Corporate Finance Through Loyalty Programs*

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Abstract

Loyalty programs (LPs) are widely prevalent and typically analyzed in economic research for their role in boosting business. This paper uncovers a novel role of LPs as financing instruments. The rewards issued to and redeemed by consumers cause shifts in firms' present and future cash flows, effectively creating a form of borrowing from consumers. We document three stylized facts about LPs in the airline and hotel industries: 1) LPs serve as significant financing sources, with cobranded credit card programs contributing much and sales directly to consumers little; 2) rewards are issued through broad consumption but are redeemed predominantly for consumption related to the issuing firm; 3) LP financing helps smooth cash flows. We then build a dynamic model of LP financing. The model features convenient rewards that do not impose restrictions on redemption. Due to convenience, consumers would demand a lower discount rate, which creates gains from trades. But the funds raised through LPs emerge endogenously in equilibrium as a result of the interplay between reward issuance and redemption. The model suggests that 1) firms supplying high-value, low-frequency services can leverage LPs more effectively for financing; 2) LP financing helps smooth cash flows and is thus attractive to firms with highly volatile business; 3) firms should aim to decouple reward issuance from their business; 4) firms should limit consumers' discretion to purchase or transfer rewards.

Keywords: loyalty programs, corporate finance, rewards, tokens, cobranded credit cards, financial resilience

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1 Introduction

Loyalty programs (LPs) are widely prevalent. According to 2017 COLLOQUY Loyalty Census, the total number of loyalty program memberships in the U.S. reached 3.8 billion. The distribution of these memberships spans various industries with 42% in retail, 29% in travel and hospitality, and 17% in financial services. Certain sectors have even embraced loyalty programs as a standard business practice. Indeed, almost all major airlines, hotel chains, banks, and top retailers offer some form of loyalty programs (Sun and Zhang, 2019). A common and crucial feature of LPs is their provision of rewards to consumers. These rewards, usually presented as points, complimentary nights, or discounts on subsequent purchases, represent promises for future services. They are generally issued to consumers based on their purchases with the brand and its partners.

Economists have offered several theories to rationalize the emergence of LPs that offer rewards. Rewards may provide psychological benefits or engender price misperceptions. They might also enable price discrimination based on quantity or purchasing history. In a market with multiple competing brands, rewards can heighten consumers' switching costs, thereby weakening competition. More recent research suggests that rewards can be used to capitalize on agency conflicts between employers and employees. Essentially, all these theories revolve around the central theme of boosting income.

In this paper, we intend to uncover a novel role of LPs as financing instruments. The idea is simple. When a firm issues rewards to consumers, it usually receives advance payments from consumers or some other benefits, boosting the firm's present cash flows. These rewards are recorded as the firm's liabilities, as it has yet to provide the promised services. Later, consumers will redeem these rewards for the firm's services at discounted prices or free of charge, reducing the firm's future cash flows. These shifts in the firm's present and future cash flows constitute de facto borrowings from consumers.

Although LPs have the potential to be used for financing in theory, do they truly serve as substantial funding sources for business in the real world? If so, how do they function in practice and what are their effects on firms? To address these questions, we document three stylized facts about LPs in the airline and the hotel industries. We focus primarily on the two industries because their LPs are central to business operations and are among the most influential programs across all industries.

First, LPs serve as significant financing sources, with cobranded credit card (CCC) programs contributing much and sales directly to consumers little. Before the Covid-19 pandemic, the total LP liabilities of the two industries reached 40 billion dollars, accounting for 30% of their debt and leases. Whether intentionally or not, LPs have become de facto significant financing sources. A noteworthy finding is that airlines and hotels issue a large fraction of rewards through their

CCC programs. In order to encourage consumers to sign up and use CCCs, bank partners purchase rewards from airlines and hotels and offer them to consumers. For instance, in 2019, approximately 70% of the cash flows of Delta's LP, SkyMiles, came from its bank partner, American Express. More broadly, we find a significantly positive relationship between a firm's LP liabilities and the size of its CCC program within the two industries. On the other hand, sales directly to consumers account for only a tiny fraction of the cash flows, largely because firms sell rewards at prices considerably higher than the actual value of rewards.

Second, rewards are issued through broad consumption, but are redeemed predominantly for consumption related to the issuing firm. American households extensively utilize CCCs for a broad spectrum of consumption, much of which bears no direct relation to the issuing firm. Moreover, airlines deliberately enable consumers to earn miles on purchases with their partners, including those unrelated to their own business like retailers and restaurants. When it comes to the redemption of these rewards, the majority are used for purchases with the issuing firm, thereby ensuring that the rewards are ultimately directed back towards the issuing firm.

Third, LP financing helps smooth cash flows. Reward redemption is closely tied to the issuing firm's business while reward issuance is linked to broad consumption. For firms with volatile business, reward issuance is less volatile than reward redemption. This pattern was prominently evident for airlines and hotel chains during the Covid-19 pandemic. As a result, their LP liabilities grow rapidly in the bad times of the firm's business. This cyclical nature of LP financing naturally enables firms to borrow more from consumers in bad times and smooths firms' overall cash flows.

Our findings indicate that airlines and hotels systematically issue rewards through broad consumption in addition to the purchases with them, but not through sales directly to consumers. This issuance pattern is not suggested by classical theories of LPs. However, we demonstrate that it can be well explained by the finance theory of LPs.

Based on these stylized facts, we develop a model of LP financing. Similar to other financings, LP financing hinges on that consumers discount the value of rewards at lower rates than the firm. The model provides reasoning on why this can be true, but it is largely an empirical question that cannot be fully addressed within the model. Instead of answering why LP financing can arise, the model basically assumes it and illustrates the features of LP financing. The power of the model lies in that its predictions align well with observations in practice, particularly those that classical theories of LPs do not suggest.

The model is an infinite-horizon continuous-time game with consumers, a single firm, and banks. All consumers are homogeneous, with an exogenous demand for a service that the firm monopolizes. This setting naturally eliminates classical motives for LPs like price discrimination and competition. The firm has the option to finance its operations through debt or a LP. By adopting a LP, the firm issues rewards to consumers and receives advance payments, which are referred to as LP liabilities. For consumers, the rewards are convenient as there are no restrictions on their redemption—a consumer can purchase a service anytime he wishes, using any combination of cash and rewards he can afford. Due to the convenience of rewards, consumers discount their value at a lower interest rate than they would for an investment like lending. This creates the gains from trade and gives rise to the financing role of LPs. However, rewards are less liquid than cash, as they can only be redeemed for the service. Hence, we assume that consumers weakly prefer holding cash to holding rewards. On the other hand, redeeming rewards is also more cumbersome than using cash and incurs a cost each time a consumer redeems. To compensate consumers for these costs, the firm must issue rewards at a discount, making LP financing costly.

We consider two types of LPs motivated by Fact 1: consumption-based LPs (CLPs) and purchase-based LPs (PLPs). Adopting a CLP , the firm cooperates with a partner bank in issuing a CCC. Since banks act competitively, it is essentially the firm that issues rewards to consumers for their consumption charged to the CCC and receives the interchange fees. Adopting a PLP, the firm issues rewards to consumers based on their purchases directly made with the firm. LPs are static in the sense that once a LP is adopted, its design does not change over time. For the baseline setup, the game is stationary, and we focus on the steady-state equilibrium paths, in which each consumer behaves optimally and the aggregate behavior of all consumers remains unchanged.

One might wonder how LP financing differs from other financings. Like demand deposits, LP financing let financiers control redemption and thus caters for their demand for convenience. Therefore, they are associated with lower required returns than financings not providing convenience such as the typical debt. But the drawback is that the firm loses direct control of the raised funds. They emerge endogenously as a result of the interplay between issuance and redemption. Consequently, the effectiveness of LP financing hinges a lot on the issuance policy and the firm's characteristics. This is not a relevant feature of demand deposits, because their issuance and redemption are exogenously determined by consumers' total income and consumption.

The model solution starts with the characterization of consumers' optimal behavior. Due to the convenience of rewards, consumers control the redemption process. Specifically, in equilibrium, when a consumer needs a service, he will purchase it using rewards alone if his reward holdings are sufficient; otherwise, he will use cash alone. Despite having the ability to do so, a consumer never opts for partial redemption for a service, as full redemption minimizes redemption costs. Additionally, a consumer does not desire to stockpile rewards beyond what are sufficient for a service, as cash is generally preferred to rewards.

The funds raised by the firm equals the product of the reward balance of all consumers and the unit price of rewards. We then explicitly determine the reward balance when either type of LPs is adopted. The reward balance consists of two parts. The first part stems from that a consumer never redeems rewards before he has accumulated sufficient ones for a service, so it equals half the value of a service. However, a consumer may also overaccumulate—hold rewards more than sufficient for a service, which generates the second part. Under a CLP, overaccumulation arises due to the asynchrony between reward issuance and reward redemption: a consumer consistently earns rewards through the CCC, even during periods when he does not need a service. However, such asynchrony is absent under a PLP, as both issuance and redemption are linked to the purchases of services and mutually exclusive. Instead, overaccumulation arises under a PLP due to the lumpiness of reward earning. Each time a consumer purchases a service with cash, he receives a chunk of rewards that are worth a fraction of the value of a service.

The analysis on the baseline setup generates three novel insights into LP financing. First, firms supplying high-value, low-frequency services can leverage LPs more effectively for financing, as these firms can accumulate high reward balance under both types of LPs. This model prediction aligns well with empirical observations across various industries. In particular, it elucidates why airlines, hotels, and department stores are the most proactive in developing LPs and expanding the issuance of rewards.

Second, to leverage LPs more effectively for financing, a firm should weaken the linkage of reward issuance to the firm's business. We obtain this implication by comparing CLPs and PLPs. We analytically show that the asynchrony under a CLP is more powerful in generating overaccumulation than the lumpiness under a PLP. We also numerically analyze the case where both types are adopted and find that the reward balance increases as the CLP gains more importance relative to the PLP. A broader implication of the comparison between CLPs and PLPs is that weakening the linkage of reward issuance to the firm's business can make LP financing more effective, as it renders issuance less synchronous with redemption. This implication resonates with Fact 2 that rewards are issued through broad consumption, including that unrelated to firms' business.

The third insight concerns transactions of rewards. We consider two benchmark cases: free purchase and free transfer. If free purchase is allowed, consumers can purchase any amount of rewards from the firm at a fixed unit price pre-specified by the firm whenever they want. If free transfer is allowed, consumers can freely transfer their rewards to others, and such transfer does not incur additional costs. We find that neither free purchase nor free transfer is desirable for the firm, as they will accelerate the redemption of rewards and dampen the accumulation of consumers' reward holdings. The driving force of the result is consumers' preference for holding cash over holding rewards. If consumers have flexible control over reward issuance or reward holdings, they will attempt to hold as few rewards as possible given that their payoffs remain the same. This result is consistent with Fact 1 that firms do not intend issue rewards through direct sales, and observations that they discourage transactions of rewards by charging high fees or restricting the scopes and frequencies of transactions.

Lastly, we examine the cyclical nature of LPs and their impact on a firm's financial resilience.

