Solution

Chapter Mid point Theorem
Exercise 2A Questions no 1b

Given: ΔABC, D is midpoint of AC

E is midpoint of BC. AB = 4.4 cm

To Find: DE

Solution: Since D and E are midpoints of side AC and BC respectively

therefore ΔABC and ΔDCE are simillar triangles $\Delta ABC \cong \Delta DCE$.

$$\frac{AC}{DC} = \frac{CB}{CE} = \frac{AB}{DE}$$

$$\frac{AC}{DC} = \frac{AB}{DE} \quad \text{— (1)}$$

AB = 4.4 cm, $DC = \frac{AC}{2}$

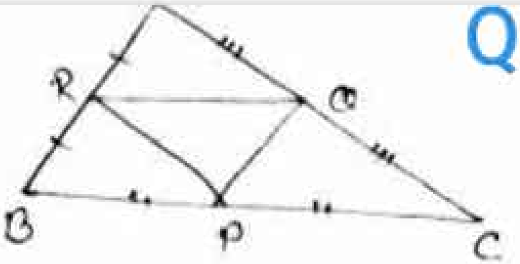
Substitute in eq (1)

$$\frac{2AC}{AC} = \frac{4.4}{DE}$$

$$\Rightarrow DE = \frac{4.4}{2} = 2.2 \text{ cm}$$

Hence value of DE is 2.2 cm.

Questions no 2



given :- P, Q, and R are the mid points of BC, CA and AB respectively.

BP = 3.5 cm , AC = 3.8 cm and PQ = 2.7 cm

To find :- (i) RQ (ii) RP (iii) AR (iv) AB

Solu^n :- Since, P, Q and R are the mid point of BC, CA and AB respectively

By mid-point theorem

$RQ = \frac{1}{2} BC$

$\Rightarrow RQ = \frac{1}{2}(BP + PC) \Rightarrow RQ = \frac{1}{2} \times 2BP$ ($\because$ P is mid point of BC)

$\Rightarrow RQ = BP \Rightarrow$ **RQ = 3.5 cm**

(ii) $RP = \frac{1}{2} AC$ (by mid point theorem)

$\Rightarrow RP = \frac{1}{2}(AQ + QC) \Rightarrow RP = \frac{1}{2} \times 2AQ$ ($\because$ Q is mid point of AC)

$\Rightarrow RP = AQ \Rightarrow RP = AC \Rightarrow$ **RP = 3.8 cm**

i) $\frac{1}{2} AB = PQ$ (by mid point theorem)

$\Rightarrow \frac{1}{2}(AR + RB) = \frac{1}{2} PQ$ ($\because$ R is mid point of AB)

$\Rightarrow PQ = \frac{1}{2} \times 2AR \Rightarrow AR = PQ \Rightarrow$ **AR = 2.7 cm**

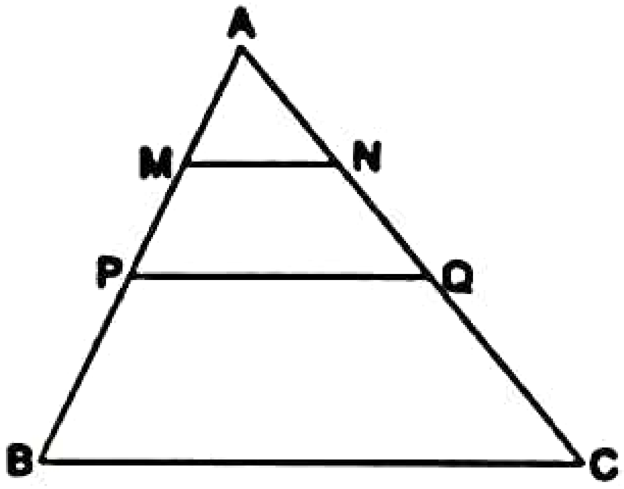
PQ = $\frac{1}{2}$AB (by mid point theorem)

$\Rightarrow AB = 2PQ \Rightarrow$ **AB = 5.4 cm**

Question

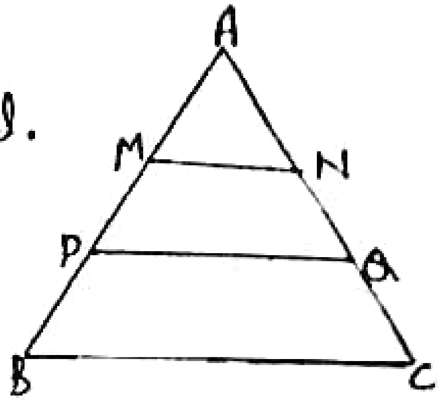
ID: 1104411788

In the triangle ABC, P and Q are the midpoints of AB and AC respectively. M and N are the midpoints of AP and AQ respectively. If $MN = 1.6$ cm, find BC.



