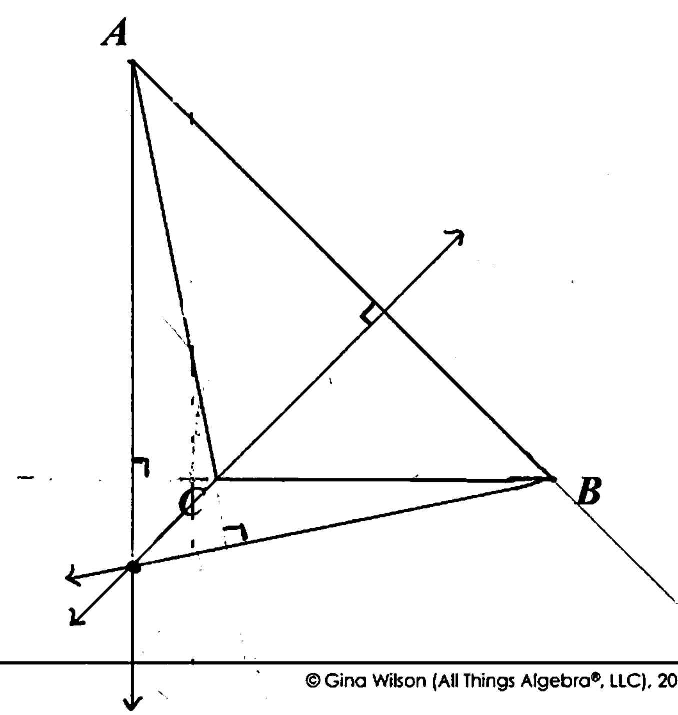


ORTHOCENTER

7



8



Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples

CIRCUMCENTER *on the Coordinate Plane*

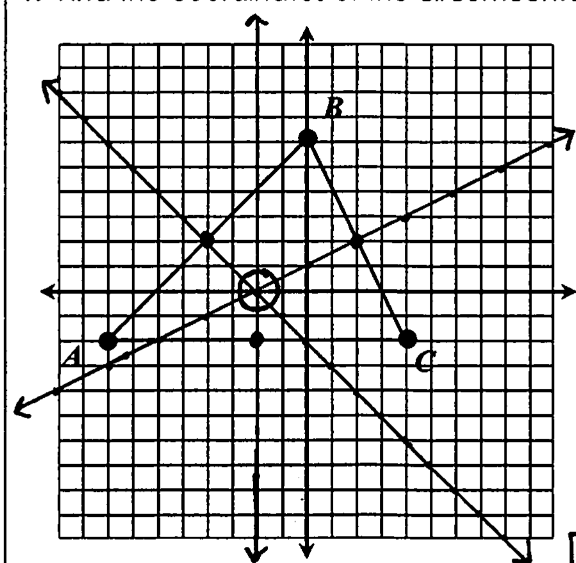
To find the **circumcenter** of a triangle on a coordinate plane, you will need to graph the **perpendicular bisectors** of each side, then find where they meet. Follow the steps below to find the circumcenter.

1	Find the **midpoint** of each side of the triangle.
2	Find the **slope** of each side of the triangle.
3	Use the midpoint and *perpendicular slope* to graph the **perpendicular bisector** of each side.
4	Find the **intersection** of the three perpendicular bisectors. This is the circumcenter!

EXAMPLES

A(-8,-2)
B(0,6)
C(4,-2)

1. Find the coordinates of the circumcenter of ΔABC below.



$M(\overline{AB}) = \left(\frac{-8+0}{2}, \frac{-2+6}{2}\right) = (-4,2)$

$m(\overline{AB}) = \frac{6+2}{0+8} = 1$

$M(\overline{BC}) = \left(\frac{0+4}{2}, \frac{6-2}{2}\right) = (2,2)$

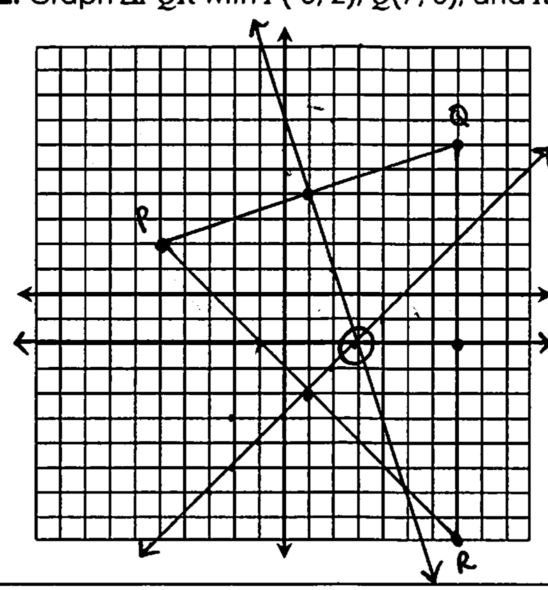
$m(\overline{BC}) = \frac{-2-6}{4-0} = \frac{-8}{4} = -2$