We consider a scenario where the economy unexpectedly enters an adverse state. In this state, consumers' demand for the service drastically declines compared with that in the normal state, resulting in negative cash flows for the firm. Consequently, the firm expends its liquidity buffers to stay afloat. If the adverse state persists for an extended period, the firm will deplete its liquidity buffers and go bankrupt. We investigate how the firm's bankruptcy risk varies with its choice of LP. We find that both types of LPs renders the firm's cash flows less volatile, thereby enhancing the firm's financial resilience. This cyclical nature of LP financing is especially appealing to firms whose business is highly volatile such as airlines, hotels, and upscale department stores.

A common mechanism that renders cash flows less volatile under both types of LPs is that LPs' financing cost is lower in bad times. LP financing is costly as consumers use rewards, issued at a discount, to purchase services instead of cash. As such, a LP effectively becomes a revenue-sharing arrangement where a fixed fraction of revenue is shared with consumers in the form of reward discounts. Consequently, when revenue decreases, the shared revenue follows suit. Additionally, a CLP also helps the firm raise more funds from consumers in the adverse state. The key is that when reward issuance is linked to broad consumption, it is less volatile than reward redemption. This mechanism is clearly revealed by Fact 3. A general implication of this mechanism is that to strengthen the desirable cyclical nature of a LP, the firm should weaken the correlation of reward issuance to its business, which resonates with Fact 2.

The rest of this paper is organized as follows. In the rest of the introduction, we review the related literature. Section 2 documents three stylized facts about LPs in the airline and the hotel industries. Section 3 outlines the baseline setup of the model. Section 4 derives the equilibrium path in the steady state of the baseline setup. Section 5 derives the implications about LP financing. Section 6 examines the cyclical nature of LPs and its impact on the firm's financial resilience. All proofs that are not provided in the main text are relegated to the Appendix.

Related literature

Our paper proposes a novel finance motive of LPs, thereby contributing to the extensive literature that seeks to rationalize LPs. Behavioral theories posit that LPs can yield psychological benefits or generate pricing misconceptions. For instance, Lim et al. (2021) discovers that certain consumers attribute more value to LP points than to actual money. In the literature of industrial organization, a traditional viewpoint is that LPs implement price discrimination based on purchase history or quantity. Cremer (1984) demonstrates that rewards can be used to differentiate between first-time and repeat buyers. Sun and Zhang (2019) postulate that the use of limited reward expiration terms could be driven by the intent to discriminate between frequent and infrequent customers. In a multi-brand environment, LPs can increase consumer switching costs, thereby weakening competition (Klemperer, 1987, 1995). Particularly, Banerjee and Summers (1987) and Caminal and Matutes (1990) first analyze rewards as a kind of endogenous switching cost. Kim et al. (2001) shows that by offering the incentives for repeat purchases, LPs increase a firm's cost to attract competing firms' current customers. Ke et al. (2024) analyzes LPs from the perspective of product search. In their framework, switching costs originate from search costs, and firms use LPs to make consumers prioritize their products when doing sequential search. However, whether switching costs have real impacts in practice is ambiguous (Hartmann and Viard, 2008; Orhun et al., 2022; Rossi and Chintagunt, 2023). Basso et al. (2009) argue that LPs, such as airlines' frequent-flier programs, can exploit agency conflicts between employers and employees. Several recent studies explore firms' optimal decisions in the context of LPs (Kim et al., 2004; Chun and Ovchinnikov, 2019; Chun et al., 2020). Taken together, the existing literature predominantly interprets LPs as instruments to boost business. Our paper document novel facts about LPs in practice, which lend support to the financing role of loyalty programs.[1] To our knowledge, our paper is the first to empirically document the importance of LPs as financing instruments and to theoretically scrutinize their nature. Particularly, we investigate how to design LPs to effectively raise funds from consumers and leverage the desirable cyclical nature.

Our paper belongs to the large and long-lasting literature of corporate finance on capital structures. We document and illustrate a new form of financing from consumers through loyalty programs. Hence, our paper is especially related to the literature studying raising funds from various stakeholder. Several explanations have been provided for why a firm would like to raise funds from stakeholders instead of investors. Strausz (2017) considers the benefit of crowdfunding as a way of acquiring information about the eventual payoff of the project if demand is uncertain (see also Astebro et al. 2017; Chemla and Tinn 2020; Ellman and Hurkens 2019; Cong and Xiao 2024). To explain the widespread trade credit, Biais and Gollier (1997) and Burkart and Ellingsen (2004) argue that suppliers are better able to mitigate the financial frictions faced by external investors such as asymmetric information and lack of commitment. Lee and Parlour (2022) find that raising fund from consumers instead of investors can be more efficient, because consumers is able to commit to paying for the consumption benefit ignored by investors. Our paper characterizes LP financing as a way to raise funds from consumers while catering for their demand for convenience. The convenience feature is reminiscent of demand deposits issued by banks. While the banking literature highlights the run risks (Diamond and Dybvig, 1983), our theory is built upon a neglected consequence of liquidity provision: liquidity provision deprives the firm of the ability to fully determine the amount of raised funds. Moreover, the firm's business model and issuance strategy become crucial to the effectiveness of LP financing.

Rewards issued by LPs are essentially tokens. Howell et al. (2020) empirically examine which

[1] Section 2.5 discusses that the existing theories are not able to explain the documented industry practice well.

issuer and ICO characteristics predict successful real outcomes. The theory literature highlights the network effect, decentralization, and tradability of tokens in the setting of platforms (Cong et al. 2021; Gryglewicz et al. 2021; Cong et al. 2022; Sockin and Xiong 2023b,a; Li and Mann 2024; Goldstein et al. 2024). On the contrary, Rogoff and You (2023) and He et al. (2024) argue that platforms can potentially earn higher revenues by making tokens non-tradable. Instead of these typical features, our paper highlights the convenience of rewards and its implication for financing.

2 Three Facts about Loyalty Programs in the Airline and the Hotel Industries

In this section, we present three stylized facts that illustrate the importance, mechanics, and features of loyalty programs as financing instruments in the travel and hospitality sector. We choose to focus on the airline industry and the hotel industry for two reasons. First, loyalty programs are prevalent in the travel and hospitality sector, which account for 29% of all loyalty memberships in the United States.[2] Second, loyalty programs are considered to be an important part of the business in this sector. All major airlines and hotel chains offer some form of loyalty programs, and disclose related information in more details than other sectors.

2.1 Data

Public firms with large loyalty programs disclose loyalty program liabilities in their annual reports, which represent what firms owe to loyalty program members. Different firms may report them using different terms such as loyalty program deferred revenue, contract liability, and gift card balance. Historically, there are two methods to calculate loyalty program liabilities. The first approach, known as the incremental cost method, recognizes a liability for the marginal costs incurred in providing services to eligible customers. On the other hand, the second approach, referred to as the deferred revenue method, recognizes a liability for advance payments received in anticipation of future service delivery. Under the deferred revenue method, loyalty program liabilities represent the amount of cash flows that the firm will forgo to fulfill its promises related to the advance payments and are close to the opportunity cost in economics. Since 2018, IFRS 15 and ASC 606 made it obligatory for companies to adopt the deferred revenue method. In our analysis, loyalty program liabilities refer to the numbers estimated using the deferred revenue method.

[2] According to the 2017 COLLOQUY loyalty census, loyalty programs are most prevalent in the retail sector, the travel and hospitality sector, and the financial services sector. They account for 42%, 29%, and 17% of all loyalty memberships in the United States, respectively.

Estimating loyalty program liabilities using the deferred revenue method is not simple because it involves the estimation of consumers' redemption behavior. For example, airlines' loyalty program liabilities mainly take the form of miles. To determine the opportunity cost induced by promises related to miles, airlines estimate the equivalent selling price of miles and the amount of miles that will be redeemed. United Airlines describe its method as follows:

> "The Company's estimated selling price of miles is based on an equivalent ticket value, which incorporates the expected redemption of miles, as the best estimate of selling price for these miles. The equivalent ticket value is based on the prior 12 months' weighted average equivalent ticket value of similar fares as those used to settle award redemptions while taking into consideration such factors as redemption pattern, cabin class, loyalty status and geographic region."
>
> "The Company's breakage model is based on the assumption that the likelihood that an account will redeem its miles can be estimated based on a consideration of the account's historical behavior. The Company uses a logit regression model to estimate the probability that an account will redeem its current miles balance. The Company reviews its breakage estimates annually based upon the latest available information. "

We collect information on the stock and the flow of loyalty program liabilities, debt, and revenue from 10-K reports. Regarding debt, we include not only long-term debt and its current maturities, but also financial leases and operating leases.[3] In our within-industry analysis, we use revenue to control for the size of firms' business. To obtain a consistent measure within industries, we use total revenue for airlines and total revenue net of reimbursement costs for hotels.[4] Since firms all adopt the deferred revenue method in 2018, we focus on the sample period spanning from 2018 to 2022. During the Covid-19 pandemic, both the airline industry and the hotel industry experienced a severe downturn, and as a response, their capital structures changed massively. To provide a picture of loyalty programs in normal times, we focus on the year 2019 when conducting cross-sectional analysis.[5] Unless otherwise specified, the scaled version of a variable refers to the ratio of the variable to revenue in 2019.

[3]Disclosure of leases is required by IFRS 16. For more details, please see https://www.pwc.com/gx/en/services/audit-assurance/assets/ifrs-16-new-leases.pdf.

[4]Hotels do not own all the properties they manage. For those purely managed properties, hotels typically cover some operating expenses such as employees' payroll and later receive full reimbursement from property owners. As a result, two reimbursement terms appear in hotels' revenue and expense, respectively, and they are almost equal. Hence, the reimbursement practice affects only hotels' revenue but not their profits and cash flows. Note that the size of the reimbursement depends on the fraction of purely managed properties and the agreement between hotels and property owners, which are heterogeneous across hotels. We think that total revenue net of reimbursement costs is a better measure of hotels' true economic revenue.

[5]Due to IFRS 16, information on leases has been universally available since 2019.

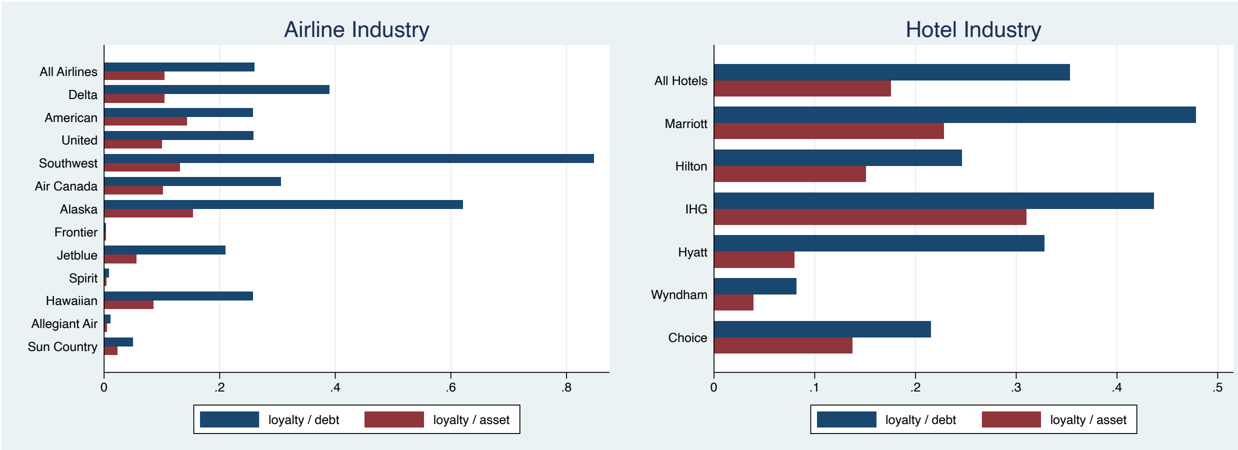

Figure 1: Loyalty program liabilities relative to total debt and total assets

Information about the operation of airlines' loyalty programs is collected from the 8-K reports of United Airlines, Delta Airlines, and American Airlines in 2020 and 2021. Severely hit by the Covid-19 pandemic, the three airlines raised money in senior secured financing collateralized by their LPs, so they made additional disclosure of LPs.

