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THE TIME PATH AND IMPLEMENTATION OF CARBON SEQUESTRATION

HONGLI FENG, JINHUA ZHAO, AND CATHERINE L. KLING

We develop a dynamic model to investigate the optimal time paths of carbon emissions, sequestration, and the carbon stock. We show that carbon sinks should be utilized as early as possible, and carbon flow into sinks should last until the atmospheric carbon concentration is stabilized. We rule out any cyclical patterns of carbon sequestration and release. We propose and assess three mechanisms to efficiently introduce sequestration into a carbon permit trading market: a pay-as-you-go system, a variable-length-contract system and a carbon annuity account system. Although the three mechanisms may not be equally feasible to implement, they are all efficient.

Key words: carbon sequestration, global warming.

Under the Kyoto Protocol, industrialized countries have pledged to reduce their carbon emissions to below their 1990 emission levels over the period 2008–12. To fulfil their commitment, some countries, including the U.S., have proposed the inclusion of three broad land management activities pursuant to Article 3.4 of the Protocol, including forest, cropland and grazing land management.[1] These activities can reduce atmospheric carbon stock by sequestering, or removing, carbon from the atmosphere and storing it in soil or biomass. For land-rich countries like the U.S., Canada, and Russia, carbon sequestration by these activities could potentially account for their significant emission reductions. For example, estimates indicate that the total carbon sequestration potential of U.S. cropland through improved management is 75–208 MMTC/year (Lal et al). Soil sinks, combined with forest sinks, could potentially be used by the U.S. to meet half of its emission reduction commitment (USDOS). However, skepticism remains among environmental groups who argue that "While preventing the emission of carbon dioxide is permanent, sequestering carbon pollution is a cheap, short-term fix that fails to address a long-term problem" (WWF).

The concerns raised by environmentalists and others relate specifically to the fact that sinks may be short run in nature and consequently, do not provide the same benefits as permanent emission reductions. This nonpermanence issue is one of the focal points in post-Kyoto negotiations on carbon sinks (IPCC, 2000a),[2] and disagreement over sinks was a major impediment to progress at the Sixth Conference of the Parties to the Framework Convention on Climate Change in Hague in November 2000 (IISD).[3]

At the heart of the debate lie two interrelated difficulties of carbon sinks due to the nonpermanence feature. The first difficulty has to do with accounting and implementation. If sequestered carbon can be easily released, governments must find

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[1] The Kyoto Protocol currently allows only forests as carbon sinks, but left the door open for soil carbon sequestration through changes in land management practices.

[2] Other issues related to sinks include measurement, accounting rules, verification procedures and leakage.

[3] The Umbrella Group, a loose alliance of Annex I Parties that includes the US, Canada, Australia, Japan, Norway, the Russian Federation, Ukraine, and New Zealand, urged the development of simple procedures that facilitate the widespread use of mechanisms across a broad range of practices (including sequestration), while the European Union insisted on imposing limitations on the use of sinks, including the exclusion of "additional activities" in the first commitment period and quantitative limit for the use of sinks in Clean Development Mechanism projects.

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ways of properly accounting for the "net value" of possibly *temporary* storage, and design mechanisms to implement carbon sinks that correctly reflect this value. For example, if a permit trading system is devised for carbon abatement, which permanently reduces carbon released into the atmosphere, the system cannot be directly applied to carbon sequestration if the sequestered carbon is only temporarily kept out of the atmosphere. The second issue concerns when carbon sinks should be utilized. While the U.S., Canada, and some other countries, arguing for cost effectiveness, prefer earlier inclusion, the EU has argued for later usage, stressing the key role of improving energy efficiency and shifting toward renewable energy resources. A related but deeper question is the optimal time path of carbon sequestration. Given that stored carbon can be easily released, thereby providing opportunities for future sequestration, the optimal time path may possibly have a cyclical pattern: sequestration and release, followed by sequestration and release, and so on.

In this article, we develop a stylized model of carbon emissions (or abatement) and carbon sequestration to investigate the optimal time patterns of sequestration, emissions, and carbon stock, and to propose three mechanisms that can efficiently implement carbon sinks in a permit trading system based on emissions and abatement. We show that a cyclical pattern is not optimal for soil sinks. In particular, both carbon emissions and stock are monotone in time: depending on the starting carbon stock level, they either increase or decrease through time monotonically. There are two possibilities for sequestration, depending again on the starting point. In one scenario, carbon is sequestered first and partially released later. In the more realistic scenario, carbon is continually being sequestered, although eventually, approaching the steady state, the scale of additional sequestration goes to zero. In both cases, we find that, if sinks are to be used at all, we should start to use them now.

We then propose three systems to implement the optimal sequestration levels: a pay-as-you-go (PAYG) system, a variable-length-contract (VLC) system, and a carbon annuity account (CAA) system. Each could be used in conjunction with a well-functioning (emission reduction-based) carbon permit trading system to efficiently include the sequestration of carbon. We show that each system can be efficient, but requires different conditions to be so. Further, the systems are likely to differ in the transaction costs associated with their implementation. Consequently, one or more may be desirable in practice and under different circumstances. These systems also indicate the proper way of accounting for the value of (possibly temporary) sequestration.

There are two studies in the literature on the optimal time patterns of carbon sequestration and emission. van Kooten et al. investigated optimal carbon sequestration for an exogenously given time path of emissions. Richards studied the optimal emission levels and carbon stock without explicitly introducing sequestration as one of the control variables. Our article extends this literature by modeling emissions and sequestration simultaneously, and studies an optimal control problem of two state and two control variables. These complications are important because abatement and sequestration are two ways of reducing the carbon concentration in the atmosphere, and their optimal time paths are bound to be interdependent.

A few studies have discussed various aspects of the implementation of carbon sinks. Recognizing the difference between abatement and sequestration, and between sequestration projects, Fearnside and Chomitz advocated a "ton-years" accounting method, which distinguishes between, say, one ton of carbon sequestered for one year and the same amount sequestered for five years. McCarl and Schneider, and Marland, McCarl, and Schneider, discussing soil carbon sinks, suggested that incentive programs for sequestration have to address the issue of "preservation of gains over time" or "longevity of agricultural carbon." Marland et al. also argued that if sequestered carbon becomes a commodity, then credits could be issued for carbon sequestered but there must be subsequent debits if the carbon is later released. To our knowledge, our article presents the first systematic study of the efficient implementation of carbon sinks that formally accounts for the nonpermanence of sinks.

Throughout the article, we use the term "abatement" to refer to reductions in carbon loadings and "sequestration" to mean the storage of carbon in soils or terrestrial biosphere in general. Thus, abatement by its nature is permanent. If a ton of carbon is not produced and emitted into the atmosphere

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today, it will not be present in the atmosphere at a later date. In contrast, a ton of carbon stored in a sink today may be only temporarily out of the atmosphere as it might be released in a future period.

An important issue that we do not address, but nevertheless provides the justification for this study, is the cost effectiveness of carbon sequestration compared with abatement. Stavins, reviewing a large body of the existing studies and providing his own analysis, argued that growing trees to sequester carbon has lower marginal costs than emission abatement for a considerable range of stored carbon. Antle et al., Pautsch et al., and Mitchell et al. assess the cost and potential of carbon sequestration by changing management practices within the agricultural sector. Although their results vary for different practices, all show that there is economic potential for soil carbon sequestration.

Model Setup

Consider the social planner's problem of maximizing the benefits of carbon emissions minus the cost of sequestration and the damage caused by global warming. Let $e(t)$ be the society's emission rate at time t and $B(e(t))$ the benefits of emissions, with $B(0) = 0$, $B'(\cdot) > 0$, $B''(\cdot) < 0$, and $\lim_{e\to 0} B'(e) = \infty$. Higher emissions represent lower levels of abatement in the economy, and the benefits are equivalent to the saved abatement costs. The monotonicity and concavity of $B(\cdot)$ then are a result of the monotonicity and convexity of the abatement cost function (in the level of abatement).[4] The marginal abatement cost approaches infinity if all of the society's emissions are to be abated, leading to the last condition on $B(\cdot)$.[5]

The emitted carbon accumulates in the atmosphere causing global warming effects. Let $C(t)$ be the total carbon stock in the atmosphere. The pollution damage of the carbon stock is $D(C(t))$ with $D'(\cdot) > 0$ and $D''(\cdot) > 0$.