Solution

Given: MN = 1.6 cm, P & Q are mid. m and N are mid point AP & AQ.
To find: BC
Solution: In $\triangle AMN$ & $\triangle APQ$



$$\frac{AM}{AP} = \frac{MN}{PQ}$$

AM is $\frac{1}{2}$ AP

$$\frac{\frac{1}{2} AP}{AP} = \frac{MN}{PQ}$$

$MN = \frac{1}{2} PQ$

$PQ = 2\ MN = 2 \times 1.6\,cm = 3.2\,cm.$

In $\triangle APQ$ & $\triangle ABC$

$$\frac{AP}{AB} = \frac{PQ}{BC}$$

P is mid point AB.

$AP = \frac{1}{2} AB$

$$\frac{\frac{1}{2} AB}{AB} = \frac{PQ}{BC}$$

$BC = 2 \times PQ = 2 \times 3.2\,cm = 6.4\,cm$

Answer: Hence length of BC is 6.4 cm.

Question

ID: 1104329157

In ΔABC (Fig. 8.27), D, E, F are the mid-points of BC, CA and AB respectively. P, Q and R are the mid-points of EF, FD and DE respectively. If AB = 4.8 cm, BC = 7.2 cm and CA = 6.8 cm, find the sides of ΔPQR.

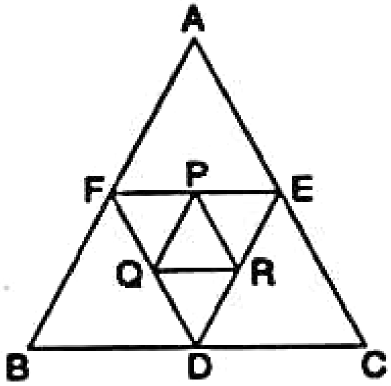


Fig. 8.27

Solution

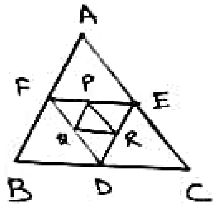
Given D, E, F are the mid points of BC, CA and AB respectively.

P, Q, R are the mid points of EF, FD and DE respectively.

AB = 4.8 cm, BC = 7.2 cm and CA = 6.8 cm.

Find the sides of $\triangle PQR$.

Solution: We know that, in a triangle mid point of two sides are joined, then joining line segment is parallel to the third side and length half of the third side.



So, Here F and D mid point of AB and DC of a $\triangle ABC$.

So, $FD = \frac{1}{2} AC = \frac{1}{2} \times 6.8 = 3.4$ cm.

Similarly, $FE = \frac{1}{2} BC = \frac{1}{2} \times 7.2 = 3.6$ cm.

and $DE = \frac{1}{2} AB = \frac{1}{2} \times 4.8 = 2.4$ cm.

Similarly, P and Q mid point of FE and FD of a $\triangle DEF$.

So, $PQ = \frac{1}{2} DE = \frac{1}{2} \times 2.4 = 1.2$ cm.

Similarly, $PR = \frac{1}{2} FD = \frac{1}{2} \times 3.4 = 1.7$ cm

$QR = \frac{1}{2} FE = \frac{1}{2} \times 3.6 = 1.8$ cm.

Hence, the sides PQ, PR and QR of a $\triangle PQR$ are 1.2 cm, 1.7 cm and 1.8 cm respectively.

Question

ID: 1106758761

Let PQR be a scalene triangle. The midpoints of PQ, QR and RP are L, M and N respectively. Join LM, MN and NL. Prove that the perimeter of ΔLMN is half the perimeter of ΔPQR.

Solution

In PQR

L and M are the mid-points of sides of PQ and QR

∴ LM||PR

similarly

NM||PQ and LN||QR

since

LN||PMand NM||PL

∴ PLNM is a parallelogram

so

$PL=MN=\frac{a}{2}$

$PM=NL=\frac{c}{2}$

similarly QLNM is a parllelogram

so

$QL=MN=\frac{a}{2}$

$NQ=LM=\frac{b}{2}$

$\therefore LM=\frac{b}{2}$

$MN=\frac{a}{2}$

$LN=\frac{c}{2}$

Perimeter of $\Delta PQR = PQ + QR + PR = a + b + c$

Perimeter of $\Delta LMN = LM+MN+LN=\frac{b}{2}+\frac{a}{2}+\frac{c}{2}$

$$=\frac{a+b+c}{2}=\frac{\text{Perimeter of }\Delta PQR}{2}$$

Question

ID: 1103535019

Prove that the triangle obtained on joining the mid-points of the sides of an equilateral triangle is also equilateral.

Solution

Sol. Since line segment joining the mid-points of two sides of a triangle is half of the third side.
Therefore, D and E are mid-points of BC and AC respectively.

$\Rightarrow \quad DE = \frac{1}{2}AB \quad \ldots(i)$

E and F are the mid-points of AC and AB respectively.