2.2 Fact 1: LPs serve as significant financing sources, with CCC programs contributing much and sales directly to consumers little.

In 2019, the LP liabilities of the airline industry amounted to \$30 billion, while those of the hotel industry amounted to \$10 billion. To put the numbers in perspective, we calculate the ratios of LP liabilities to total debt and total assets, which are shown in Figure 1. On average, LP liabilities constitute approximately 30% of total debt and around 15% of total assets of the two industries. Here, total debt includes long-term debt and its current maturities, operating leases, and finance leases. Whether intentionally or not, LPs are de facto important sources of financing for the two industries.

Given the large size of LP liabilities, one might wonder how firms accumulate them. Figure 2 illustrates the operation of Delta's LP, SkyMiles, in 2019.[6] Every time a customer flies with Delta and chooses to earn Delta miles, Delta purchases miles from SkyMiles and grants to him. This part accounts for 32% of SkyMiles' cash flows. The other 68% of the cash flows are mostly due to sales to Amex, Delta's partner bank. Amex issues a collection of cobranded credit cards with Delta and grants the miles to cardholders when they sign up for or spend with the cards.

Delta provided a more detailed breakdown over time. As shown in the lower half of Figure 2,

[6]During the COVID-19 pandemic, United, Delta, and American Airlines borrowed against the cash flows of their LPs and disclosed information on the inner workings of LPs.

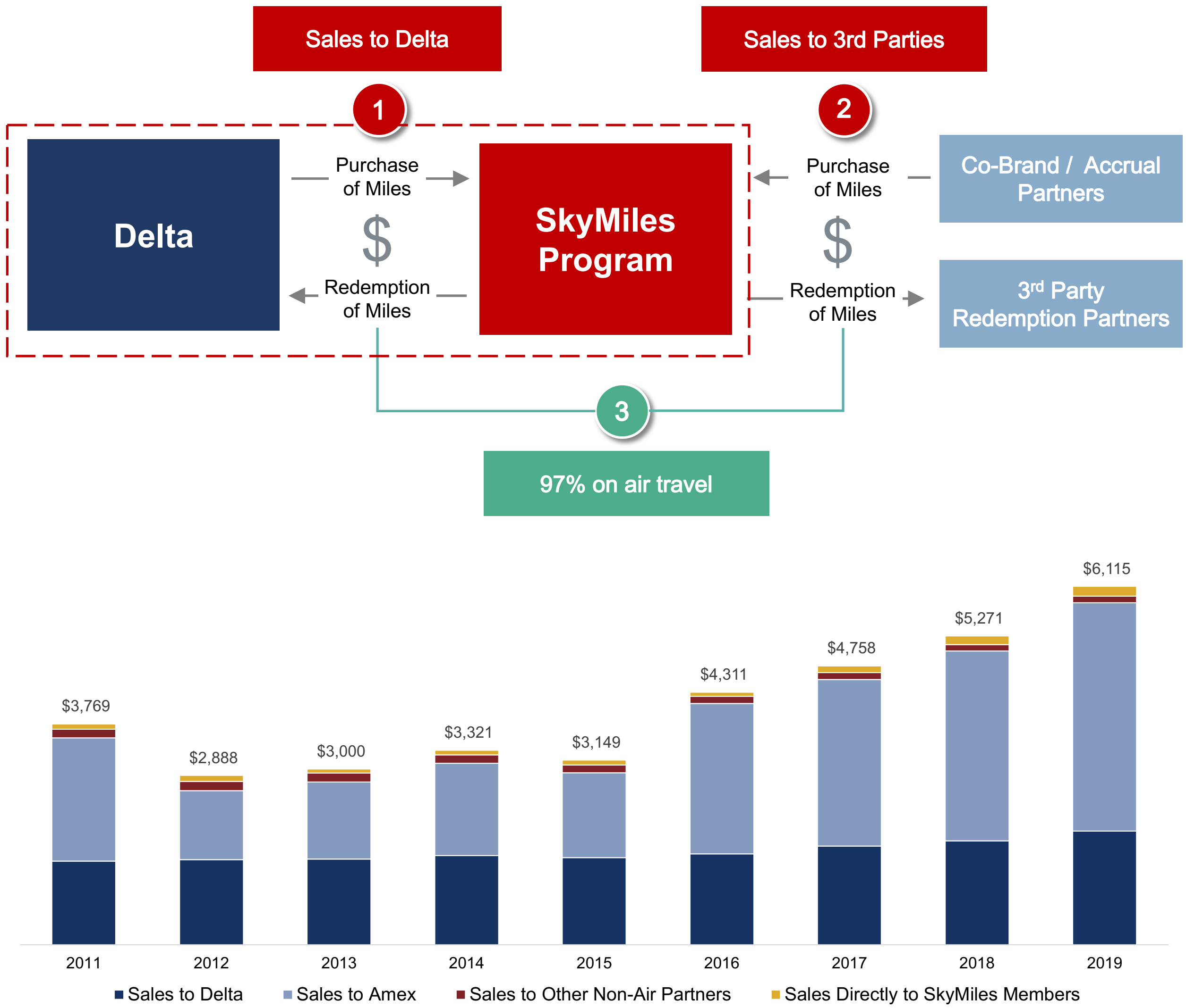

Figure 2: Delta's loyalty program

sales to Amex have been the largest source of SkyMiles' cash flows and increasingly important in recent years, followed by sales to Delta. United and American Airlines are in similar situations. In 2019, sales to third parties accounted for 71% of the loyalty program's cash flows for United and 65% for American. Such breakdown of cash flows is not available for other firms. Nevertheless, we do observe a significantly positive relationship between firms' LP liabilities and the sizes of their CCC programs.[7]

[7] We use the variety of a firm's CCCs—the number of the types of a firm's CCCs—as a proxy for the sizes for the CCC program, as we do not observe the number of cardholders or card spend. The rationale is that when a firm's CCC program is larger and has more cardholders, the firm and its partner bank will develop a richer product line to better attract and serve customers. Figure 9 shows that there is a clear positive relationship between the LP liabilities scaled by revenue and the variety of CCCs for both industries. Table 5 confirm that this positive relationship is statistically significant at 5% level.

Another noteworthy observation from Figure 2 is that sales directly to consumers are tiny and do not constitute a way to systematically issue rewards to consumers for Delta. This observation is likely true for other firms in the airline and hotel sector as well. Airlines and hotels do sell rewards directly to consumers, but they typically sell rewards at a premium rather than a discount compared with the actual values of rewards to consumers. Table 1 shows the best selling prices and valuations of rewards of major airlines and hotels as of 2024 November. The best selling prices are collected from the firms' official website,[8] and the valuations are calculated by The Points Guy (TPG).[9] It is widely acknowledged among experienced travelers that directly buying rewards is rarely a good idea for consumers, unless the firm is offering special promotions or a consumer needs only a small amount of additional rewards to reach for a good redemption deal.[10]

Table 1: The selling prices and valuations of rewards as of 2024 November

Airlines	Best selling price (¢)	TPG valuations (¢)	Hotels	Best selling price (¢)	TPG valuations (¢)
Delta	3.5	1.2	Marriott	0.93	0.85
American	2.45	1.6	Hilton	0.55	0.6
United	2	1.35	IHG	1.2	0.5
Southwest	3	1.4	Hyatt	1.92	1.7
Air Canada	2.45	1.5	Wyndham	1.3	1.1
Alaska	1.65	1.4	Choice	0.64	0.6
Frontier	2.5	1.1			
Jetblue	1.43	1.3			
Spirit	1.61	1.1			
Hawaiian	2.5	1.2			

2.3 Fact 2: rewards are issued through broad consumption but are redeemed predominantly for consumption related to the firm

CCCs usually offer the highest rewards for purchases directly made with the issuing firm, so we naturally expect that cardholders will use CCCs a lot for such purchases. In fact, consumers also use them extensively for purchases with other firms. Figure 3 shows the annual transaction volume

[8] the best selling prices may vary across time and consumers, as firms sometimes roll out promotions.

[9] The Points Guy is an American travel website and blog that produces sponsored news and stories on travel, means of accumulating and using airline points and miles, politics, and credit cards. The valuations of rewards are updated on a monthly basis at https://thepointsguy.com/loyalty-programs/monthly-valuations/.

[10] For example, see https://www.nerdwallet.com/article/travel/when-does-it-make-sense-to-buy-points-miles

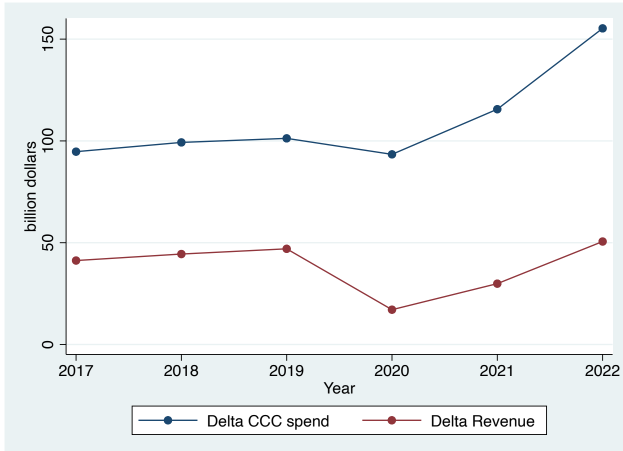

Figure 3: Delta's CCCs

of Delta's CCCs. The numbers are disclosed by Amex in its 10-K reports because Delta is its most important partner. It was 101 billion dollars and accounted for 12% of Amex US transaction volume in 2019; it later increased to 155 billion dollars and accounted for 14% of Amex US transaction volume. Meanwhile, Delta's annual revenue was close to 50 billion dollars. Also note that a large fraction of Delta's revenue is not billed to its CCCs. Therefore, a dominant fraction of the spend with Delta's CCCs was not related to Delta's business. This pattern is not unique to Delta. In 2019, the spend with American's CCCs was 109 billion dollars and accounted for 12% of Mastercard U.S. volume, while American's annual revenue was 45.8 billion dollars. Given the importance of CCC programs to LPs indicated by Fact 1, big U.S. airlines actually issue a large fraction of rewards through broad consumption, which is not necessarily related to their business. This statement will be further confirmed in the next subsection. There, we will see that the issuance of airline miles exhibit much lower correlation with the airline business than the redemption.