$M(\overline{AC}) = \left(\frac{-8+4}{2}, \frac{-2-2}{2}\right) = (-2,-2)$

$m(\overline{AC}) = 0$

$\boxed{\text{Circumcenter} = (-2,0)}$

2. Graph ΔPQR with $P(-5, 2)$, $Q(7, 6)$, and $R(7, -10)$, then find the circumcenter.



$M(\overline{PQ}) = \left(\frac{-5+7}{2}, \frac{2+6}{2}\right) = (1,4)$

$m(\overline{PQ}) = \frac{1}{3}$

$M(\overline{QR}) = \left(\frac{7+7}{2}, \frac{6-10}{2}\right) = (7,-2)$

$m(\overline{QR}) = \text{undef.}$

$M(\overline{PR}) = \left(\frac{-5+7}{2}, \frac{2-10}{2}\right) = (1,-4)$

$m(\overline{PR}) = -1$

$\boxed{(3,-2)}$

CENTROID
on the Coordinate Plane

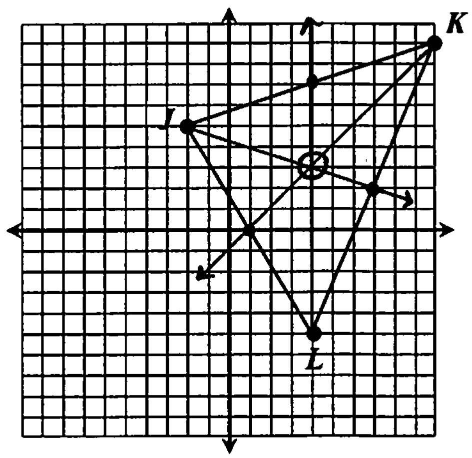
To find the **centroid** of a triangle on a coordinate plane, you will need to graph the three **medians**, then find where they meet. Follow the steps below to find the centroid.

1	Find the **midpoint** of each side of the triangle.
2	Connect the midpoint of each side to the vertex opposite the side. These three lines are your **medians.**
3	Find the **intersection** of the three medians. This is the centroid!

EXAMPLES

J(-2,5)
K(10,9)
L(4,-5)

3. Find the coordinates of the centroid of ΔJKL below.



$M(\overline{JK}) = \left(\frac{-2+10}{2}, \frac{5+9}{2}\right)$

$= (4,7)$

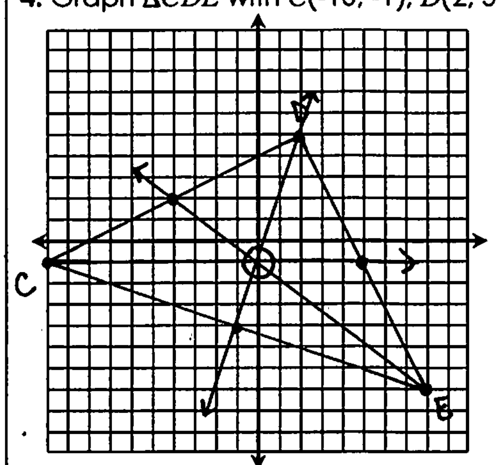
$M(\overline{KL}) = \left(\frac{10+4}{2}, \frac{9-5}{2}\right)$

$= (7,2)$

$M(\overline{JL}) = \left(\frac{-2+4}{2}, \frac{5-5}{2}\right) = (1,0)$

$\boxed{\text{centroid} = (4,3)}$

4. Graph ΔCDE with $C(-10,-1)$, $D(2,5)$, and $E(8,-7)$, then find the centroid.



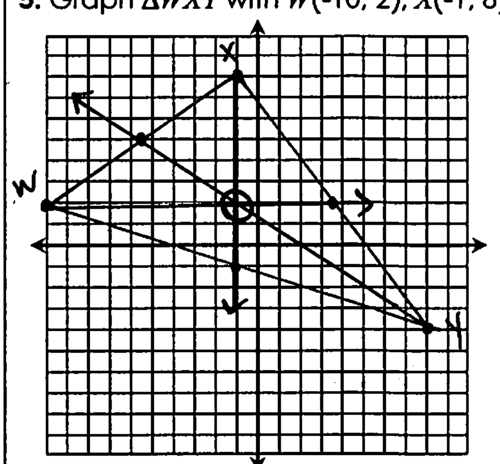
$M(\overline{CD}) = \left(\frac{-10+2}{2}, \frac{-1+5}{2}\right) = (-4,2)$