We assume that the carbon stock $C(t)$ decays at an exponential rate $\delta \geq 0$. By decay we mean the process by which atmospheric carbon is "sunk" into the ocean. There is a constant process of carbon flow between the atmosphere and the ocean, the direction and speed of which depend on the temperature and carbon concentration in both media. Typically, carbon flow is not exponential. Our assumption simplifies the model and captures the notion that carbon flow into the ocean increases as the carbon stock rises in the atmosphere. The assumption also indicates that carbon is not a pure stock pollutant, and theoretically, its concentration level *can* go down, and thus can be stabilized (through sequestration and reduced emissions).

Let $A(t)$ be the total units of land that are enrolled in carbon sequestration programs at time t. To simplify notation, we define one unit of land as the acreage that is needed to sequester one ton of carbon. For example, if one acre of land can sequester α tons of carbon, one unit of land is equal to $1/\alpha$ acres. Let $Q(A(t))$ be the cost of enrolling $A(t)$ with $Q'(\cdot) > 0$, $Q(0) = 0$, and $Q''(\cdot) > 0$. The cost of carbon sequestration can be interpreted in two ways. If sequestration requires changing agricultural production practices, the cost may be the agricultural profit foregone for doing so. For example, switching from conventional to conservation tillage may reduce a farmer's profit (Pautsch et al.; Antle and Mooney), and some amount of profit may also be lost if cropland is converted to forestland (Plantinga, Mauldin, and Miller). In the case of improved management of an existing forest stand, the cost of carbon sequestration is the expenditure incurred to enhance management, e.g., fertilization (Hoen and Solberg; Boscolo, Buongiorno, and Panayotou).

The cost function $Q(\cdot)$ can be convex for a variety of reasons. Different land may incur different sequestration costs: some highly productive land is best kept in conventional tillage and some land can be converted to forest without much economic loss. Typically, land with low sequestration cost is converted first. As the land area A increases, the cost $Q(A)$ will increase at a faster rate when land of higher sequestration cost is converted.[6] Let $\bar{A}$ be the total land units. We assume that $\lim_{A\to\bar{A}} Q(A) = \infty$, implying that all land will never be converted.

[4] Montgomery formally establishes the monotonicity and convexity of the abatement cost function.

[5] We can relax the monotonicity assumption by allowing $B'(e)$ to be negative. Then in our article, the relevant domain of $B(\cdot)$ is $[0, \bar{e}]$, where $\bar{e}$ is the optimal emission level in the absence of any regulation, i.e., $B'(\bar{e}) = 0$. We ignore this domain restriction because it is never binding.

[6] If a substantial amount of land is diverted from agricultural production, agricultural output prices may increase and the profit reduction $Q(A)$ would be even greater. Then $Q(A)$ is likely to be convex even with homogeneous land.

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Let $a(t)$ be the units of land newly enrolled ($a(t) > 0$) or withdrawn ($a(t) < 0$) in period t. For simplicity, we assume that when land is newly enrolled, carbon is *immediately* removed from the atmosphere, up to its full capacity (of one ton per unit). Likewise, all of the stored carbon is completely and *immediately* released if the land is converted back to its original use. In truth, soil carbon sequestration is a gradual process, and it may take up to fifty years for certain soil to reach its full sequestration capacity. Our assumption simplifies the model, and incorporates a key feature of sequestration: a piece of land can only hold a certain amount of carbon, all of which could be released back to the atmosphere during a very short period. To capture in a simple way the fact that there are costs (or physical limits) of converting land, we place bounds on the amount of land that can be converted each period: $\underline{a} \le a(t) \le \bar{a}$, with $\underline{a} < 0$ and $\bar{a} > 0$.

The equations of motion for $C(t)$ and $A(t)$ are

$$(1) \quad \begin{aligned} &\dot{C}(t) = e(t) - a(t) - \delta C(t), \\ &C(0) = C_0 > 0 \end{aligned}$$

$$(2) \quad \begin{aligned} &\dot{A}(t) = a(t), \quad A(0) = A_0 \ge 0, \\ &0 \le A(t) \le \overline{A}, \quad \underline{a} \le a(t) \le \bar{a}. \end{aligned}$$

Equation (1) indicates that the change in the stock of carbon each period equals new emissions less the amount sequestered and the amount of natural decay. Let r be the social discount rate. Then the social planner's net payoff function is

$$(3) \quad \begin{aligned} &V^0(A, C, e, a) \\ &= \int_0^\infty e^{-rt}[B(e(t)) - D(C(t)) \\ &\qquad - Q(A(t))]\,dt. \end{aligned}$$

Maximizing (3) subject to (1) and (2) yields the optimal carbon sequestration and emission levels over time.

Optimal Paths of Sequestration and Emissions

Since $\lim_{A \to \overline{A}} Q(A) = \infty$, the constraint $A(t) \le \overline{A}$ is never binding along the optimal path. So is the constraint $A(t) \ge 0$, as we will show later (Remark 3). This observation is intuitive: since the marginal cost of sequestration $Q'(A)$ is low when A is close to zero, and is typically lower than the marginal cost of emission reduction $B'(e)$, it makes economic sense to use some sinks to store a positive amount of carbon.

The current value Hamiltonian for the social planner's problem is

$$(4) \quad \begin{aligned} &H(C, A, e, a, \lambda, \mu) \\ &= B(e(t)) - D(C(t)) - Q(A(t)) \\ &\quad - \lambda(t)[e(t) - a(t) - \delta C(t)] - \mu(t)a(t) \end{aligned}$$

where $\lambda(t)$ and $\mu(t)$ are the negative of the costate variables which are continuously differentiable, and are assumed to be twice continuously differentiable almost everywhere. The necessary conditions are

$$(5) \quad \begin{aligned} &\frac{\partial H}{\partial e} = -\lambda(t) + B'(e(t)) = 0, \text{ or} \\ &\lambda(t) = B'(e(t)) \end{aligned}$$

$$(6) \quad \max_a H \quad \text{or} \quad a(t) \begin{cases} = \bar{a} \\ = \underline{a} \\ \in [\underline{a}, \bar{a}] \end{cases} \quad \text{if} \quad \lambda(t) - \mu(t) \quad \begin{cases} > 0 \\ < 0 \\ = 0 \end{cases}$$

$$(7) \quad \begin{aligned} \dot{\lambda}(t) &= r\lambda(t) + \frac{\partial H}{\partial C} \\ &= (r + \delta)\lambda(t) - D'(C(t)) \end{aligned}$$

$$(8) \quad \dot{\mu}(t) = r\mu(t) + \frac{\partial H}{\partial A} = r\mu(t) - Q'(A(t))$$

$$(9) \quad \lim_{t \to \infty} e^{-rt}\lambda(t) = 0, \quad \lim_{t \to \infty} e^{-rt}\mu(t) = 0.$$

From (7) and (8), we know

$$(10a) \quad \lambda(t) = \int_t^\infty e^{-(r+\delta)(s-t)} D'(C(s))\,ds,$$

$$(10b) \quad \mu(t) = \int_t^\infty e^{-r(s-t)} Q'(A(s))\,ds.$$

Thus $\lambda(t)$ measures the total discounted (to period t) future damages caused by one more unit of atmospheric carbon in period t. Notice the symmetric role played by the natural decay rate, δ, and the social discount rate, r. Equation (5) simply says that the marginal benefit of emitting one unit of carbon must equal its marginal cost. Similarly, $\mu(t)$ measures the discounted costs of

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maintaining one unit of land in sequestration, and (6) indicates that land should be converted at its maximum speed whenever the benefit of land conversion $\lambda(t)$ is different from the cost $\mu(t)$. This feature of land conversion is due to the linearity of the Hamiltonian in $a(t)$.