One may think that such issuance strategy is simply a by-product of CCCs and not intended by firms. A more direct indication of firms' intention is that they allow consumers to earn rewards on purchases with their partners. For example, consumers can earn Delta miles even without Delta CCCs, when they book hotel stays and car rental on Delta's website, shop at over 1000 retailers on SkyMiles Shopping, and dine out at over 10000 restaurants listed on SkyMiles Dining. Although issuance through hotel stays and car rental can potentially be ascribed to the synergy between different travel products, shopping and dining are not connected to Delta' business in particular ways. In fact, U.S. airlines clearly describe this issuance strategy as a key strength of their loyalty programs. For example, in its investors presentation, Delta states,[11]

"SkyMiles has an extensive network of longstanding partner relationships, allowing

[11] United and American also made similar statements.

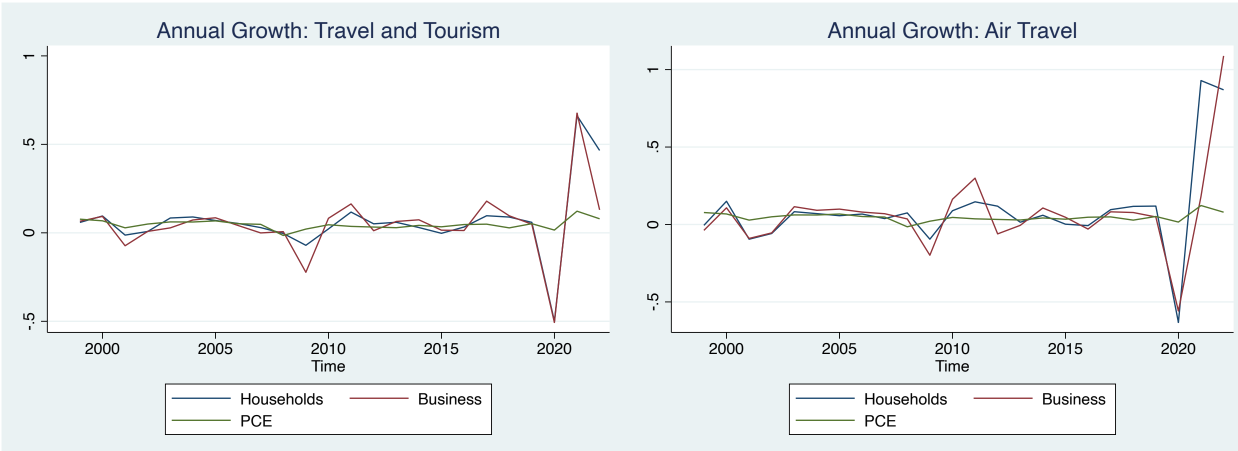

Figure 4: Volatility of travel demand and aggregate consumption

members to earn miles on car rental, hotel, retail, dining and other partners."

As for redemption of miles, around 95% of miles are redeemed for air travel for the three airlines. United further disclosed that it provided around 90% of the air travel that its miles are redeemed for.[12] The concentration of redemption on air travel is not surprising, as the value of miles is set to be the highest when they are redeemed in this way. For the same reason, we speculate that most of hotel points are redeemed for hotel stays.[13]

2.4 Fact 3: LP financing helps smooth cash flows

Travel consumption is highly volatile, more than most consumption categories. Figure 4 demonstrates the time series of the annual growth rates of US travel demand and personal consumption expenditures. US travel demand is obtained from Tourism Satellite Accounts Data of Bureau of Economic Analysis, and personal consumption expenditures are obtained from St. Louis Fed. The left panel plots the demand for the whole travel and tourism sector, while the right panel plots the demand for air travel. Clearly, both households' travel demand (blue) and business' (red) are more volatile than the aggregate consumption (green), and business' demand is even more volatile than households'.

On the one hand, most rewards are redeemed for travel services, meaning that reward redemption tends to move in tandem with travel consumption, which is highly volatile. On the other hand, reward issuance through CCC programs and partners tends to move in tandem with broad consumption, which is relatively stable. Taken together, for firms with highly volatile business such as airlines and hotels, reward issuance is less volatile than reward redemption. In fact, United reported

[12]The number increased from 86% in 2018 to 92% in 2022.

[13]In many cases, hotel rewards are directly free nights instead of hotel points that may be redeemed for other things.

that during 2008-2009 recession, its revenue declined 19% while its loyalty program revenue only declined 2%. Due to Covid-19, Delta's revenue dropped by 63.6% in 2020, but the transaction volume of its CCCs dropped only by 7.8%, as shown in Figure 3. The different cyclical nature of reward issuance and reward redemption implies that LPs help smooth cash flows: in the bad states of their business, firms can receive more cash flows through issuance than those foregone due to redemption. Notably, what drives the cyclical nature is not that firms have higher sales in bad times but that they raise more funds through LP financing, which is reflected by increases in LP liabilities.

To see more direct evidence of the cyclical nature of LP financing, we investigate how Covid-19 affects its flows and balances. Figure 5 presents the time series of the issuance and redemption of airline miles scaled by the revenue in the same year from 2018 to 2022. The airlines are placed in descending order of scaled LP liabilities in 2019, where Alaska Airlines has the largest scaled LP liabilities. For most of the 12 airlines depicted, the scaled redemption remains stable, implying that redemption of miles closely follows the overall airline business. Meanwhile, the scaled issuance experienced a pronounced spike in 2020, for the airlines with large scaled LP liabilities, e.g. the first seven airlines from Alaska to United. The logic here is that a firm issuing rewards in more diverse ways will have larger LP liabilities relative to its business, and its reward issuance is less correlated with its business.

Figure 6 plots the time series of LP liabilities. To control for the cross-sectional variations in the size of business and demonstrate the change in LP liabilities over time, we scale them by revenue in 2019. LP liabilities experienced a significant increase in 2020 for the seven airlines with large scaled LP liabilities, while this trend was not observed for other airlines. Similarly, Marriott and Hilton, which have the largest scaled LP liabilities among hotels, also saw a dramatic increase in their LP liabilities in 2020. We run OLS regressions to test this hypothesis in a more formal manner. We construct an indicator of large loyalty programs that equals 1 for the seven airlines from Alaska to United, Marriott, and Hilton, and 0 otherwise. We regress the change in scaled LP liabilities from 2019 to 2020 on the indicator. We also add the same change from 2018 to 2019 to control for potential time trends. As shown in Table 6, the coefficient of the indicator is positive at 5% significance level for both industries. Note that this analysis is essentially a joint test of two statements: 1) firms issuing rewards in more diverse ways have larger LP liabilities compared with its business, and 2) their LP liabilities increase in the bad times of their business.

One may wonder how the industries view these observations. Do they know, and do they care? Delta Airlines described one strength of its LP to investors as follows:

> "SkyMiles offers significant diversity of cash flows with a long-term track record of stable and growing performance through cycles."

United and American have almost the same description. Clearly, big U.S. airlines realize that more

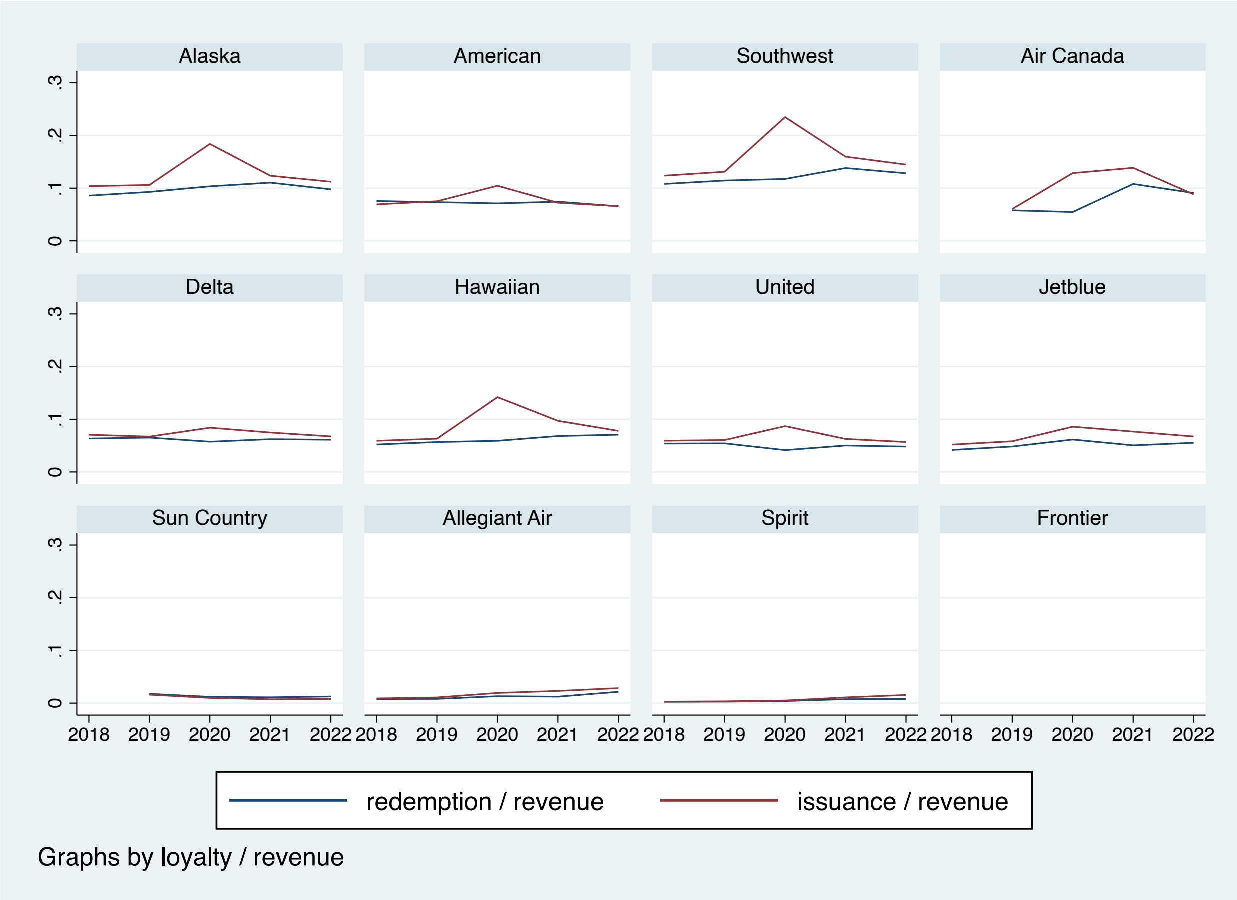

Figure 5: The time series of issuance and redemption

diverse and thus more stable issuance of miles can help them better survive business cycle fluctuation. They attempt to achieve it by expanding their partnership networks and allow consumers to earn miles on unrelated consumption.

2.5 Discussion

Traditionally, economists tend to characterize loyalty programs as issuing rewards through purchases with the issuing firm. However, it turns out that for airlines and hotels, rewards are issued through broad consumption with a large fraction not directly related to issuing firms. In addition, it is completely feasible to issue rewards through direct sales, but firms choose to discourage this type of issuance by setting high selling prices for rewards.