$M(\overline{DE}) = \left(\frac{2+8}{2}, \frac{5-7}{2}\right) = (5,-1)$

$M(\overline{CE}) = \left(\frac{-10+8}{2}, \frac{-1-7}{2}\right) = (-1,-4)$

$\boxed{(0,-1)}$

5. Graph ΔWXY with $W(-10,2)$, $X(-1,8)$, and $Y(8,-4)$, then find the centroid.



$M(\overline{WX}) = \left(\frac{-10-1}{2}, \frac{2+8}{2}\right) = (-5.5, 5)$

$M(\overline{XY}) = \left(\frac{-1+8}{2}, \frac{8-4}{2}\right) = (3.5, 2)$

$M(\overline{WY}) = \left(\frac{-10+8}{2}, \frac{2-4}{2}\right) = (-1,-1)$

$\boxed{(-1,2)}$

ORTHOCENTER
on the Coordinate Plane

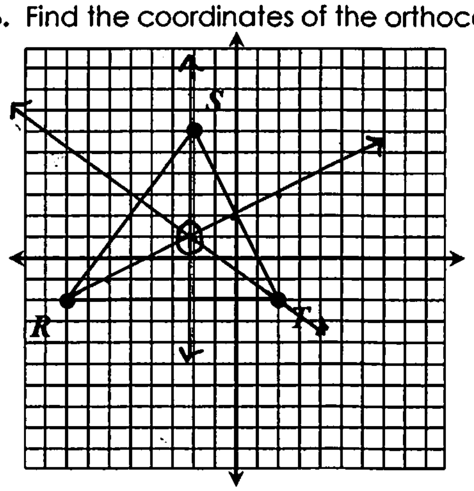
To find the **orthocenter** of a triangle on a coordinate plane, you will need to graph the three **altitudes**, then find where they meet. Follow the steps below to find the orthocenter.

❶	Find the **slope** of each side of the triangle. Then determine the *perpendicular slope* for each side.
❷	Graph each **altitude** by starting at each vertex and using its corresponding perpendicular slope from the opposite side.
❸	Find the **intersection** of the three altitudes. This is the orthocenter!

EXAMPLES

6. Find the coordinates of the orthocenter of ΔRST below.

R(-8, -2)
S(-2, 6)
T(2, -2)



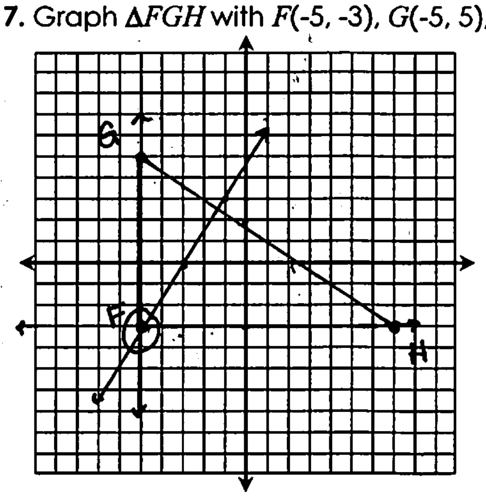
$m(\overline{RS}) = \frac{4}{3}$

$m(\overline{ST}) = -2$

$m(\overline{RT}) = 0$

$\boxed{\text{Orthocenter} = (-2, 1)}$

7. Graph ΔFGH with $F(-5, -3)$, $G(-5, 5)$, and $H(7, -3)$, then find the orthocenter.



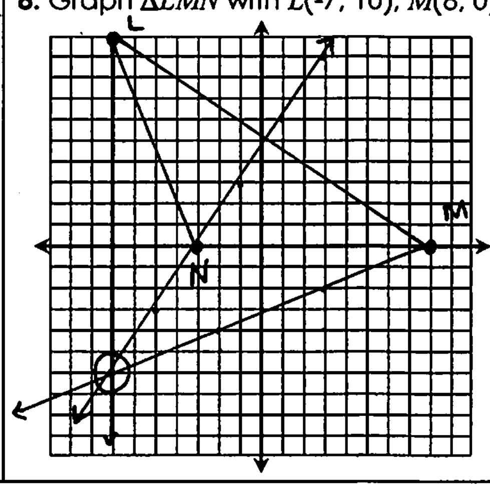
$m(\overline{FG}) = \text{undef.}$

$m(\overline{GH}) = \frac{-2}{3}$

$m(\overline{FH}) = 0$

$\boxed{(-5, -3)}$

8. Graph ΔLMN with $L(-7, 10)$, $M(8, 0)$, and $N(-3, 0)$, then find the orthocenter.



$m(\overline{LM}) = \frac{-2}{3}$

$m(\overline{MN}) = 0$

$m(\overline{LN}) = \frac{-5}{2}$

$\boxed{(-7, -6)}$

Name:

Date:

Topic:

Class:

Main Ideas/Questions	Notes/Examples
TRIANGLE INEQUALITY Theorem	A triangle exists if the sum of the two smaller sides is larger than the third side.

Can it form A TRIANGLE?

Determine if the following side lengths could form a triangle. Prove your answer with an inequality.

1. 8, 17, 24 $8+17>24$ $25>24$ Yes	**2.** 3, 3, 7 $3+3>7$ $6>7$ No
3. 25, 35, 12 $12+25>35$ $37>35$ Yes	**4.** 52, 37, 42 $37+42>52$ $79>52$ Yes
5. 28, 50, 22 $22+28>50$ $50>50$ No	**6.** 6, 18, 14 $6+14>18$ $20>18$ Yes
7. 24, 12, 11 $11+12>24$ $23>24$ No	**8.** 41, 7, 35 $7+35>41$ $42>41$ Yes

FINDING A RANGE for the third side

(Let x = the third side)

$a < x < b$

difference ↑ (a) ↑ sum (b)

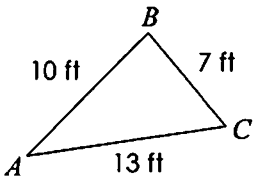
Given two sides of a triangle, you can set up an inequality using the sum and difference to show the range of possible lengths for the third side.