The Steady State

We assume that a steady state exists (thus the transversality conditions in (9) are naturally satisfied). Later we will show that the steady state is a saddle point. Setting $\dot{C}=0$, $\dot{A}=0$, $\dot{\lambda}=0$, and $\dot{\mu}=0$, from the conditions in (7), (8) and the two state equations (1), (2), we obtain the following description of the steady state:

$$(11)\quad \begin{aligned} &(i) && e^*=\delta C^*,\\ &(ii) && B'(e^*)=\tfrac{D'(C^*)}{r+\delta},\\ &(iii) && \lambda^*=\tfrac{D'(C^*)}{r+\delta}\\ &(iv) && \mu^*=\lambda^*=\tfrac{Q'(A^*)}{r}, \text{ and}\\ &(v) && a^*=0. \end{aligned}$$

The steady-state levels of emission and stock e^* and C^* are uniquely determined by (11-i) and (11-ii), and are *independent* of the sequestration activities. In particular, they are independent of the cost of sequestration $Q(\cdot)$. That is, once the steady-state emission and stock levels are attained, there is no role for additional carbon sequestration activities. In the very long run, emissions have to be kept at such a level that it is just offset by the reduction in carbon stock due to the natural decay. Thus, in setting the *targets* for the long-run control of global warming, the government only needs to consider the costs and benefits of emission abatement and atmospheric carbon concentration. The option of carbon sequestration should not matter.

However, from (11-iii) to (11-iv), we know $\lambda^*>0$ and $A^*>0$. Thus, a certain amount of carbon in fact is sequestered in the biomass in the steady state. This amount is higher if the marginal cost of sequestration $Q'(\cdot)$ is lower, or as the sequestration becomes more effective. Then the positive stock A^* must be the result of using sequestration during the transition path toward the steady state. That is, sequestration does affect the *process* of reaching the long-run targets. We will investigate this process in the rest of this section.

In summary,

REMARK 1. *The long-run targets of controlling global warming are independent of the sequestration possibilities. That is, carbon sequestration cannot efficiently provide a long-run solution to global warming on its own. However, it may be efficiently employed "in the process," resulting in a permanent level of sequestered carbon.*

The Transition Paths

Analyzing the transition paths of the system and the stability of the steady state is complicated because there are two state variables and the problem is singular in one of the control variables. In principle, we have to study a system of four differential equations, and the eigenvalues are difficult to characterize without assuming special functional forms for the cost and benefit functions. We develop an alternative method to analyze the system, and partly rely on the (quasi) phase diagrams in the space of $e(t)$ and $C(t)$, shown in figure 1. Setting $\dot{C}=0$ in (1), we get

$$(12)\quad e(t)=a(t)+\delta C(t).$$

Thus the $\dot{C}=0$ locus is linear and upward slopping, and its location depends on the value of $a(t)$. This locus is shown for three constant levels of $a(t)$, $\bar{a}$, 0, and $\underline{a}$, in figure 1.

To derive the equation of motion for $e(t)$, we differentiate both sides of (5), and get

$$(13)\quad \dot{\lambda}(t)=B''(e(t))\dot{e}(t).$$

Plugging (5) and (13) into (7) and rearranging, we know

$$(14)\quad \dot{e}(t)=\frac{r+\delta}{-B''(e(t))}\times\left[\frac{D'(C(t))}{r+\delta}-B'(e(t))\right].$$

Setting $\dot{e}(t)=0$ leads to

$$(15)\quad \frac{D'(C(t))}{r+\delta}=B'(e(t)).$$

Finally, totally differentiating equation (15), we obtain

$$(16)\quad \frac{de}{dC}=\frac{D''(C(t))}{(r+\delta)B''(e(t))}<0.$$

Thus the $\dot{e}=0$ locus is downward slopping and is independent of the level of $a(t)$. In

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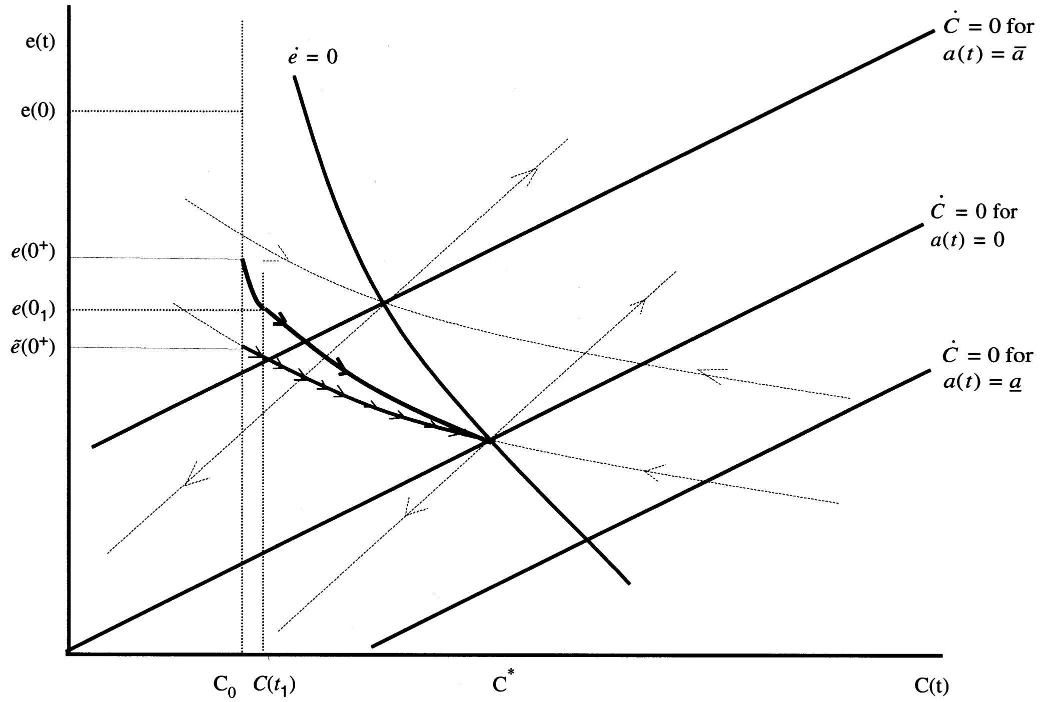

Figure 1. Phase diagram for carbon emission and stock

figure 1, from (13), we know $\dot{e}(t) < 0$ when $\{C(t), e(t)\}$ is to the left of the $\dot{e} = 0$ locus, and $\dot{e}(t) > 0$ when $\{C(t), e(t)\}$ is to the right, independent of the value of $a(t)$.

The dotted lines with arrows in figure 1 represent the stable and unstable branches of the system for *fixed* levels of a. It is obvious that when there is no or constant rate of sequestration, the system of $e(t)$ and $C(t)$ should approach the steady state on a saddle path.

The derivation of the optimal paths (characterized in Propositions 1–5) when $a(t)$ is endogenous is given in Appendix A. Here we only present the results and discuss the intuition. The rate of sequestration $a(t)$ is determined by (6). The system is thus on a singular path when $\lambda(t) = \mu(t)$. This path is important because it contains the steady state, which is associated with $\underline{a} < a(t) = 0 < \bar{a}$.

Intuitively, once the system $\{C(t), A(t), e(t), a(t)\}$ reaches the singular path, it should stay on the path until the steady state is reached in the limit. The reason is that before the singular path is reached, the marginal benefit and cost of sequestration $\lambda(t)$ and $\mu(t)$ are not equal. Thus land should be converted (either into or out of the sequestration programs) at the maximum rate to mitigate the inequality. Once they are equal, the social planner has no incentive to break the equality because any future inequality can be avoided by adjusting the conversion rate $a(t)$ now while on the singular path to improve welfare.

This intuition suggests that the transition path resembles a most rapid approach path, except that the system "quickly" approaches the singular path, rather than the steady state. Before the singular path is reached, $a(t)$ equals either $\bar{a}$ or $\underline{a}$. The values of $e(t)$ and $C(t)$ are then determined jointly by (1) and (14) with appropriate boundary conditions (discussed later). The following proposition shows that our intuition is indeed correct.

PROPOSITION 1. *Given the starting point* $\{C_0, A_0\}$, *the optimal path will move to the singular path as soon as possible, by setting* $a(t)$ *to be either* $\bar{a}$ *or* $\underline{a}$ *and choosing* $e(t)$ *accordingly. The system will then stay on the singular path forever, approaching the steady state.*

The proposition describes what happens before the singular path is reached, we next characterize the features of the singular path.