In fact, this issuance pattern does not align well with classical theories on loyalty programs. Under the theory of price discrimination, rewards serve as a summary of a consumer's purchase history that is informative about the consumer's relevant characteristics. A firm should not issue rewards based on broad consumption beyond purchases with the firm, unless the former contains

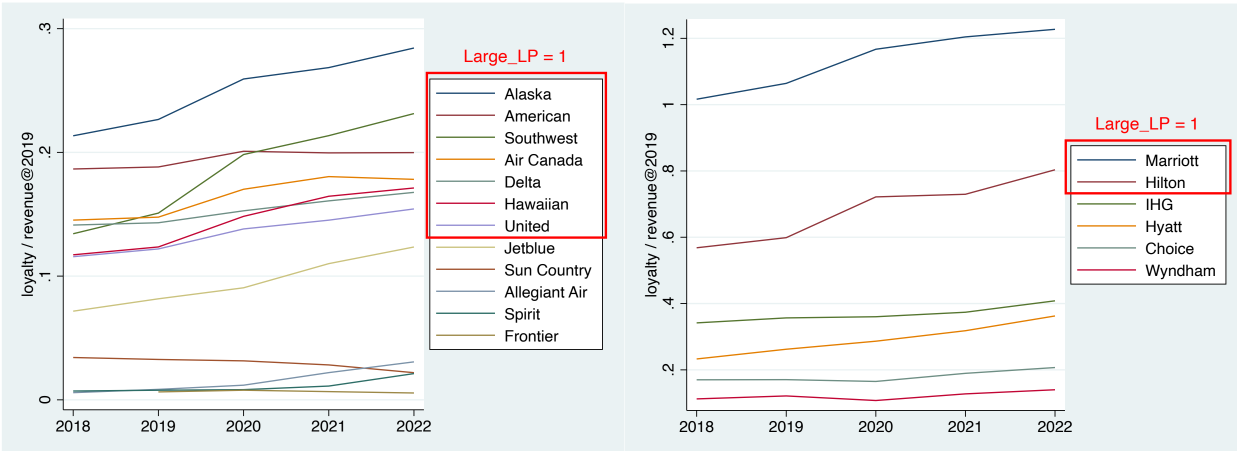

Figure 6: The time series of scaled loyalty program liabilities

more relevant information than the latter, which is a bit far-fetched. Under the theory of switching cost, rewards are used to generate or take advantage of switching cost and soften competition. The theory does not have particular implications for how rewards should be issued, so it does not explain why rewards should be issued through broad consumption and should not through direct sales at the same time. The agency theory would suggest that rewards should be issued through business consumption instead of personal consumption. Later, we show that this issuance strategy is implied by the finance theory of loyalty programs. Specifically, to leverage the financing function of LPs, a firm would find it more desirable to issue rewards through broad consumption than through purchase with the firm, and undesirable to issue rewards through direct sales.

3 The Baseline Setup

In this section, we build a dynamic model of LP financing. It is helpful to clarify that the main purpose of the model. Conceptually, LP financing hinges on that consumers discount the value of rewards at lower rates than the firm. The model provides some reasoning on why this can be true, but it is largely an empirical question that cannot be fully addressed within the model. Instead of answering why LP financing can arise, the model basically assumes it and illustrates the features of LP financing. The power of the model lies in that its predictions align well with observations in practice, particularly those that classical theories of LPs cannot explain.

3.1 Players

We consider an infinite-horizon continuous-time game. The two main players in the game are consumers and a firm, while banks play minor roles. Players are all risk neutral. The return of investment in the game is uniformly r_0, and there is no tax.

Consumers are infinitesimal and homogeneous. Their total mass is normalized to 1. Each consumer needs a service, e.g., a flight or a hotel night, at the rate of q_0. The arrival of the need is independent across consumers and over time. A service is worth p_0 to a consumer when he needs it and 0 when he does not need. Taken together, the demand for the service is $p_0 q_0$. In addition to the service, each consumer also has exogenous consumption of other goods. Each consumer's total consumption, including the service and other goods, is completely smoothed out, so his consumption flow is constant, which is C_0 per unit of time.

The firm monopolizes the supply of the service that consumers need, so the model abstracts from competition. To extract all the surplus, it sets the price of a service to p_0. To provide services, the firm needs to raise funds to buy and maintain certain assets and hire workers. Here, we do not specify the details of the firm's operation.[14] Instead, we assume that the firm's cash flows are positive in the normal state for all cases we consider, and the firm pays out the cash flows as dividend, so its balance sheet does not change. The firm's operating expense is then constant over time, which is OE_0 per unit of time. As a benchmark, we first consider the case where the firm adopts no LP and consumers can make purchases only with cash. A consumer will buy one service whenever he needs, and the firm receives p_0.

Proposition 1. *If no loyalty program is adopted, consumers buy services with cash whenever they need, and the firm's cash flows are*

$$CF_0 \triangleq p_0 q_0 - OE_0. \tag{1}$$

Banks play passive roles in the game. Each bank can issue a bank credit card (BCC) to consumers and earn interchange fees paid by merchants. Interchange fees are ϕ of the consumption.[15] For the baseline setup, we assume that banks do not charge any fee to consumers. To competitively attract consumers' usage of its BCC, a bank offers each consumer cashback as much as the fees it earns from him.

3.2 Loyalty programs

The firm can choose to adopt a LP and issue rewards. By assuming homogeneous consumers and a monopolistic supplier, the model naturally shuts down LPs' typical functions discussed in economics, marketing and operation literature, and financing is the only possible function.

[14] In Section 6, we introduce shocks to the game and consider the possibility of bankruptcy. There we specify more details.

[15] Interchanges fees are roughly 2% to 3% of the consumption in the U.S. and lower in other countries. The exact fraction varies with consumption categories and merchants.

As observed in practice, we assume that rewards are convenient to use. First, although a service is indivisible, a consumer can purchase it with any combination of cash and rewards that are worth one service. Second, unlike debt whose payout follows a fixed schedule, consumers can redeem rewards whenever they want. Consumers enjoy substantial convenience when holding rewards, which makes it more similar to holding demand deposits than doing investment. To capture the benefits of convenience, we assume that consumers treat rewards like a checking account term and do not discount its future redemption value. We do not consider the possibility that the firm issues nonconvenient rewards because consumers will require more compensation for holding rewards, making LP financing unattractive.

However, rewards are not as desirable as cash. On the one hand, cash can be redeemed for all kinds of consumption but rewards only for the service. Hence, we assume that consumers weakly prefer holding cash to holding rewards. On the other hand, redeeming rewards involves a more troublesome procedure than using cash, so each redemption incurs a *redemption cost* ε, which is smaller than p_0. Since the firm needs to compensate consumers for the use costs, rewards must be issued at a discount in equilibrium.

We assume one reward is always worth $1/p_0$ of one service. Hence, a consumer can purchase one service with x rewards and $p_0 - x$ cash for any $x \in [0, p_0]$. Essentially, the real value of one reward is 1, since one service is worth p_0 when a consumer needs it. The exact value of one reward does not matter, as the firm determines the amount of rewards issued to consumers. The essence of the assumption is that this value is constant over time.

3.3 CLPs and PLPs

Motivated by Fact 1, we consider two types of LPs: consumption-based LPs (CLPs) issue rewards through CCCs, while purchase-based LPs (PLPs) issue rewards through purchases with the firm. We assume that consumers cannot purchase rewards from the firm or transfer rewards to other consumers. In Section 5.3, we consider the impact of these kinds of transactions and discuss the observations in practice.

Adopting a CLP, the firm cooperates with a bank in issuing a cobranded credit card (CCC). For the CCC, the bank purchases rewards from the firm and offer them to consumers instead of cashback. Due to competition among banks for the cooperation, the firm receives all the fees earned by the bank from this CCC and determines rewards offered to consumers. The CCC is determined by the scope of the reward category γ and the reward rate a_c. The firm classifies a fraction γ of consumption as the reward category. For each dollar of consumption in the reward category charged to the CCC, a consumer receives a_c rewards, and 0 rewards for other consumption. In practice, a CCC typically offers multiple tiers of rewards to cardholders. For example, Delta

SkyMiles Gold Amex Card offers

- 2 miles per dollar for purchases made directly with Delta,
- 2 miles per dollar for purchases at restaurants and U.S. supermarkets, and
- 1 miles per dollar for other purchases.

The firm can determine γ by choosing the categories with high rewards, which are Delta, restaurants and supermarkets in this case; the firm can determine a_c by choosing the amount of miles consumers earn for consumption in these categories, which is 2 miles per dollar of consumption in this case. For each dollar of consumption in the reward category, consumers receive a_c rewards instead of ϕ cash. Hence, the price of a reward is

$$\theta_c \triangleq \phi / a_c.$$

Certainly, a consumer does not use the CCC for consumption not in the reward category. If he decides to use the CCC for all consumption in the reward category, he will earn rewards at the rate of $A_c \triangleq \gamma C_0 a_c$. We focus on the cases with $\gamma C_0 a_c < p_0 q_0$. This condition implies that a consumer never worries that he may accumulate too many rewards and some are left unused. It generally holds in practice, as most consumers still use cash for some purchases.[16]

Adopting a PLP, the firm offers a consumer an option to buy rewards when he makes purchases. For each dollar spent on services, a consumer can choose to buy π rewards at the unit price of θ_p. In practice, it is more common to see that the firm requires consumers to buy rewards by selling a bundle of services and rewards. For example, an air ticket is a bundle of a flight and some miles. Here, bundling and offering an option are equivalent. Consider any bundling under which a consumer can earn π' rewards for each dollar spent on services, and because of the rewards, the firm sets the price of a service to p' that is higher than p_0. The firm can always replicate this bundling by offering an option with $\theta_p = (1 - p_0/p')/\pi'$ and $\pi = \pi' p'/p_0$. We choose the language of offering an option to draw a better parallel between the two types of LPs. We focus on the cases with $\pi \leq 1$, since $\pi = 1$ is already "buy one get one free". In practice, consumers usually need to accumulate rewards through multiple purchases to get one service for free.

We say a consumer fully earn rewards, if he charges all consumption in the reward category to the CCC under a CLP or always exercises the option to buy rewards under a PLP. When we do not distinguish between the two types of LPs, we use θ to represent the unit price of rewards, and

[16]On the one hand, the issuance rate of rewards of a CCC is typically small because it is related to the rate of interchange fees and the consumption paid for with credit cards. Interchange fees are roughly 2% to 3% of the consumption. A large fraction of consumption such as rents for housing is usually required to be paid for with cash in order to avoid interchange fees. On the other hand, consumers in the model are basically frequent users of the service and thus have high expected demand.

	A: the issuance rate of rewards	θ: the unit price of rewards
CLP	$\gamma C_0 a_c$	ϕ / a_c
PLP	$\frac{p_0 q_0}{1/\pi+1}$	θ_p

Table 2: The issuance rate and the unit price of rewards

A to represent the issuance rate of rewards if a consumer fully earns rewards and redeem rewards optimally. Apparently, $A = A_c$ under a CLP. As for a PLP, Lemma 1 shows that a consumer will optimally redeem almost all his rewards. If a consumer fully earns rewards, then cash will be used to cover a fraction $\frac{1}{\pi+1}$ of the total costs, and rewards a fraction $\frac{\pi}{\pi+1}$. This implies $A = A_p \triangleq \frac{p_0 q_0}{1/\pi+1}$. Given it is either a CLP or a PLP, a LP is completely characterized by θ and A. The firm can determine them through the parameters of each type of LPs following Table 2.