9. 14 and 22 $8<x<36$	**10.** 31 and 28 $3<x<59$
11. 3 and 11 $8<x<14$	**12.** 19 and 45 $26<x<64$
13. 24 and 7 $17<x<31$	**14.** 8 and 17 $9<x<25$

Given two of the side lengths, check all possible lengths for the third side.

15. 15 ft and 27 ft $12<x<42$ ☑ 34 ft ☐ 12 ft ☑ 29 ft ☑ 18 ft ☐ 43 ft	**16.** 45 cm and 9 cm $36<x<54$ ☐ 35 cm ☐ 58 cm ☑ 47 cm ☐ 54 cm ☑ 39 cm

ORDERING
Angles



The **angles of a triangle** can be put in order by **comparing the sides.**

* The **smallest angle** is always **opposite the** shortest side.

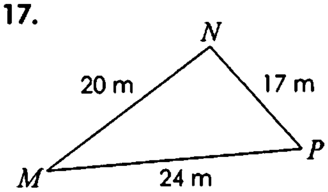
* The **largest angle** is always **opposite the** longest side.

Order the angles of $\triangle ABC$ from least to greatest:

$\angle A, \angle C, \angle B$

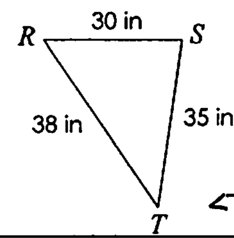
Order the angles measures from least to greatest.

17.



$\angle M, \angle P, \angle N$

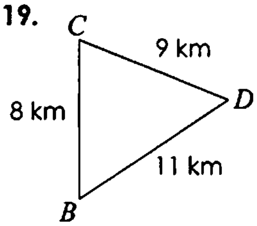
18.



$\angle T, \angle R, \angle S$

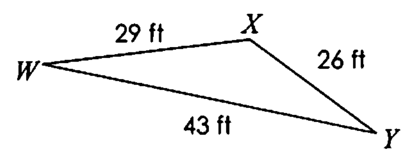
Order the angles measures from greatest to least.

19.



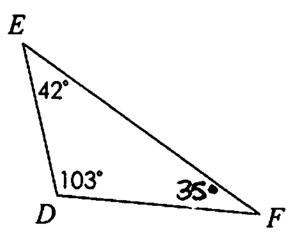
$\angle C, \angle B, \angle D$

20.



$\angle X, \angle Y, \angle W$

ORDERING
Sides



The **sides of a triangle** can be put in order by **comparing the angles.**

* The **smallest side** is always **opposite the** Smallest angle.

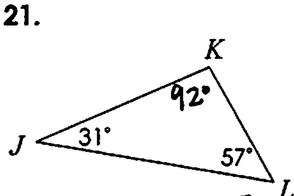
* The **largest side** is always **opposite the** largest angle.

Order the sides lengths of $\triangle DEF$ from least to greatest:

$\overline{DE}, \overline{DF}, \overline{EF}$

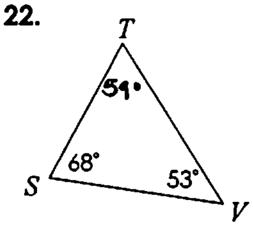
Order the sides lengths from least to greatest.

21.



$\overline{KL}, \overline{JK}, \overline{JL}$

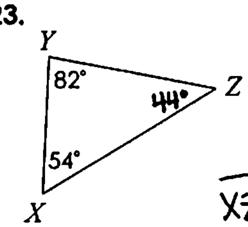
22.



$\overline{ST}, \overline{SV}, \overline{TV}$

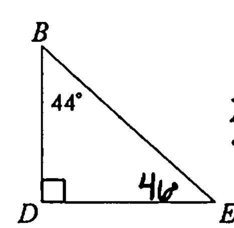
Order the sides lengths from greatest to least.

23.



$\overline{XZ}, \overline{YZ}, \overline{XY}$

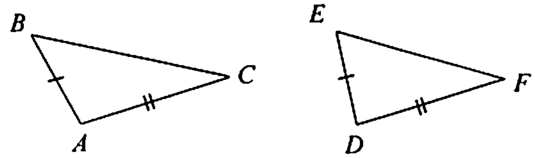
24.



$\overline{BE}, \overline{BD}, \overline{DE}$

Name:	Date:
Topic:	Class:

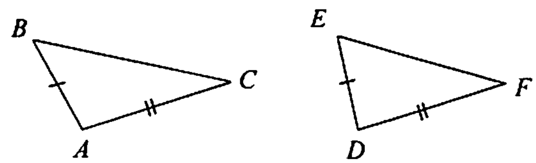
Main Ideas/Questions	Notes/Examples
INEQUALITIES in Two Triangles	**HINGE THEOREM:** If two sides of a triangle are congruent to two sides of another triangle, and the included angle of the first triangle is larger than the included angle of the second triangle, then the side opposite the angle of the first triangle is larger than the side opposite the angle of the second triangle.