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Proposition 2. *The land conversion rate on the singular path is given by*

$$(17)\quad a(t) = [-\delta(r+\delta)B'(e(t)) + \delta D'(C(t)) + D''(C(t))(e(t) - \delta C(t))]/[Q''(A(t)) + D''(C(t))].$$

As emissions, carbon stock, and land stock $A(t)$ change overtime, the value of $a(t)$ is likely to change as well. In fact, it is generic that $a(t)$ is not a constant on any time interval on the singular path. This observation highlights the limitations of studying carbon emissions assuming fixed levels of sequestration.

Next we show that the emission and carbon levels are monotonic.

Proposition 3. *Along the singular path, both $e(t)$ and $C(t)$ are monotone. That is, except at the steady state, neither $\dot{e}(t)$ nor $\dot{C}(t)$ can be zero.*

The proposition says that, if the steady-state carbon stock level is lower (higher) than the starting level, then the carbon stock will increase (decrease) steadily through time. Since the damage function $D(\cdot)$ of the stock is convex, the planner has an incentive to "smooth out" $C(t)$ overtime and avoid cyclical variations such as raising the stock for a while only to abate it later. Similarly, the benefit function of $B(e)$ is concave, and emission smoothing implies monotone $e(t)$. The proposition implies that depending on the starting point, the transition path can only stay on one side of the isocline $\dot{e} = 0$ in figure 1 (recall that $\dot{e}(t)$ is a function of $e(t)$ and $C(t)$ only). The next proposition shows that, under a certain condition, $A(t)$ is monotone as well, i.e., $a(t)$ does not change signs on the singular path.

Proposition 4. *$A(t)$ is monotone along the singular path if and only if*

$$(18)\quad \ddot{\lambda}(t)/\dot{\lambda}(t) < r.$$

When (18) *is satisfied, if the singular path approaches the steady state from the left of the isocline $\dot{e} = 0$ in figure 1, $a(t) > 0$ and $A(t)$ monotonically rises. If the path approaches the steady state from the right, $a(t) < 0$ and $A(t)$ monotonically decreases.*

Condition (18) requires that the marginal emission damage $\lambda(t)$ is not "too convex" in time. For example, suppose the system approaches the steady state from the left so that $\dot{\lambda}(t) > 0$. Then condition (18) requires that either $\dot{\lambda}(t)$ is decreasing over time, i.e., $\lambda(t)$ is increasing but concave in t, or is not increasing at a rate higher than the discount rate r. In other words, eventually the rate of increase of $\lambda(t)$ cannot be too high.

From (5), we know $r\dot{\lambda} - \ddot{\lambda} = r\ddot{e}B'' - \dddot{e}B'' - (\ddot{e})^2B'''$. Again, consider the system to the left of the steady state. Then sufficient conditions for (18) are $\dddot{e} \geq 0$ and $B''' \leq 0$, or the absolute values of these two variables are low regardless of their signs. The following proposition presents two additional sufficient conditions.

Proposition 5. (i) *The condition* (18) *is satisfied when the system is sufficiently close to the steady state.* (ii) *It is also satisfied if the rate of increase of the marginal damage of atmospheric carbon, $dD'(\dot{C}(t))/dt = D''(C)\dot{C}$, is constant or decreasing overtime.*

Sudden and drastic reductions in the emission level may occur at time zero as the system moves to the optimal trajectory. Afterwards, emissions tend to stabilize toward the steady-state level. That is, the absolute value of $\ddot{e}$ tends to be small. Further, as the system moves toward the steady state, carbon stock will slowly approach its steady-state level, or $\dot{C}$, and $D''\dot{C}$, will be decreasing. Thus, in general, we expect that (18) is satisfied. We assume this is the case in the article.[7] Propositions 3 and 4 rule out any cyclical patterns in the transition path and any spiral (the steady state being a spiral point) or orbital stability (such as limit cycles). In fact, because there is a unique singular path passing through the steady state, the steady state must be a saddle point.

Remark 2. *The optimal emission level and atmospheric carbon concentration are not cyclical: they should monotonically increase or decrease overtime. Further, under a rather general condition, the optimal path does not involve any cyclical patterns of carbon sequestration, or repeated sequestration and release activities.*

Proposition 4 and Remark 2 imply that the constraint $A(t) \geq 0$ is never binding. Since A_0 is low, $a(t) = \bar{a}$ before the singular path is reached. Thus, if the system approaches the steady state from the left of $\dot{e} = 0$, $a(t) > 0$

[7] In the rare cases, where (18) is not satisfied, $A(t)$ may not be monotone and there may be cyclical patterns of carbon sequestration and release. Such patterns need further study.

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for all t, and thus $A(t) > 0$ for all t. If the system starts from the right of $\dot{e} = 0$, carbon is sequestered first, and since $A^* > 0$, only part of it is released later. Thus $A(t) > 0$ for all t. Therefore,

REMARK 3. *On the optimal trajectory,* $A(t) > 0$ *for all* $t \geq 0$.

Effects of Carbon Sequestration

To completely characterize the paths of carbon emissions and sequestration, and to evaluate the effects of the availability of sinks on the optimal emission and stock, we need to specify the starting point, or the levels of $\{C_0, A_0, e(0), a(0)\}$, in particular their relative positions to the steady state. It is safe to assume that $A_0 < A^*$: because no mechanism exists to encourage carbon sequestration yet, the current use of carbon sinks is likely below the socially optimal long-run level. In addition, as discussed in the introduction, the marginal cost of carbon sequestration is low (close to zero) if only a small amount of carbon is sequestered. We assume a low current rate of land conversion $a(0)$. This rate may even be negative given widespread deforestation in many parts of the world.

We refer to the recent IPCC reports to specify C_0 and $e(0)$.[8] IPCC (2000b, 2001) projects the atmospheric CO_2 concentrations by year 2100 to be about 540–970 ppm for a wide range of emission scenarios. In contrast, the current CO_2 concentration is about 360 ppm. In these scenarios, IPCC lists 450, 650, and 1000 ppm as possible alternative targets of CO_2 concentration levels in the long run. These numbers seem to indicate that $C_0 < C^*$.[9] The IPCC (2001) further noted that to reach the three targets, CO_2 emissions have to "drop below the 1990 levels within a few decades, about a century, or about two centuries, respectively, and continue to decrease steadily thereafter. Eventually, CO_2 emissions have to decline to a very small fraction of current emissions." We therefore assume that $e_0 > e^*$, and that e_0 is above the optimal emission level given C_0 and A_0.

With our specification of the starting condition, the optimal transition path is represented by the heavy solid line in figure 1, with the arrows indicating the direction of movement. Before the singular path is reached at time t_1, $a(t) = \bar{a}$, and the motion of the system is dictated by the locus of $\dot{e} = 0$ and $\dot{C} = 0$ for $a = \bar{a}$. To guarantee that the path reaches the singular path, $e(0^+)$ must fall below the stationary arm of the steady state associated with $a = \bar{a}$. Along the entire transition path, $e(t)$ decreases and $C(t)$ increases.

Land is converted at its maximum rate $\bar{a}$ before t_1, and is converted at a lower, but positive rate afterwards. Sequestered carbon is never released. Sequestration will be utilized as early and as extensively as possible. The intuition for this is as follows. At the beginning, the marginal cost of sequestration is low, lower than the marginal damage of an additional unit of carbon in the atmosphere, thus it makes economic sense to reduce (or eliminate) the difference between the marginal cost and marginal benefit of sequestration. The sooner this is done the better. However, the amount of carbon that can be sequestered at any point of time is constrained. So the best we can do is to sequester the maximum amount of carbon that can be sequestered to bridge the difference between the marginal cost and benefit of sequestration.[10] Of course, early use of sinks should also be accompanied by (possibly drastic) emissions reduction. After the system reaches the singular path, $a(t)$ should be set so as to maintain the equality of marginal cost and benefit of sequestration, as the carbon stock approaches its steady-state level.