$\therefore \quad EF = \frac{1}{2}BC \quad \ldots(ii)$

F and D are the mid-points of AB and BC respectively.

$\therefore \quad FD = \frac{1}{2}AC \quad \ldots(iii)$

Now, ΔABC is an equilateral triangle.

$\Rightarrow \quad AB = BC = CA$

$\Rightarrow \quad \frac{1}{2}AB = \frac{1}{2}BC = \frac{1}{2}CA$

$\Rightarrow \quad DE = EF = FD$ [using (*i*), (*ii*) and (*iii*)]

Hence, DEF is an equilateral triangle.

Question

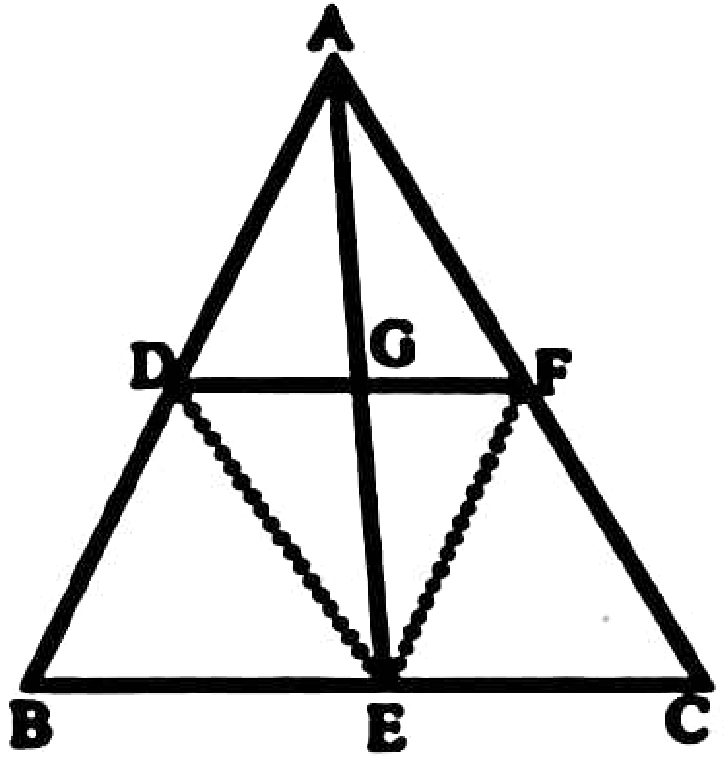
ID: 1104411858

ABC is a triangle. *D*, *E* and *F* are the midpoints of *AB*, *AC* and *BC* respectively. Prove that *DE* and *AF* bisect each other.

Solution

Construction : Join EF, and ED

Proof : ∵ E and F are the mid points of BC and AC respectively



∴ EF ∥ AB ...(*i*)

Similarly D and E are the mid points of AB and BC respectively

∴ DE ∥ AC ...(*ii*)

From (*i*) and (*ii*), we have

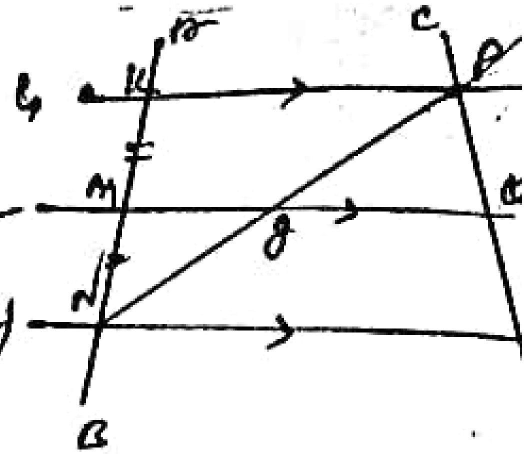
ADEF is a parallelogram

But the diagonals AE and DF of a parallelogram bisect each other at G

∴ AG = GE and DG = GF

Hence AE and DF bisect each other at G.

Solution,



(1) if PQ = 1.8 find QR

$L_1 \parallel L_2 \parallel L_3$

KM = MN PG = GN (by intercept theorem)

∴ PQ = QR

hence QR = 1.8

(2) If PN = 5.8 cm. find GN

$L_1 \parallel L_2 \parallel L_3$

PG = GN PQ = QR (by Intercept theorem)

$\frac{PN}{2} = GN$

$GN = \frac{5.8}{2} = 2.9$ cm.

(3) if PR = 4.6 cm, find PQ

$L_1 \parallel L_2 \parallel L_3$ and KM = MN

PQ = QR (by Intercept theorum)

$PQ = \frac{PR}{2} = \frac{4.6}{2} = 2.3$ cm.

(4) if PG = 2.4 cm. fin PN

$L_1 \parallel L_2 \parallel L_3$ KM = MN.

hence PG ∥ GN (by Intercept theorem)

PN = 2 PG

PN = 4.8 cm

2b 1