If m∠A > m∠B,

then BC > EF.

CONVERSE OF THE HINGE THEOREM: If two sides of a triangle are congruent to two sides of another triangle, and the side opposite the included angle of the first is larger than the side opposite the included angle of the second triangle, then the included angle of the first triangle is greater than the included angle of the second triangle.

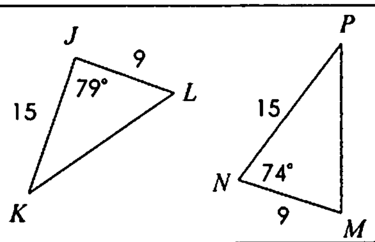


If BC > EF,

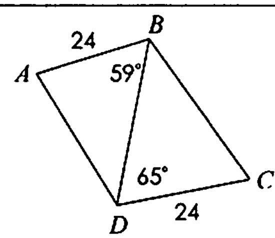
then m∠A > m∠B.

Directions: Compare the sides and angles by filling in the blank with a < or > symbol.

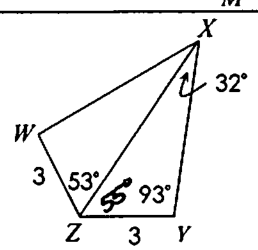
1. KL > PM



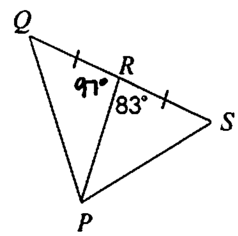
2. AD < BC



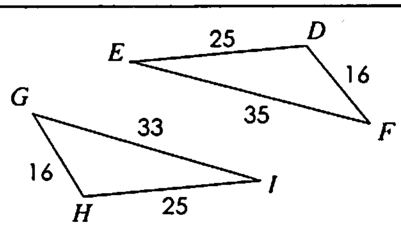
3. WX < XY



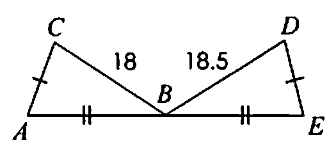
4. PQ > PS



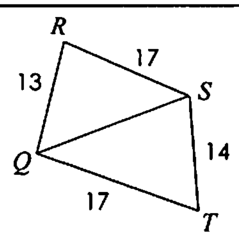
5. $m\angle D$ > $m\angle H$



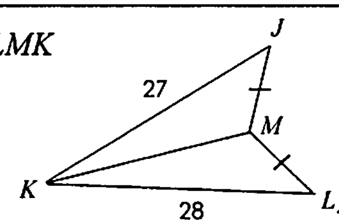
6. $m\angle A$ < $m\angle E$



7. $m\angle RSQ$ < $m\angle TQS$



8. $m\angle JMK$ < $m\angle LMK$



TRIANGLE INEQUALITIES & Algebra

Review! Solve the following inequalities. Watch out for the flippers!

1. $5x - 18 > 2x + 3$

$3x > 21$

$\boxed{x > 7}$

2. $8x + 7 > 10x - 15$

$-2x > -22$ *Flip!

$\boxed{x < 11}$

3. $9x - 26 > 14x - 40$

$-5x > -14$ *Flip!

$\boxed{x < 2.8}$

4. $9 - 2x > 57 - 10x$

$8x > 48$

$\boxed{x > 6}$

Directions: If the sides of a triangle have the given lengths, find a range of possible x- values.
Since you do not know which two sides are the shortest, you must account for all possiblilities.

5. $AB = 4x + 25$, $BC = 3x - 2$, $AC = 9x - 5$

$AB + BC > AC$	$AB + AC > BC$	$AC + BC > AB$
$4x+25 + 3x-2 > 9x-5$	$4x+25 + 9x-5 > 3x-2$	$9x-5 + 3x-2 > 4x+25$
$7x+23 > 9x-5$	$13x + 20 > 3x-2$	$12x-7 > 4x+25$
$-2x > -28$	$10x > -22$	$8x > 32$
$\boxed{x < 14}$	$x > -2.2$	$\boxed{x > 4}$

Range of x-values: $4 < x < 14$

6. $MN = x - 1$, $NP = 9x - 68$, $MP = 5x - 4$

$MN + NP > MP$	$MN + MP > NP$	$MP + NP > MN$
$x-1 + 9x-68 > 5x-4$	$x-1 + 5x-4 > 9x-68$	$5x-4 + 9x-68 > x-1$
$10x-69 > 5x-4$	$6x-5 > 9x-68$	$14x-72 > x-1$
$5x > 65$	$-3x > -63$	$13x > 71$
$\boxed{x > 13}$	$\boxed{x < 21}$	$x > 5.46$

Range of x-values: $13 < x < 21$

7. $JK = x + 7$, $KL = 3x + 25$, $JL = 7x - 22$

$JK + KL > JL$	$JK + JL > KL$	$JL + KL > JK$
$x+7 + 3x+25 > 7x-22$	$x+7+7x-22 > 3x+25$	$7x-22 + 3x+25 > x+7$
$4x+32 > 7x-22$	$8x-15 > 3x+25$	$10x+3 > x+7$
$-3x > -54$	$5x > 40$	$9x > 4$
$\boxed{x < 18}$	$\boxed{x > 8}$	$x > 0.\overline{4}$