To further study the effects of carbon sinks on the optimal emission levels, consider the optimal emission trajectory when sinks are not available, or when $a(t) = 0 \; \forall t$, denoted by $\tilde{e}(t)$. At any stock level C, the fact that $a(t) > 0$ when sinks are available indicates that $\lambda(C) < \tilde{\lambda}(C)$, where $\tilde{\lambda}(C)$ is the marginal

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[8] Intergovernmental Panel on Climate Change (IPCC) was established in 1988 by the World Meteorological Organization (WMO) and the United Nations' Environmental Program (UNEP). It organizes scientists from all over the world to conduct rigorous surveys of the latest technical and scientific literature on climate change. The IPCC's assessment reports are widely recognized as the most credible sources of information on climate change.

[9] On the other hand, IPCC (2001) cites an increasing body of observations supporting the notion that global warming is already happening (and that most of the warming observed over the last 50 years is attributable to human activities). Depending on the damage of the warming (which may takes some years to realize) and the costs of reducing the current emissions (which we do not consider in this article), it is also possible that the steady-state C^* should be lower than the current C_0. Our analysis can be easily extended to analyze this situation.

[10] If the system starts from the right of $\dot{e} = 0$, $a(t) = \bar{a}$ before the singular path is reached, after which $a(t) < 0$. However, since $A^* > 0$, only part of sequestered carbon is released. In this situation, sinks are utilized early and to a great extent, so much so that part of the stored carbon has to be released.

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damage of emissions without the sinks.[11] The reason is that sequestration offers an additional way of reducing the carbon stock, thereby reducing the marginal damage of emissions. Since $B(\cdot)$ is concave, we know from (5) that $e(C) > \tilde{e}(C)$ for all $C < C^*$. Figure 1 shows the relative positions of the two paths: the optimal trajectory with sinks lies strictly above that without sinks, before reaching the steady state. In summary,

REMARK 4. (*i*) *Sequestration should be utilized as early as possible, accompanied by a reduction of the emissions.* (*ii*) *The availability of carbon sinks raises the optimal emissions, or decreases the degree of emission reduction that is needed to reach the steady-state level of carbon stocks.*

The remark further shows the role of carbon sequestration: sinks only affect the *processes*, but not the steady-state levels, of carbon emission and stock. This result, of course, is consistent with the steady-state analysis in Remark 1.

Implementation Mechanisms of Carbon Sinks

We have shown above that sequestration can be used to reduce the pressure on emission abatement. In this section, we propose and assess three distinct trading mechanisms, each of which can implement the socially optimal level of carbon sequestration. We refer to the three mechanisms as PAYG, VLC, and CAA. All three mechanisms are designed to be implemented within a well-functioning permit market for carbon emission reductions. Thus, we assume there is a carbon permit trading system, and that the permit price in the system is efficient: $P(t) = B'(e(t)) = \lambda(t)$. We analyze how trade between sources and sinks can take place efficiently, yielding the optimal amount of sequestration. We also discuss some of the potential advantages and drawbacks of the three mechanisms in terms of ease of implementation. Throughout this discussion, one "carbon credit" means a unit of carbon that is permanently removed from the atmosphere and the carbon price is the payment for one full carbon credit.

[11] To simplify notation, we write λ as a function of C, instead of t, because the systems with and without carbon sequestration will arrive at C at different times. Strictly speaking, we should write $\lambda(t_1) < \tilde{\lambda}(t_2)$, where t_1, t_2 are such that $C(t_1) = C$ and $\tilde{C}(t_2) = C$. In this paragraph, we will use similar notations for $e(\cdot)$ and $\tilde{e}(\cdot)$.

PAYG System

In a PAYG system, owners of sinks sell (and repurchase) emission credits based simply on the permanent reduction of carbon. For example, in the first year, a farmer who adopts conservation tillage practices on 100 acres may earn 200 permanent carbon reduction credits which he can then sell at the going rate. If, in the fifth year, the farmer plows the field and releases all of his stored carbon, he would be required to purchase carbon credits from the market at the going price to cover his emissions.

In a world of certainty, the price trajectory $P(t)$ is known. Suppose there is perfect competition in the sink credit market. Then the competitive solution is equivalent to the problem of maximizing the present discounted revenue from carbon sequestration, $a(t)P(t)$, minus the sequestration cost $Q(A(t))$. Mathematically, the problem can be written as,

$$\max_{a(t)} \int_0^\infty [P(t)a(t) - Q(A(t))]e^{-rt}\,dt$$
$$\text{(19)}\quad \text{s.t.}\quad \dot{A}(t) = a(t), \quad 0 \le A(t) \le \overline{A},$$
$$\underline{a} \le a(t) \le \bar{a}.$$

As in the last section, we first ignore the constraint $A(t) \ge 0$, and derive the optimality conditions. We show that these conditions replicate the social planner's problem. Then by Remark 3, we know the constraint is not binding. Thus in the balance of this section, we will ignore the constraint $A(t) \ge 0$.

The Hamiltonian is $H^1 = P(t)a(t) - Q(A(t)) - \mu(t)a(t)$, and the first-order necessary conditions are

$$\text{(20)}\quad \max_a H^1 \text{ or } a(t) \begin{cases} = \bar{a} \\ = \underline{a} \\ \in [\underline{a}, \bar{a}] \end{cases} \quad \text{if } P(t) - \mu(t) \quad \begin{cases} > 0 \\ < 0 \\ = 0 \end{cases}$$
$$\dot{\mu}(t) = r\mu(t) - \frac{\partial H^1}{\partial A} = r\mu(t) - Q'(A(t))$$
$$\lim_{t\to\infty} e^{-rt}\mu(t) = 0.$$

The first-order conditions are the same as (6), (8), and the transversality condition for μ in (9). Together with the efficient permit price $P(t) = \lambda(t)$, these conditions

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exactly replicate the social planner's choice of sequestration together with abatement. Therefore, given that the permit price equals the present discounted value of marginal damage, the PAYG is efficient. Given the obvious practical difficulties (i.e., the obligation of purchasing credits upon release, intentional or accidental) of implementing and enforcing such a system, we present the efficiency results in large part as a basis of comparison for the following two systems.

The VLC System

The VLC system might evolve through independent broker arrangements. If a broker wishes to buy permits from sink sources and sell them to emitters, the broker must contract with sink sources to achieve a permanent reduction in carbon. This could be accomplished by making a contract with one farmer to adopt conservation tillage for, say 3 years before plowing the field, contracting with a second farmer to plant trees beginning in year 4 for a certain number of years and so on. In each period, the broker might offer farmers a menu of prices associated with different contract lengths. In this system, private brokers provide the service of generating "permanent" carbon reductions from a series of separate temporary reductions.

Formally, suppose that a broker offers farmers a menu of prices for different contract lengths in each period. Let $q(t,\tau)$ be the price offered at time t for a contract with length τ. Then given this price menu, a farmer's decision is to maximize the net gain from carbon sequestration by choosing units of land for contracts of different lengths. Let $a(t,\tau)$ be the units of land enrolled at time t for a contract length of τ periods. The farmer's problem is

$$\operatorname*{Max}_{a(t,\tau)} \int_0^\infty e^{-rt}\left[\int_0^\infty q(t,\tau)a(t,\tau)\,d\tau - Q(A(t))\right]dt$$

$$\text{(21)}\quad \text{s.t.}\quad \dot{A}(t) = \int_0^\infty a(t,\tau)\,d\tau - \int_0^t a(t-\tau,\tau)\,d\tau,$$

$$0 \le A(t) \le \overline{A}, \quad \underline{a} \le \dot{A}(t) \le \bar{a}$$

where $\int_0^\infty [q(t,\tau)a(t,\tau)]\,d\tau$ is the sum of total revenue at time t from contracts of all lengths; $\int_0^\infty a(t,\tau)\,d\tau$ is the total units at time t of newly enrolled land under contracts of all lengths, and $\int_0^t a(t-\tau,\tau)\,d\tau$ is the total unit of contracts expiring at time t.

Proposition 6. *The VLC system is efficient if*

$$\text{(22)}\quad q(t,\tau) = P(t) - e^{-r\tau}P(t+\tau).$$

The proof is given in Appendix B. The condition in (22) is intuitive: for the VLC to be efficient, the price paid to the temporary storage should equal the difference between the damage that is reduced when carbon "flows into" the sinks and the added (discounted) damage when carbon is released into the atmosphere.