3.4 The steady state

For the baseline setup, we consider that the firm can choose to adopt either a CLP or a PLP, which allows for analytical solutions of the model.[17] Also, we focus on a static LP—once chosen, its design does not change over time. Since there are infinite horizons and a consumer does not discount his future payoff, a consumer cares about his average payoff flow. Note that a consumer may not use rewards immediately after he earns them. As a result, a consumer may hold a substantial amount of rewards. Since the LP is static and does not respond to consumers' behavior, it is without loss of generality to assume that consumers play Markov strategies that depends only on their current reward holdings. Further, the state of the game can be summarized by the distribution of all consumers' reward holdings, due to homogeneous consumers. Therefore, we define the steady state under a static LP as follows.

Definition 1. In a steady state under a static LP,

- each consumer's decision maximizes his expected average payoff flow;
- the distribution of consumers' reward holdings stays unchanged over time.

For the baseline setup, we focus on equilibrium paths in steady states. In a steady state, the aggregate behavior of all consumers remains constant, and the firm's cash flows are constant. Our analysis centers around how a LP affects the constant cash flows.

[17] In Section 5.2, we numerically demonstrate the case when both types are adopted.

3.5 Discussion about model assumptions

Constant total consumption of a consumer. The essence of this assumption is that a consumer's overall consumption expenditure is stable and not strongly correlated with his idiosyncratic demand for the service. This feature holds by and large in practice. Moreover, consumer credit products such as credit card loans and buy-now-pay-later are widely used and help smooth out the consumption expenditure. Here, we assume constant total consumption for simplicity.

Zero discount rate for rewards. The essence of this assumption is that a consumer discounts the payment of rewards at lower rates than the firm. This is a simple way to generate gains from trade and commonly used in the finance and token literature (Rogoff and You, 2023). However, given that consumers can earn a fair interest rate by investing in debt, why are they willing to accept a lower rate? The model suggests that rewards can provide convenience for consumers like demand deposits. Some other factors may also affect consumers' discount rates. For example, with mental accounting, how people view rewards psychologically makes a difference; some people may gain positive utility from earning and holding rewards (Lim et al., 2021). The paper does not take a particular stance on how consumers' discount rates are determined, besides that convenience must play an important role. We assume a exactly zero discount rate instead of a merely lower one for simplicity of illustration. With a zero discount rate, a consumer will redeem for a service only when he owns at least p_0 rewards, as shown in Lemma 1. If the discount rate is positive, a consumer may redeem even when he owns slightly fewer than p_0 rewards. This difference does not affect the qualitative features of LP financing, but complicates the illustration.

Constant valuation of rewards. The model assumes constant valuation of rewards for simplicity. In practice, the values of rewards are set by firms and can change over time. Many people complained that airlines and hotels devalued their rewards. Firms benefit from devaluation as it reduces the actual payment to the existing reward holders, similar to dilution of creditors. In a model with devaluation, consumers will demand a lower unit price of rewards to offset the loss incurred by devaluation. In addition, consumers will accelerate the redemption of rewards to avoid being hurt by devaluation, leading to smaller reward balance. Both effects reduce the funds raised from consumers. Therefore, devaluation will render LP financing less attractive. In practice, several forces may restrain firms from rampant devaluation. First, devaluation hurts firms' reputation for caring about customers and especially enrages frequent customers, who tend to accumulate sizable balances. Second, devaluation compromises the attractiveness of cobranded credit cards and hurts both the firm's and its bank partner's interests. Third, devaluation at the cost of consumers will draw regulators' attention. As a response to the complains about devaluation of airline miles, the U.S. Department of Transportation (DOT) initiated an investigation into airlines' rewards practices in September 2024.

3.6 How LP financing differs from other financings

Readers may wonder how LP financing differs from other common financings. In general, effective financing should allow a firm to raise sufficient funds at low costs. Typical debt financing stipulates a fixed repayment schedule, which does not cater for investors' liquidity demand, so debt financing is associated with high required returns. Meanwhile, with the fixed repayment schedule, the firm can directly determine the raised funds. On the contrary, LP financing and demand deposits let financiers control the redemption, so they are associated with low required returns. But the drawback is that the firm loses direct control of the raised funds. They emerge endogenously as a result of the interplay between issuance and redemption.

For LP financing, the issuance is determined by the firm, and the redemption is determined by the consumption of the particular service supplied by the firm. Then there are two natural questions: how firms should issue rewards, and how the nature of the service affects financing. But the two questions are not so relevant to demand deposits, since the issuance and the redemption are exogenously determined by consumers' total income and total consumption. The model mainly addresses the two questions.

4 Model Solution

In this section, we characterize the steady state under a static CLP or PLP.

4.1 Consumers' behavior and the firm's cash flows

We first characterize consumers' behavior, assuming that they find it desirable to use the service. Since a consumer cares about his average payoff flow, a consumer's strategy can be generally characterized as earning rewards at the rate of z and redeeming x rewards for a service at the rate of y on average. Here, $0 < x \leq p_0$, since a service takes at most p_0 rewards, and $xy \leq z$ due to the budget constraint of rewards. For a service he buys with only cash, he pays p_0 cash. For a service he buys with a positive amount of rewards, he pays the redemption cost ε and $p_0 - x$ cash in addition to x rewards on average. The opportunity cost of earning rewards at the rate of z is $z\theta$. Taken together, a consumer's average payoff flow is

$$(q_0 - y)(p_0 - p_0) + y[p_0 - \varepsilon - (p_0 - x)] - z\theta = y(x - \varepsilon) - z\theta. \tag{2}$$

Given z, the average payoff flow is maximized at $x = p_0$ and $y = z/p_0$. The intuition is as follows. Each redemption incurs a redemption cost ε. Given that a consumer uses a fixed amount of rewards, he would like to use more rewards in each redemption and redeem less frequently to save redemption costs. For one service, he can redeem at most p_0 rewards, so he would like to redeem p_0 rewards whenever he decides to redeem.

On the other hand, a consumer weakly prefers holding cash than holding rewards, so with at least p_0 rewards, he must redeem p_0 rewards when he needs a service. The two observations lead to the following lemma.

Lemma 1. *A consumer redeems p_0 rewards for a service when he needs a service and have at least p_0 rewards, and does not redeem rewards otherwise.*

Lemma 1 implies that if a consumer fully earns rewards under a PLP, he will optimally redeem almost all rewards, because he never holds more than $p_0 + p_0\pi$ rewards. We then readily obtain the issuance rate of a LP as in Table 2.

Lemma 2. *If a consumer fully earns rewards, he receives rewards at a rate of A on average, where A follows the characterization in Table 2.*

Combining Lemmas 1 and 2, Proposition 2 characterizes consumers' behavior.

Proposition 2. *Suppose that the firm adopts a LP with the unit price of rewards θ and the issuance rate A. If $\theta \leq 1 - \frac{\varepsilon}{p_0}$, each consumer fully earns rewards, and when needing a service, he redeems p_0 rewards if he has at least p_0 rewards and pays p_0 cash otherwise. If $\theta > 1 - \frac{\varepsilon}{p_0}$, a consumer does not earn rewards and pays p_0 cash when needing a service.*

Under consumers' optimal redemption strategy, each redemption always involves p_0 rewards. The average benefit of a reward after the redemption cost is $1 - \frac{\varepsilon}{p_0}$. Therefore, a consumer would like to earn and redeem rewards if and only if this average benefit is not lower than the unit price of rewards:

$$\theta \leq 1 - \frac{\varepsilon}{p_0}. \tag{CC}$$

This condition is referred to as the consumer constraint.

Next, we calculate the firm's cash flows in the steady state if each consumer uses the service. Consumers as a whole redeem p_0 rewards for a service at the rate of A/p_0, and buys a service with purely cash at the rate of $q_0 - A/p_0$. The firm receives cash flows $A\theta$ from issuing rewards and $(q_0 - A/p_0)\,p_0$ from selling services to consumers. Since the distribution of consumers' reward holdings stays unchanged, consumers as a whole hold a constant reward balance. Denote it by B. Essentially, the firm pre-sells this amount of rewards to consumers and receives an amount $B\theta$ of cash in advance. With the $B\theta$ cash, the firm can reduce debt financing or increase investment by $B\theta$, which generates cash flows $r_0 B\theta$. Taken together, the firm's cash flows are

$$\begin{aligned} CF &\triangleq A\theta + \left(q_0 - \frac{A}{p_0}\right) p_0 + r_0 B\theta - OE_0 \\ &= \underbrace{r_0 B\theta}_{\text{interest from reward balance}} - \underbrace{A(1-\theta)}_{\text{discount of rewards}} + CF_0. \end{aligned} \tag{3}$$

Although the funds raised through the LP do not incur the typical interest cost, they are also costly because the firm needs to issue rewards at a discount to compensate consumers for use costs. The financing cost is effectively

$$\frac{A(1-\theta)}{B\theta} = \frac{A}{B} \cdot \frac{1-\theta}{\theta} \tag{4}$$

per unit of funds. Whether LP financing is desirable hinges on the discount at which the firm issues rewards, $1-\theta$, and the balance-to-flow ratio, B/A. LP financing is more effective if the firm can accumulate a greater reward balance given a certain issuance rate. While A and θ are chosen by the firm, B emerges endogenously in equilibrium as a result of the interplay between reward issuance and redemption. Next, I characterize the steady-state distribution of consumers' reward holdings and derive B.

4.2 Reward balance under a CLP

Suppose that the firm adopts a CLP. Let $\Gamma(N)$ be the mass of consumers holding rewards fewer than N, and $\gamma(N)$ be the density of consumers holding N rewards. Lemma 3 characterizes $\gamma(\cdot)$ in the steady state under a CLP.

Lemma 3. *In the steady state under a CLP, the density of consumers' reward holdings must satisfy the equation system*

$$\gamma'(N) = \begin{cases} \gamma(N+p_0)\frac{q_0}{A}, & \text{if } N \in [0, p_0), \\ -\gamma(N)\frac{q_0}{A} + \gamma(N+p_0)\frac{q_0}{A}, & \text{if } N \in [p_0, +\infty), \end{cases} \tag{5}$$

and three boundary conditions

$$\lim_{x \uparrow N} \gamma(x) = \lim_{x \downarrow N} \gamma(x), \forall N \tag{6}$$

$$\gamma(N) = 0, \text{ if } N < 0 \tag{7}$$

and

$$\int_0^{+\infty} \gamma(x)\, dx = 1. \tag{8}$$

The steady-state distribution of consumers' reward holdings consists of two parts. For N smaller than p_0, a consumer with N rewards does not redeem. The density is increasing in N, since a consumer with $N + p_0$ rewards may redeem once and end up with N rewards. For N greater than or equal to p_0, a consumer with N rewards will redeem if he needs services, so an attrition term appears in the equation. The boundary conditions stem from that the distribution is continuous everywhere, no consumer holds a negative number of rewards, and the total mass of consumers is 1, respectively.