Range of x-values: $8 < x < 18$

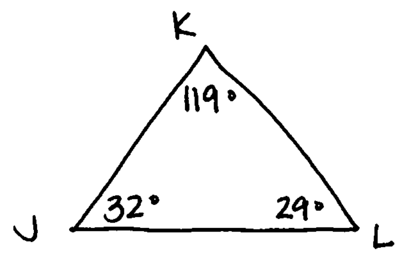
8. List the sides of ΔJKL in order from least to greatest if $m\angle J = (2x + 6)^\circ$, $m\angle K = (10x - 11)^\circ$, and $m\angle L = (3x - 10)^\circ$.

$2x+6 + 10x-11 + 3x-10 = 180$

$15x - 15 = 180$

$15x = 195$

$x = 13$



$\boxed{\overline{JK}, \overline{KL}, \overline{JL}}$

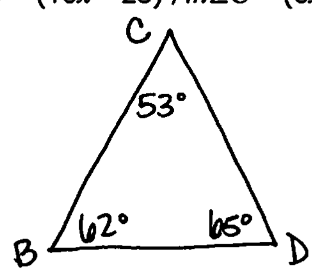
9. List the sides of ΔBCD in order from least to greatest if $m\angle B = (10x - 28)^\circ$, $m\angle C = (6x - 1)^\circ$, and $m\angle D = (8x - 7)^\circ$.

$10x - 28 + 6x - 1 + 8x - 7 = 180$

$24x - 36 = 180$

$24x = 216$

$x = 9$



$\boxed{\overline{BD}, \overline{CD}, \overline{BC}}$

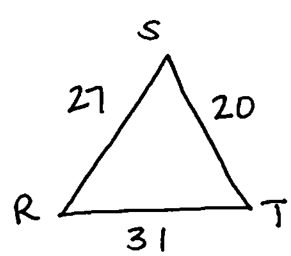
10. List the angles of ΔRST in order from least to greatest if $RS = 5x + 7$, $ST = 9x - 16$, $RT = 2x + 23$, and the perimeter of $\Delta RST = 78$.

$5x+7 + 9x-16 + 2x+23 = 78$

$16x + 14 = 78$

$16x = 64$

$x = 4$



$\boxed{\angle R, \angle T, \angle S}$

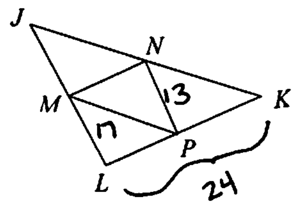
Unit 5 Test Study Guide
(Relationships in Triangles)

Name: ______________________

Date: ________________ **Per:** ________

Topic 1: Midsegments

In the diagram below, $\overline{MN}$, $\overline{NP}$, and $\overline{PM}$ are midsegments.



1. Name all parallel segments:

$\overline{MN} \parallel \overline{LK}$, $\overline{MP} \parallel \overline{JK}$, $\overline{NP} \parallel \overline{JL}$