The condition in (22) will always be satisfied if there is no arbitrage in the trading of VLCs and emission permits. To see this, suppose a certain contract, say $\tilde{q}(t,\tau)$, is offered that is different from (22), and without loss of generality, suppose $\tilde{q}(t,\tau) > q(t,\tau)$. Then a broker can earn strictly positive profits by buying at time t an emission permit at $P(t)$, selling at t a VLC for the length of τ at $\tilde{q}(t,\tau)$, and selling at $t+\tau$ the emission permit at $P(t+\tau)$. The strategy clearly covers the broker's position: at each moment, the broker's balance of net emission is zero. However, the broker's loss in buying and selling the emission permit, $-P(t) + e^{-r\tau}P(t+\tau) = -q(t,\tau)$, is more than covered by the gain in selling the VLC, $\tilde{q}(t,\tau)$, leading to the arbitrage opportunity.

Arbitrage opportunities are not likely to arise if the emission permit and VLC trading markets are perfectly competitive. For a global pollutant-like carbon with countless emission sources, the emission permit market is likely to be competitive. The nature of the VLC market will depend on the geographical distribution of the sinks and the brokers. It can be competitive if multiple brokers operate in each geographical area of carbon sinks. Since the owners of the sinks (i.e., farmers) do not have to directly "pay out" when carbon is released, the VLC approach is likely to be more feasible to implement compared with the PAYG system.

The "ton-years" accounting method mentioned in the introduction section can be made equivalent to the VLC if the correct discount factor is used. According to the "ton-years" accounting method, the amount of carbon sequestered is directly discounted,

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while in the VLC system, the price of sequestration is discounted. In both methods, the "correct" discount factor (either for quantity or price), depends on the duration of sequestration, the discount rate for future damage, and the natural decay rate of carbon.

The CAA System

Finally, a CAA system may be the most straightforward to implement of all three systems. Similar to the PAYG system, in a CAA system, the generator of a sink is paid the full value of the permanent reduction in the GHG's stored in the sink. However, CAA is also different from PAYG in that the payment, in stead of being paid to a farmer (or whoever sequesters carbon), is put directly into an annuity account. The payment deposited in the annuity account works as a "bond"—with the money in the account, the farmer is discouraged to release her stored carbon, and if she releases it, it is guaranteed that she will be able to pay at least partly for the released carbon. As long as the sink remains in place, the owner can access the earnings of the annuity account, but not the principal. The principal is reduced at the on-going permit price when and if the sink is removed (e.g., the soil is tilled or other change is made to release the stored carbon). If the sink remains permanently, the sink owner eventually earns all of the interest payments, the discounted present value of which equals the principal itself—the permanent permit price. We now show that a CAA system is efficient.

Let $M(t)$ be the balance in the CAA account. Then in each period, $M(t)r$ will be the farmer's revenue, and $Q(A(t))$ will be her cost. The farmer's objective is to maximize the present discounted value of net revenue.

$$\underset{a(t)}{\text{Max}} \int_0^{\infty} [M(t)r - Q(A(t))]e^{-rt}\,dt$$

$$\text{(23)}\quad \text{s.t.}\quad \dot{A}(t) = a(t), \quad 0 \le A(t) \le \overline{A},$$

$$\underline{a} \le a(t) \le \bar{a}, \quad \dot{M}(t) = a(t)P(t).$$

Let $\theta(t)$ be the costate variable for $M(t)$. Again, we first ignore the constraint $A(t) \geq 0$. Then the current value Hamiltonian is $H^2 = M(t)r - Q(A(t)) + \theta(t)a(t)P(t) - \mu(t)a(t)$, and the necessary conditions are,

$$\text{(24)}\quad \max_a H^2 \quad \text{or} \quad a(t)\begin{cases} = \bar{a} \\ = \underline{a} \\ \in [\underline{a}, \bar{a}] \end{cases} \quad \text{if} \quad \theta(t)P(t) - \mu(t) \begin{cases} > 0 \\ < 0 \\ = 0 \end{cases}$$

$$\text{(25)}\quad \dot{\theta}(t) = r\theta(t) - \frac{\partial H^2}{\partial M} = r\theta(t) - r = r(\theta(t) - 1)$$

$$\text{(26)}\quad \dot{\mu}(t) = r\mu(t) - \frac{\partial H^2}{\partial A} = r\mu(t) - Q'(A(t)).$$

Rearranging (25), we know $d/dt[\theta(t) - 1] = r[\theta(t) - 1]$, which implies that $\theta(t) - 1 = e^{rt}[\theta(0) - 1]$. However, since $\theta(0) = 1$, that is, the marginal value of money in period zero is equal to one, we know $\theta(t) = 1$ for all t. Then the necessary conditions are the same as those in the PAYG system. Thus the CAA system is efficient.

Discussion and Final Remarks

Resolving the permanence issue will be key to introducing carbon sequestration into the Kyoto Protocol or any other international agreement concerned with global warming. In this article, we have addressed this issue directly with a model of carbon emissions and sequestration dynamics. Several valuable policy insights come directly from the framework. First, the view that carbon sequestration should not be used to address global warming is not warranted from a theoretical perspective. Ultimately, as long as there is less carbon in the air, it does not matter whether the reduction is done by sequestration or emission abatement. We showed that carbon sequestration should be used as early as possible (as long as it is ever efficient to use it) to reduce the pressure on emission abatement, and the carbon flow into sinks lasts until the atmospheric carbon concentration is stabilized. We also ruled out any cyclical patterns of carbon sequestration and release in the utilization of sinks.

The insights concerning the efficient and early use of sequestration shown in this article are particularly interesting in light of the current policy forum about global

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warming. Some businesses and even some nations, including the U.S., are very reluctant to take actions to reduce carbon emissions. Sequestration can reduce the pressure on emission abatement in current periods, providing time to develop political support for and the technological capability to reduce carbon emissions.

However, despite the clear theoretical role for carbon sequestration, it is equally clear that it should not be treated the same as carbon emission reductions. Sequestration, by its nature, always has the potential to be temporary; consequently, it cannot be attributed the same value that emission reductions have if an efficient solution is to be obtained. The correct view is that sequestration has value, but the value is different from (and less than) the value of direct emission reduction. Therefore, special mechanisms should be used to address the difference. We define three such systems and demonstrate the efficiency properties of each of them.

To properly implement any of the three systems, we will need accurate approaches to measure the amount of carbon stored in sinks. Likewise, for carbon trading to occur between sinks and emission sources, all three systems need price information from outside the agricultural and forest sector. PAYG and CAA both require the current permit prices and VLC requires prices of temporary carbon storage for all lengths of duration. Note that there is nothing preventing the simultaneous use of all systems.

Given that all three systems can be demonstrated to yield the theoretically efficient solution, the choice between which, if any, of these systems to actually implement may largely depend on the costs involved of implementation as well as the general acceptability of the approach to all involved. On this score, we suspect that the repayment obligations inherent in the PAYG system will render it politically infeasible. The CAA system might be more appealing because it partially solves the repayment problem. The comparison of the systems also depends on how their efficiency will be altered when the carbon prices are not efficient. This issue is an interesting topic for future research.

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Appendix A

Derivation of the Optimal Transition Paths

We prove the propositions used in deriving the optimal transition paths.

Proof of Proposition 1

We follow a similar approach used by Tsur and Zemel, who extend the Spence and Starrett methodology to more than one state variable.

Suppose the time path of the costate variable $\lambda(t)$ is given. Thus from the necessary conditions (5) and (7), we know $e(t)$ and $C(t)$ are completely determined by $\lambda(t)$. In particular, we write them as $e(\lambda(t))$ and $C(\lambda(t), \dot{\lambda}(t))$.

Then, by (3) and (4), we can rewrite the optimal control problem for $a(t)$ as

$$\text{(A1)}\quad \max_a \int_0^\infty e^{-rt} \times [B(e(\lambda)) - D(C(\lambda(t), \dot{\lambda}(t))) - \lambda[e(\lambda) - a - \delta C(\lambda(t), \dot{\lambda}(t))] - Q(A)]dt$$
$$\text{s.t.}\quad \dot{A} = a, \quad \underline{a} \le a \le \bar{a}.$$

We can check that the necessary conditions in (A1) replicate the original necessary conditions for a in (6) and (8).