Solving the equation system in Lemma 3, we explicitly characterize the steady-state distribution $\gamma(\cdot)$ and pin down the reward balance through

$$B = \int_0^\infty N\gamma(N)\, dN.$$

Proposition 3. *The steady-state distribution of consumers' reward holdings is*

$$\gamma(N) = \begin{cases} 0, & \text{if } N \in (-\infty, 0), \\ \frac{1}{p_0} \cdot \left(1 - e^{-N/\lambda}\right), & \text{if } N \in [0, p_0), \\ \frac{1}{p_0\left(\lambda \frac{q_0}{A} - 1\right)} e^{-N/\lambda}, & \text{if } N \in [p_0, +\infty), \end{cases} \tag{9}$$

where λ is the unique positive root of

$$\left(1 - e^{-p_0/\lambda}\right) \cdot \lambda = \frac{A}{q_0}. \tag{10}$$

Further, the reward balance is

$$B = \frac{p_0}{2} + \lambda, \tag{11}$$

The reward balance consists of two parts. The first part is simply a half of p_0. It stems from that consumers are not willing to redeem partially. Consider the case that consumers always need services. Their reward holdings follow a cycle, as shown in the left panel of Figure 7. Starting from 0, a consumer's reward holdings increase at a constant rate up to p_0, then consumers redeem rewards, and they drop back to 0. In this case, the average reward holdings are a half of p_0. The second part stems from that consumers do not always need services but always earn rewards. Since the issuance rate is smaller than the expected demand, i.e., $A = \gamma C_0 a_c < p_0 q_0$, a consumer never worries that some rewards are left unused, so he always fully earns rewards and accumulates rewards at the rate of A. That means, the issuance of rewards to a consumer is constant and asynchronous with his redemption of services. Therefore, a consumer may accumulate substantially more than p_0 rewards if he does not need services for a while. The right panel of Figure 7 shows a possible path of rewards balance. The second part is referred to as overaccumulation. Put intuitively, the

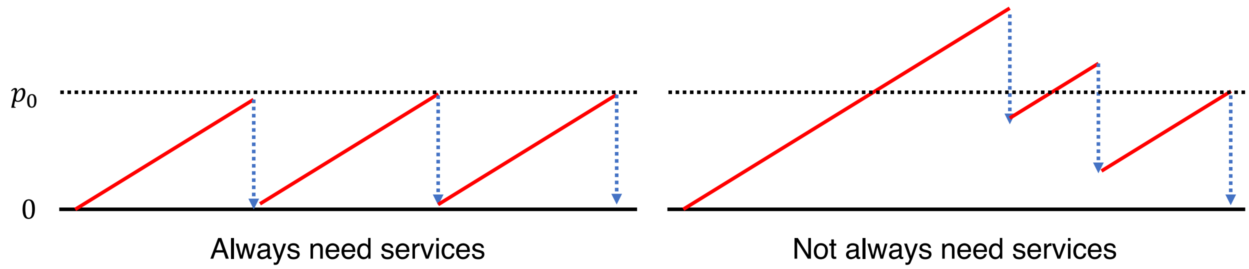

Figure 7: The dynamics of reward balance under CLPs

accumulation of reward balance relies on both consumers' unwillingness to redeem partially and the asynchrony between reward issuance and reward redemption in equilibrium.

4.3 Reward balance under a PLP

Suppose that the firm adopts a PLP. A consumer's reward holdings change only when he buys a service. We can then represent a consumer's reward holdings by a sequence $\{h_i\}_{i\in\mathbb{N}}$, where h_i is the amount between his ith purchase and his $(i+1)$th purchase. By Lemma 1, if he holds fewer than p_0 rewards, he will pay p_0 cash for the next purchase and earn $p_0\pi$ rewards; if he holds at least p_0 rewards, he will redeem p_0 rewards for the next purchase. That means,

$$h_i = \begin{cases} 0, & \text{if } i = 0, \\ h_{i-1} + p_0\pi, & \text{if } h_{i-1} < p_0, \\ h_{i-1} - p_0, & \text{if } h_{i-1} \geq p_0. \end{cases}$$

Notice that a redemption can be considered as that a consumer still earns $p_0\pi$ rewards but meanwhile gives up $p_0 + p_0\pi$ rewards. Therefore, we can write the sequence in modular arithmetic as follows:

$$h_i = i \cdot p_0\pi \pmod{(p_0 + p_0\pi)}.$$

Since the need for a service arrives independently over time, a consumer stays at each point of the sequence for the same expected length of time. This implies that the reward balance in the steady state equals the average of the sequence. Bases on this formulation, Proposition 4 characterizes the steady-state distribution of consumers' reward holdings and the reward balance.

Proposition 4.
- *If $1/\pi$ is a rational number, let m and n be the positive coprime integers such that $\mu/n = (1+\pi)/\pi$. Consumers are uniformly distributed over the set $\left\{\frac{i}{\mu}(p_0 + p_0\pi)\right\}_{i=0}^{\mu-1}$ in terms of reward holdings.*
- *If $1/\pi$ is an irrational number, consumers are uniformly distributed over the interval $[0, p_0 + p_0\pi)$*

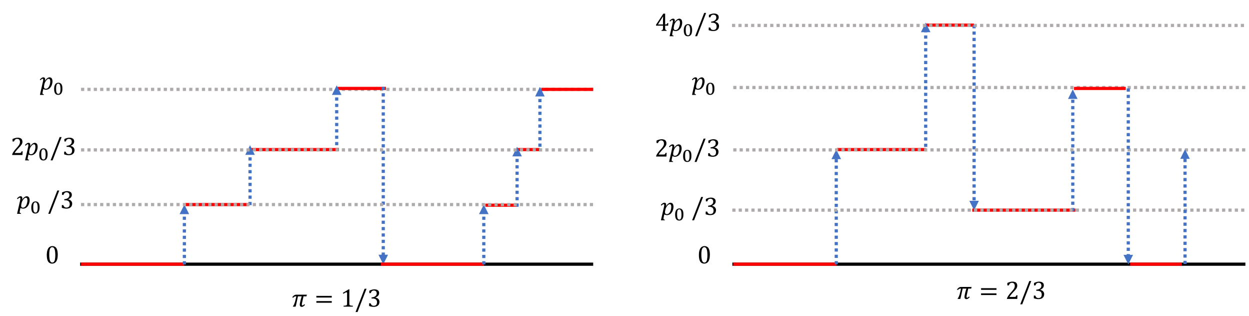

Figure 8: The dynamics of reward balance under CLPs

in terms of reward holdings.

- *Let* μ *be* $+\infty$*, if* $1/\pi$ *is an irrational number. The reward balance is*

$$B = \frac{\mu - 1}{2\mu}\left(1+\pi\right)p_0, \tag{12}$$

which must be in the interval $\left[p_0/2, p_0\left(1+\pi\right)/2\right]$

The intuition of Proposition 4 is as follows. We start with the case that $1/\pi$ is an integer. As demonstrated in the left panel of Figure 8, a consumer's reward holdings follow a cycle:

- Starting with zero rewards, a consumer will pay for the next $1/\pi$ purchases with cash, so his reward holdings consistently increase as he make purchases until he accumulates p_0 rewards.

- He will then redeem all his rewards for the next purchase, and his reward holdings go back to 0.

In the steady state, all consumers are uniformly distributed over $\{0, p_0\pi, 2p_0\pi, \ldots, p_0 - p_0\pi, p_0\}$ in terms of reward holdings, and the reward balance is $p_0/2$. This steady-state distribution is driven by two forces. First, a consumer is not willing to redeem partially. Second, his reward holdings never exceed p_0. Since $1/\pi$ is an integer, his reward holdings will exactly hit p_0 when they reach or exceed p_0, and he will then redeem rewards for the next purchase instead of earning more.

When $1/\pi$ is not a integer, the case becomes more complicated. Under consumers' optimal strategy, reward earning is lumpy whenever it occurs, which is $p_0\pi$. Due to the lumpiness, consumer's reward holdings may exceed p_0. For example, as demonstrated in the right panel of Figure 8, if $\pi = 2/3$, a consumer's reward holdings are $4p_0/3$ when they first reach or exceed p_0; moreover, when the consumer redeems p_0 rewards for the next purchase, his reward holdings decrease to a positive amount instead of 0. To characterize the sequence of reward holdings in a general way, the formal proof resorts to simple number theory.

The reward balance is generally characterized by equation (12). Similar to what under CLPs, the reward balance is at least $p_0/2$, because of consumers' unwillingness to redeem partially. Over-accumulation arise when $1/\pi$ is not a integer, due to the lumpiness of reward earning discussed above, and the size of the overaccumulation is capped by the half of each reward earning, $p_0\pi/2$.

5 Implications About LP Financing

As discussed above, the convenience of rewards renders the reward balance endogenous, and the desirability of LP financing depends on the balance-to-flow ratio, B/A. In this section, we derive the implications about LP financing based on this feature.

5.1 The firms' characteristics

In the model, the firm is characterized by the value of a service p_0 and the frequency of the need for a service q_0. We examine how the desirability of LP financing depends on these characteristics. To control for the size of the firm's business, we fix the demand for the service p_0q_0 and vary p_0 (q_0). Intuitively, we are comparing firms providing high-value, low-frequency services with those providing low-value, high-frequency services.

Proposition 5. *Given the demand for the service and the issuance rate, the reward balance is greater under both CLPs and PLPs if p_0 is higher and q_0 is lower.*

For both types of LPs, the reward balance consists of one half of p_0 and overaccumulation. Under a CLP, the overaccumulation, λ, is determined by equation (10). It is decreasing in p_0, as each redemption reduces the balance by p_0. It is decreasing in q_0, as less frequent demand means that the likelihood that a consumer does not need a service for a long time is greater. When p_0q_0 is held fixed, a higher p_0 is associated with a lower q_0, and the impact of q_0 turns out to dominate.

Under a PLP, the overaccumulation derives from and increases with the lumpiness of reward earning, which is $p_0\pi$. By $A = \frac{p_0q_0}{1/\pi+1}$, π is fixed given p_0q_0 and A. Hence, a higher price leads to greater lumpiness.

In fact, Proposition 5 aligns well with empirical observations across different industries. Table 3 shows the balances and flows of LPs for firms in different industries. Marriott and Delta provides upscale travel services and have the highest balance-to-flow ratios. Macy's, Target, GAP and ULTA Beauty retail middle-end goods and have lower but still sizable balance-to-flow ratios. TJX is an off-price retailer, and Starbucks mainly sells coffee and bakery, which features low-value and high-frequency. No wonder they have very low balance-to-flow ratios.

Firms in 2019	B (deferred revenue*)	A (deferral of revenue*)	B/A (balance-to-flow ratios)
Marriott	5718	2468	2.32
Delta	6728	3156	2.13
Macy's	839	554	1.51
Target	935	729	1.28
GAP	226	187	1.21
ULTA Beauty	230	206.7	1.11
TJX	501	1690	0.30
Starbucks	1114	10984	0.10

*in million dollars

Table 3: The balances and flows of LPs

Proposition 5 suggests that firms supplying high-value, low-frequency services can leverage LPs more effectively for financing. Such firms can borrow more from consumers without compromising the convenience of rewards or incurring more redemption costs. In practice, airlines, hotels, and department stores provide high-value, low-frequency services. Consistent with the model prediction, they are the most active in developing LPs and expanding the issuance of rewards.