2. If $MP = 17$, $LK = 24$ and $PN = 13$, find each measure.

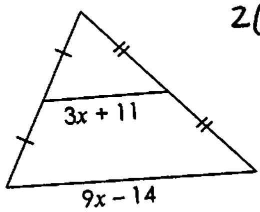
a) $JK =$ 34

b) $MN =$ 12

c) $JL =$ 26

d) Perimeter of ΔJKL: 84

3. Solve for x.



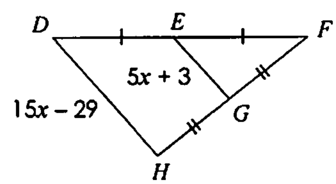
$2(3x+11) = 9x-14$

$6x+22 = 9x-14$

$36 = 3x$

$\boxed{x=12}$

4. Find DH.



$2(5x+3) = 15x-29$

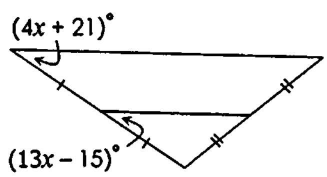
$10x+6 = 15x-29$

$35 = 5x$

$x=7$

DH: $15(7)-29$ $\boxed{=76}$

5. Solve for x.

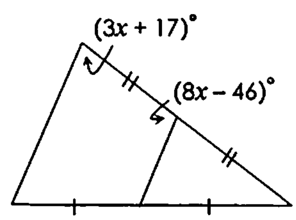


$4x+21 = 13x-15$

$36 = 9x$

$\boxed{x=4}$

6. Solve for x.



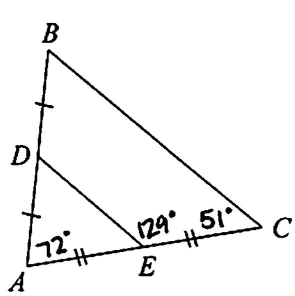
$3x+17+8x-46 = 180$

$11x-29 = 180$

$11x = 209$

$\boxed{x=19}$

7. If $m\angle DEC = (12x-3)°$, $m\angle BCE = (7x-26)°$, and $m\angle DAE = 72°$, find each angle measure.



$12x-3+7x-26 = 180$

$19x-29 = 180$

$19x = 209$

$x = 11$

$m\angle DEC =$ 129°

$m\angle BCE =$ 51°

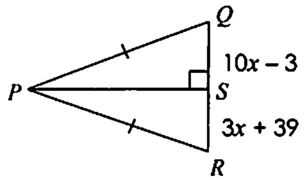
$m\angle ADE =$ 57°

$m\angle EDB =$ 123°

$m\angle DBC =$ 57°

Topic 2: Perpendicular Bisectors & Angle Bisectors

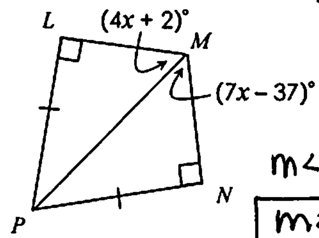
8. Find SR.



$10x-3 = 3x+39$
$7x = 42$
$x=6$

$SR: 3(6)+39$
$\boxed{=57}$

9. Find $m\angle LMN$.



$4x+2 = 7x-37$
$39 = 3x$
$x=13$
$m\angle LMP = 4(13)+2 = 54°$
$\boxed{m\angle LMN = 108°}$

Topic 3: Centers of Triangles (Circumcenter, Incenter, Centroid, Orthocenter)

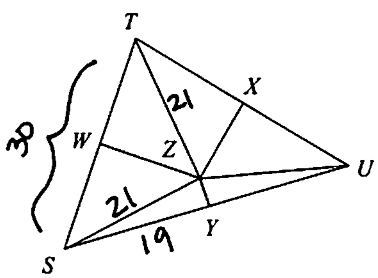
10. The perpendicular bisectors of a triangle intersect at the **circumcenter.**

11. The angle bisectors of a triangle intersect at the **incenter.**

12. The medians of a triangle intersect at the **centroid.**

13. The altitudes of a triangle intersect at the **orthocenter.**

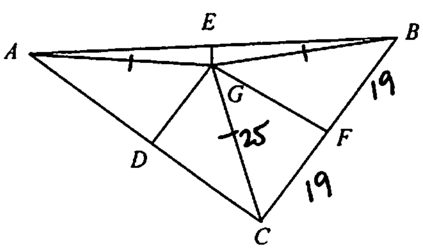
14. If Z is the **circumcenter** of ΔSTU, $SY = 19$, $TZ = 21$, and $ST = 30$, find each measure.



$ZY: x^2+19^2=21^2$
$x^2+361=441$
$x^2=80$
$x=8.9$

$SZ =$ 21
$YU =$ 19
$WT =$ 15
$ZY =$ 8.9

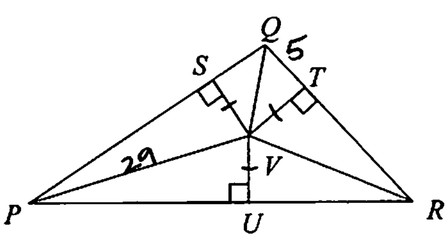
15. If G is the **circumcenter** of ΔABC, $AG = 6x + 1$, $GC = 9x – 11$, and $BC = 38$, find GF.



$6x+1 = 9x-11$
$12 = 3x$
$x=4$
$AG = 6(4)+1$
$=25$

$GF: x^2+19^2=25^2$
$x^2+361=625$
$x^2=264$
$\boxed{x=16.2}$

16. If V is the **incenter** of ΔPQR, $QT = 5$, $VU = 7$, and $PV = 29$, find each measure.

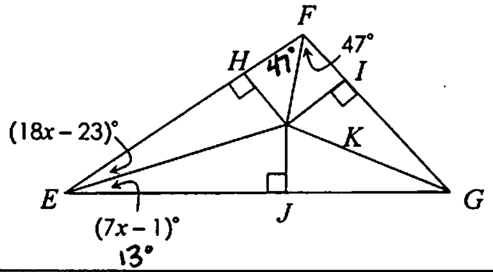


$PS: 7^2+x^2=29^2$
$49+x^2=841$
$x^2=792$
$x=28.1$

$5^2+7^2=x^2$
$74=x^2$
$x=8.6$

$SV =$ 7
$QS =$ 5
$QV =$ 8.6
$PS =$ 28.1

17. If K is the **incenter** of $\triangle EFG$, find x and each angle measure.



$18x - 23 = 7x - 1$

$11x = 22$

$x = 2$

$x =$ 2

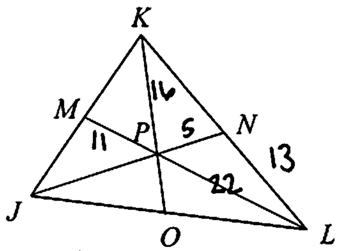
$m\angle KEJ =$ 13°

$m\angle EFG =$ 94°

$m\angle FGE =$ 60°

$m\angle KGJ =$ 30°

18. If P is the **centroid** of $\triangle JKL$, $PN = 5$, $LM = 33$, $KP = 16$, and $NL = 13$, find each measure.



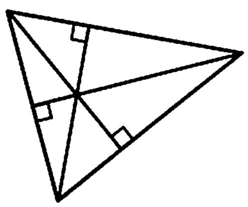
$PM =$ 11 $JP =$ 10

$PL =$ 22 $JN =$ 15

$PO =$ 8 $KL =$ 26

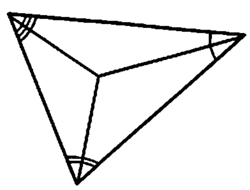
Classify each triangle center.