Replacing a by $\dot{A}$ in the objective function, and integrating $\int_0^\infty e^{-rt}\lambda\dot{A}\,dt$ by parts, we know (A1) can be rewritten as

$$\text{(A2)}\quad \max_A \int_0^\infty e^{-rt} \times [B(e(\lambda)) - \lambda e(\lambda) - D(C(\lambda(t), \dot{\lambda}(t))) + \lambda\delta C(\lambda(t), \dot{\lambda}(t))]dt. + \int_0^\infty [-A(\delta\lambda - D'(C(\lambda, \dot{\lambda}))) - Q(A)]dt - \lambda(0)A_0.$$

Thus given $\lambda(t)$, the objective function depends only on A. We only need to choose $A(t)$ to maximize the second integrand in (A2) for each time t, and the first-order condition is

$$\text{(A3)}\quad -\delta\lambda + D'(C) = Q'(A).$$

With $\lambda(t) = \mu(t)$ and $\dot{\lambda} = \dot{\mu}$ on the singular path, we know from (7) and (8) that condition (A3) is satisfied on the singular path. Thus, the objective function in (A2) is maximized on the singular path.

Since (A2) depends only on A, we would want to move to the optimal path of A, or the singular path, as soon as possible. Thus the optimal solution involves choosing $a(t)$ to be either $\underline{a}$ or $\bar{a}$ until the singular path is reached. Once the arc is reached, the system stays on it forever.

Proof of Proposition 2

From (7) and (8), we know $\ddot{\lambda} = (r+\delta)\dot{\lambda} - D''\dot{C}$ and $\ddot{\mu} = r\dot{\mu} - Q''a$. However, $\ddot{\lambda} = \ddot{\mu}$ and $\dot{\lambda} = \dot{\mu}$ on the singular path. Thus we know $Q''a = -\delta[(r+\delta)\lambda - D'] + D''(e - a - \delta C)$, which implies (17).

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Proof of Proposition 3

Part (i). We first prove that $e(t)$ is monotonic. Suppose this is not true. In particular suppose there exists a time $t < \infty$ such that $\dot{e}(t) = 0$. In the phase diagram, if the path crosses $\dot{e} = 0$ from the left, $e(t)$ must be convex in time: it first decrease and then increases. Similarly, if the path crosses $\dot{e} = 0$ from the right, $e(t)$ must first increase and then decrease. The two possibilities are described in figure A1 by the four short arrowed curves.

Therefore, if a path has ever crossed the $\dot{e} = 0$ curve, there are only two scenarios in which the path will approach the steady state. In the first case, the *last* crossing of $\dot{e} = 0$ is from the right, and thus the system approaches the steady state from the left. Since $\dot{e}(t) < 0$ before reaching the steady state, the last crossing must have occurred above the steady state along the $\dot{e} = 0$ line. This path is depicted as Path One in figure A1. In the second scenario, the last crossing is from the left, and the path finally approaches the steady state from the right. Thus $e(t)$ is increasing after the last crossing, and the crossing point is below the steady state on the $\dot{e} = 0$ isocline.

Consider path one. Since the $\dot{e} = 0$ isocline is downward slopping, the carbon stock $C(t_1)$ at the crossing time t_1 is lower than the steady-state level C^*. Thus after t_1, $C(t)$ will eventually increase. However, immediately after t_1, $C(t)$ is decreasing since the path crossed the $\dot{e} = 0$ isocline from the right. Therefore, there exists a time $t_2 > t_1$ at which $\dot{C}(t_2) = 0$. Further, at this time, $\{C(t_2), e(t_2)\}$ is to the northwest of the steady state. This observation implies that $a(t_2) > 0$, since the $\dot{C} = 0$ isoclines are higher as a is higher.

Taking a time derivative of (7) and (8), and using $\lambda(t) = \mu(t)$, $\dot{\lambda} = \dot{\mu}$, and $\ddot{\lambda} = \ddot{\mu}$ on the singular path, we know $\delta\dot{\lambda}(t_2) = -Q''(A(t_1))a(t_2)$. However, since $\dot{e}(t_2) < 0$, we know $\dot{\lambda}(t_2) > 0$, violating the fact that $a(t_2) > 0$ (since $Q'' > 0$). Thus Path One never arises.

Similarly, we can show that Path Two should not arise either, establishing the monotonicity of $e(t)$ on the singular path.

If there are infinite number of crossings so that there is not a *last* crossing, a point like $\{C(t_2), e(t_2)\}$ (or its counterpart to the right of $\dot{e} = 0$) still exists when the system is close to the steady state. Thus, our proof still carries through.

Part (ii). We next prove that $C(t)$ is monotone. Without loss of generality, consider the singular path that is to the left of the $\dot{e} = 0$ isocline so that $\dot{e}(t) < 0$. (we have just shown that the path should never cross the isocline). Suppose there exists a time $\infty > t_3 > 0$ such that $\dot{C}(t_3) = 0$. Differentiating (7) and (8) and adjusting, using $\lambda(t) = \mu(t)$, we know $\delta\dot{\lambda}(t_3) = -Q''a(t_3)$. However, since $\dot{\lambda} = B''\dot{e} > 0$, we know $a(t_3) < 0$.

Therefore, in figure 1, the system at t_3 must be at a point on the $\dot{C} = 0$ isocline for a negative a level. Because this isocline is upward slopping, we know this point must be below the steady state.