19.



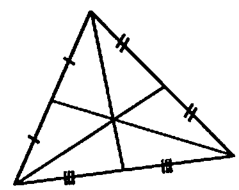
orthocenter

20.



incenter

21.



centroid

Topic 4: Inequalities in Triangles

Determine whether the side lengths could form a triangle. Prove your answer with an inequality.

22. 5 ft, 2 ft, 10 ft

$5 + 2 > 10$

$7 > 10$

No

23. 37 in, 18 in, 25 in

$18 + 25 > 37$

$43 > 37$

Yes

24. 15 m, 50 m, 37 m

$15 + 37 > 50$

$52 > 50$

Yes

25. 7 cm, 24 cm, 31 cm

$7 + 24 > 31$

$31 > 31$

No

Given the measures of two sides of a triangle, find the range of values for the third side.

26. 3 km, 48 km

$45\text{ km} < x < 51\text{ km}$

27. 11 ft, 24 ft

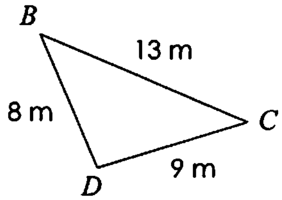
$13\text{ ft} < x < 35\text{ ft}$

28. If two sides of a triangle measure 19 cm and 34 cm, check all possible values for the third side.

$15 < x < 53$

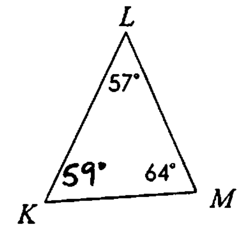
☐ 13 ☐ 15 ☑ 21 ☑ 38 ☑ 52 ☐ 59

29. Give the angles measures in order from least to greatest.



$\angle C, \angle B, \angle D$

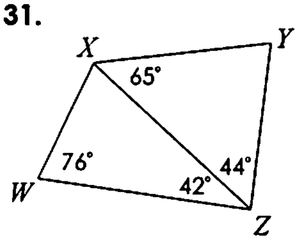
30. Give the side lengths in order from least to greatest.



$\overline{KM}, \overline{LM}, \overline{KL}$

Compare the sides by filling in the blank with a < or > symbol.

31.

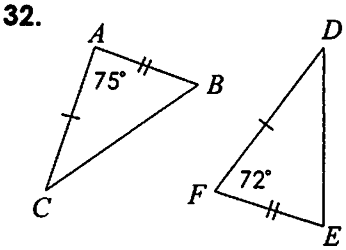


WZ <u> < </u> XZ

XY <u> < </u> YZ

XZ <u> > </u> YZ

32.



BC <u> > </u> DE

Topic 5: Triangle Inequalities & Algebra

33. If the sides of a ΔQRS are $QR = 10x - 6$, $RS = 6x - 15$, $QS = x + 24$, find a range of possible values for x.

$QR + RS > QS$
$10x - 6 + 6x - 15 > x + 24$
$16x - 21 > x + 24$
$15x > 45$
$\boxed{x > 3}$

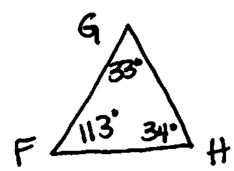
$QR + QS > RS$
$10x - 6 + x + 24 > 6x - 15$
$11x + 18 > 6x - 15$
$5x > -33$
$x > -6.6$

$QS + RS > QR$
$x + 24 + 6x - 15 > 10x - 6$
$7x + 9 > 10x - 6$
$-3x > -15$
$\boxed{x < 5}$

Range of x-values: $\boxed{3 < x < 5}$

34. List the sides of ΔFGH in order from least to greatest if $m\angle F = (15x - 7)°$, $m\angle G = (6x - 15)°$, and $m\angle H = (4x + 2)°$.

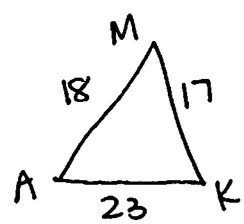
$15x - 7 + 6x - 15 + 4x + 2 = 180$
$25x - 20 = 180$
$25x = 200$
$x = 8$



$\boxed{\overline{FH}, \overline{FG}, \overline{GH}}$

35. List the angles of ΔAMK in order from least to greatest if $AM = x + 13$, $MK = 4x - 3$, $AK = 9x - 22$, and the perimeter of $\Delta AMK = 58$.

$x + 13 + 4x - 3 + 9x - 22 = 58$
$14x - 12 = 58$
$14x = 70$
$x = 5$



$\boxed{\angle A, \angle K, \angle M}$