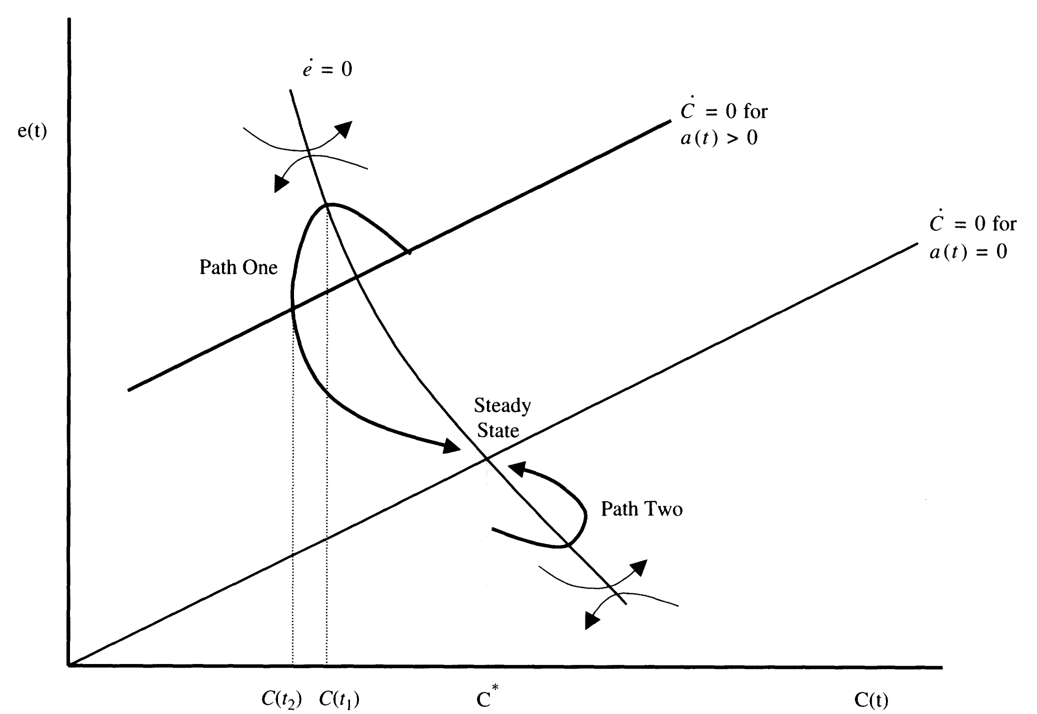

Figure A1. Possible singular paths

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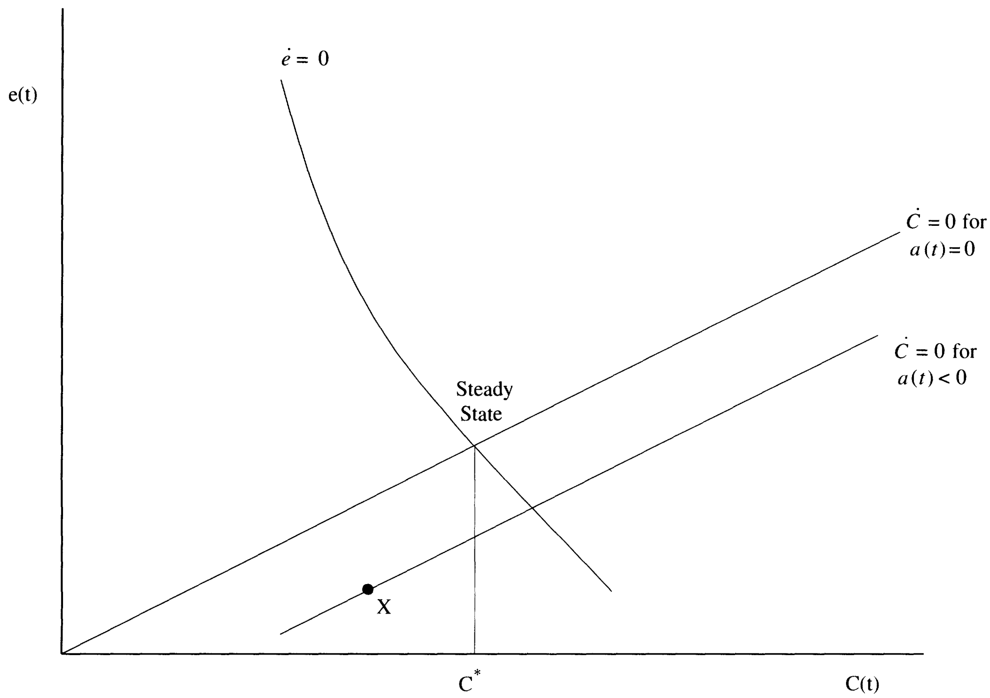

Figure A2. The position of the system at t_2

The point is represented by x in figure A2. Thus, to reach the steady state, $e(t)$ must eventually increase. However, this condition contradicts the fact that we are to the left of the $\dot{e} = 0$ isocline, or $e(t)$ cannot increase.

Proof of Proposition 4

From (8), we know $Q''a = r\dot{\mu} - \ddot{\mu}$. Substituting λ for μ along the singular path, we know $a = (r\dot{\lambda} - \ddot{\lambda})/Q''$. To the left of the steady state, $\dot{\lambda} > 0$. Thus $a > 0$ if and only if $r\dot{\lambda} > \ddot{\lambda}$, or $\ddot{\lambda}/\dot{\lambda} < r$. To the right of the steady state, $\dot{\lambda} < 0$. Then $a < 0$ if and only if $r\dot{\lambda} < \ddot{\lambda}$, or $\ddot{\lambda}/\dot{\lambda} < r$.

Proof of Proposition 5

Without loss of generality, we consider the case when the optimal path is to the left of the $\dot{e} = 0$ isocline, or when $\dot{\lambda} > 0$.

Differentiating (7) with respect to time, we know $\ddot{\lambda}/\dot{\lambda} = r + \delta - D''(C)\dot{C}/\dot{\lambda}$. Thus (18) is true if and only if

$$\text{(A4)} \qquad \delta\dot{\lambda} < dD'(C(t))/dt.$$

At the steady state, $(r+\delta)\lambda^* = D'(C^*)$ (cf. (11)). Before the steady state, $\dot{\lambda} > 0$, or $(r+\delta)\lambda(t) > D'(C(t))$. Therefore, when the system is sufficiently close to the steady state, the left-hand side, $(r+\delta)\lambda(t)$, must be increasing at a smaller rate than the right-hand side $D'(C(t))$. That is, $(r+\delta)\ \dot{\lambda}(t) < dD'(C(t))/dt$, which implies (A4). This proves the first part of the proposition.

Differentiating (10a) with respect to t, and integrating the resulting right-hand side by parts, we can show that

$$\text{(A5)} \qquad \dot{\lambda} = \int_t^\infty \exp\left[-(r+\delta)(s-t)\tfrac{d[D'(C(s))]}{ds}\right] dt.$$

If $d[D'(C(s))]/ds = D''(C)\dot{C}(s)$ is constant, then the right-hand side equals $d[D'(C(t))]/dt/(r+\delta)$, which together with (A5) implies (18). If $d[D'(C(s))]/ds$ is decreasing, then (A5) implies $\dot{\lambda} < d[D'(C(t))]/dt/(r+\delta)$, and (18) follows from (A4).

Appendix B

Proof of Proposition 6

Define $\tilde{a}(t)$ as follows,

$$\text{(B1)} \quad \tilde{a}(t) \equiv \dot{A}(t) = \int_0^\infty a(t,\tau)\,d\tau - \int_0^t a(t-\tau,\tau)\,d\tau.$$

We show below Problem (21) is just the same as Problem (19) with $\tilde{a}(t)$ in place of $a(t)$. We have shown the solution to Problem (19) is efficient. If both problems are the same, then the solution to Problem (21) also has to be efficient. Plugging (22) into the objective function of problem (21), we get

$$\int_0^\infty e^{-rt}\left[\int_0^\infty q(t,\tau)a(t,\tau)\,d\tau - Q(A(t))\right] dt$$

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$$
\begin{aligned}
&= \int_0^\infty e^{-rt}\Big[\int_0^\infty [P(t) - e^{-r\tau}P(t+\tau)] \\
&\qquad \times a(t,\tau)\,d\tau - Q(A(t))\Big]dt \\
&= -\int_0^\infty e^{-rt} Q(A(t))]\,dt \\
&\quad + \int_0^\infty e^{-rt}\Big[\int_0^\infty (P(t)a(t,\tau)\,d\tau\Big]dt \\
&\quad - \int_0^\infty e^{-rt}\Big[\int_0^\infty e^{-r\tau}P(t+\tau)a(t,\tau)\,d\tau\Big]dt \\
&= -\int_0^\infty e^{-rt} Q(A(t))]\,dt \\
&\quad + \int_0^\infty e^{-rt}\Big[\int_0^\infty (P(t)a(t,\tau)\,d\tau\Big]dt \\
&\quad - \int_0^\infty e^{-rt}\Big[\int_0^t P(t)a(t-\tau,\tau)\,d\tau\Big]dt \\
&= -\int_0^\infty e^{-rt} Q(A(t))]\,dt \\
&\quad + \int_0^\infty e^{-rt}\Big[\int_0^\infty (P(t)a(t,\tau)\,d\tau \\
&\qquad - \int_0^t P(t)a(t-\tau,\tau)\,d\tau\Big]dt \\
&= -\int_0^\infty e^{-rt} Q(A(t))]\,dt \\
&\quad + \int_0^\infty e^{-rt} P(t)\tilde{a}(t)\,dt
\end{aligned}
$$

It is easy to see that the above expression is just the same as that in Problem (19) with $a(t)$ replaced by $\tilde{a}(t)$.

The third line follows because of the following,

$$
\begin{aligned}
&\int_0^\infty e^{-rt}\Big[\int_0^\infty e^{-r\tau}P(t+\tau)a(t,\tau)\,d\tau\Big]dt \\
&\quad = \int_0^\infty \int_0^\infty e^{-r(t+\tau)}P(t+\tau))a(t,\tau)\,dt\,d\tau \\
&\quad \text{(By change of integration order)} \\
&\quad = \int_0^\infty \int_\tau^\infty (e^{-rx}P(x)a(x-\tau,\tau)\,dx\,d\tau \\
&\quad \text{(By change of variable,} \\
&\qquad x = t+\tau,\ y = \tau) \\
&\quad = \int_0^\infty \int_0^x e^{-rx}P(x)a(x-\tau,\tau)\,d\tau\,dx \\
&\quad \text{(By change of integration order)} \\
&\quad = \int_0^\infty e^{-rt}\Big[\int_0^t P(t)a(t-\tau,\tau)\,d\tau\Big]dt.
\end{aligned}
$$

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