Remark 1. The firm's characteristics potentially affect the redemption cost as well. A lower redemption cost allows for a higher price of rewards, making LP financing more attractive. Unfortunately, we do not have clear observation regarding redemption costs across firms. A conjecture that is reasonable in general would be that the redemption cost is concave in p_0. The concavity can result from common cost structures in economics. For example, the redemption cost consists of a fixed part and a variable part that is proportional to the value of a service. Under this conjecture, ε/p_0 is decreasing in p_0. That means, the consumer constraint is more relaxed if p_0 is higher and q_0 is lower.

5.2 Ways to issue rewards

We compare the two types of LPs with respect to the reward balance.

Proposition 6. *Given the issuance rate, a CLP results in a greater reward balance than a PLP.*

Under a CLP, overaccumulation arises due to the asynchrony between reward issuance and reward redemption—a consumer consistently earns rewards, even during the period that he does not need a service. However, such asynchrony is missing under a PLP, because issuance and

A_p	200	199	150	120	100	80	50	0
A_c	0	1	50	80	100	120	150	200
B	150	180	184	186.6	188	191	194	200

Table 4: The reward balance when both types are adopted

This table presents the reward balance predicted by the model when both types are adopted. A_c and A_p are the issuance rates under the CLP and the PLP respectively. For all calculation, $p_0 = 300$, $q_0 = 4$, and the total issuance rate is set to 200.

redemption are both linked to purchases of services. Opportunities to earn rewards and redeem rewards are synchronous and exclusive. This is why overaccumulation is zero when $1/\pi$ is an integer. A consumer with p_0 rewards will redeem rewards instead of earning more. When $1/\pi$ is not an integer, overaccumulation arises due to the lumpiness of reward earning and could reach $p_0\pi/2$ at maximum. Proposition 6 rests on that the asynchrony under a CLP is more powerful in generating overaccumulation than the lumpiness under a PLP.

Numerical analysis suggests that such superiority of CLPs over PLPs also holds when the firm adopts both types simultaneously. Suppose that the issuance rates under the CLP and the PLP are A_c and A_p respectively. Similar to Lemma 3, the steady-state distribution of consumers' reward holdings satisfies the equation system

$$\gamma'(N) = \begin{cases} -\gamma(N)\frac{q_0}{A_c} + \gamma(N+p_0)\frac{q_0}{A_c}, & \text{if } N \in [0, p_0\pi), \\ -\gamma(N)\frac{q_0}{A_c} + \gamma(N+p_0)\frac{q_0}{A_c} + \gamma(N-p_0\pi)\frac{q_0}{A_c}, & \text{if } N \in [p_0\pi, p_0+p_0\pi), \\ -\gamma(N)\frac{q_0}{A_c} + \gamma(N+p_0)\frac{q_0}{A_c}, & \text{if } N \in [p_0+p_0\pi, +\infty), \end{cases}$$

and three boundary conditions in Lemma 3. We numerically calculate the steady-state distribution and the reward balance. We set p_0 to 300, q_0 to 4, and the total issuance rate to 200. Table 4 presents the reward balance for different combinations of A_c and A_p. Clearly, given the total issuance rate, as more rewards are issued through the CLP and fewer through the PLP, the reward balance increases.

Combining Proposition 6 and the numerical analysis, we obtain a general implication regarding reward issuance. Redemption occurs only when a consumer purchases a service, so it is naturally tied to the firm's business. A weaker linkage between reward issuance and the firm's business will render issuance less synchronous with redemption. Therefore, to leverage LP financing more effectively (i.e., a higher balance-to-flow ratio), the firm should weaken the linkage of reward issuance to the firm's business. This implication is consistent with Fact 2 in Section 2 that in practice, rewards are issued through broad consumption, including that unrelated to the firm's business.

5.3 Transactions of rewards

For the baseline setup, we assume that consumers cannot obtain rewards in ways other than the two types of LPs, either from the firm or from other consumers. In this subsection, we consider the impact of these alternative ways and discuss the observations in practice. Specifically, we analyze two benchmark cases. The first is that consumers can purchase any amount of rewards from the firm whenever they want, which we refer to as free purchase. The unit price of rewards in a free purchase follows a fixed price determined by the firm, $\hat{\theta}$. The second is that consumers can transfer any amount of their rewards to others at an agreed price, and such transfer does not incur additional costs, which we refer to as free transfer.

Proposition 7. *Free purchase is always undesirable for the firm.*

Proposition 7 rests on two points. First, free purchase increases the total discount of rewards enjoyed by consumers. A consumer chooses to do free purchase only when the price of free purchase $\hat{\theta}$ is so low that redeeming the rewards purchased through free purchase is a better deal than using cash. In that case, a consumer redeems rewards for all services he needs. This channel always lowers the firm's cash flows.

Second, free purchase does not increase the raised funds and actually decreases them when a LP is adopted. Due to the preference for holding cash over holding rewards, a consumer purchases rewards through free purchase only when he needs a service and has fewer than p_0 rewards, and he redeems the rewards immediately. Hence, the firm does not raise funds from the rewards sold through free purchase. If no LP is adopted or a LP is adopted but consumers choose not to earn any reward from the LP, the reward balance will be exactly 0. If consumers choose to earn rewards from the LP, free purchase will accelerate reward redemption: when needing a service, a consumer with fewer than p_0 rewards will get the rest through free purchase and redeem all rewards. The more frequent redemption dampens the accumulation of a consumer's reward holdings.

Taken together, we obtain that allowing for free purchase is always undesirable for the firm. This prediction is consistent with Fact 1 in Section 2 that firms discourage consumers from directly buying rewards by charging high prices, and sales directly to consumers do not serve as an important way to issue rewards.

Proposition 8. *Free transfer renders the reward balance 0 and is always undesirable for the firm.*

Proposition 8 follows a simple observation. If free transfer is allowed, the rewards issued at each instant will be transferred to the consumers who need services and redeemed immediately. This type of transactions is desirable for consumers because it reduces consumers' reward holdings without reducing their payoffs. As a result of free transfer, a consumer holds 0 rewards on average. That means, free transfer will completely eliminate the financing function of LPs. This observation resonates with Rogoff and You (2023). In the setting of platforms issuing tokens, they find that platforms can benefit from making tokens non-tradable unless they can generate a sufficiently high outside-platform convenience yield.

In practice, airlines and hotels discourage transfers of rewards between consumers in general and want to restrict the transfers to being between family members and friends. Some firms directly impose pecuniary costs on transfers. For example, for each transfer of miles, Delta charges 1 cent per mile plus a fixed processing fee of $30, and United charges 1.5 cent per mile plus a fixed processing fee of $30. That means, the cost of transferring miles is actually close to the redemption value of the transferred miles shown in Table 1. Some firms restrict the scope or the frequency of transfers. For example, Cathay Pacific Airlines asks a consumer to nominate up to five individuals to his Redemption Group and does not allow him to transfer miles to or redeem his rewards for people outside the group. Marriott imposes restrictions on the frequency of transfers. Some firms clearly require that transfers not be used for commercial purposes. For example, the terms and conditions of American Airlines' LP—AAdvantage program—has the following content:

> "At no time may AAdvantage mileage credit or award tickets be purchased, sold or bartered (including but not limited to transferring, gifting, or promising mileage credit or award tickets in exchange for support of a certain business, product or charity and/or participation in an auction, sweepstakes, raffle or contest). Any such mileage or tickets are void if transferred for cash or other consideration. Violators (including any passenger who uses a purchased or bartered award ticket) may be liable for damages and litigation costs, including American Airlines attorneys' fees incurred in enforcing this rule."

6 The Cyclical Nature of LP Financing

In this section, we examine the cyclical nature of LPs and its impact on the firm's financial resilience. We refer to the state in the baseline setup as the normal state. The economy may enter an adverse state in which consumers' demand for the service drops dramatically. The firm's cash flows cannot cover its expenses, so it has to expend liquidity buffers to maintain the business. If the liquidity buffers are depleted before the economy returns to the normal state, the firm will go bankrupt and shut down the business. Since bankruptcy is value-destroying, the firm's financial resilience in the adverse state is valuable. In practice, the firm may raise funds from outside investors to obtain additional liquidity. But such emergency financing is far from perfect, due to various financial frictions, especially in adverse states of the economy. This leaves the room for LP financing to make improvement. We will illustrate that LP financing helps smooth the firm's cash flows and improve its financial resilience.

6.1 The adverse state

To consider the possibility of bankruptcy, we need more details about the firm's balance sheet and operation in the normal state. To provide services, the firm needs to hold an illiquid asset and operate it at a flow cost of O_0. In practice, the flow cost contains depreciation of equipment, employees' compensation, and any other operating expense. Purchasing and deploying the illiquid asset incurs a one-time cost I_0 to the firm. Since the variable cost of providing one service must be smaller than consumers' reservation value p_0, it does not affect the firm's decision in the model. Hence, we assume it is zero. The firm also holds liquidity buffers, L_0, and earns interest at the rate of r_0. Liquidity buffers are used for daily operation as well as to help withstand temporary operating loss in adverse states. The firm raises funds through debt and potentially through LPs. We assume zero equity for simplicity. Debt requires a coupon rate of r_0.[18] I_0, O_0, L_0, and r_0 are exogenously given. Since the financial cost of the liquidity buffers equal their interest income, the operating expense in the baseline setup equals the operating and financing cost of the illiquid asset:

$$OE_0 = O_0 + r_0 I_0.$$

The firm's cash flows are positive in the normal state. It pays out the cash flows as dividend and maintain the balance sheet.

Suppose that the economy is in the normal state, and the firm is in the steady state, whether it adopts a LP. At $t = 0$, the economy is hit by an unexpected shock, for example 9-11 or Covid-19, and enters an adverse state. A fraction σ of consumers are affected by the shock: they do not need the service, and their consumption in all categories decreases to δC_0, where $\delta \in (0,1]$. Other consumers remain unchanged. I refer to σ as the magnitude of the shock. The economy will return to the normal state at some unknown points. The assumption about the adverse state implies that the firm's business is volatile: the demand for the service decreases by a fraction of σ, while the aggregate consumption decreases by a fraction of $\sigma(1-\delta)$.

The shock σ is so large that the firm's cash flows are negative in all cases we consider. It enters the adverse state with the liquidity buffers L_0 and goes bankrupt if the buffers are depleted. To avoid costly bankruptcy, the firm pays no dividend and rollovers all debt. To manifest the impact of the LP and simplify the illustration, we assume that the firm can respond to the shock only in a limited way. It cannot change operating expense or the design of the LP, which we refer to as operational inflexibility. It can roll over existing debt but cannot issue new debt, which we refer to as financial constraint.

Consumers' behavior does not change and still follows the characterization in Proposition 2.

[18] We assume equal interest rates on liquidity buffers and debt purely for simplicity. All results remain if they are different.

This assumption is motivated by the observation that during the Covid pandemic, although consumers did not fly much with airlines and airlines suffered tremendous losses, they did not change the use of airlines' CCCs significantly. There are two potential reasons for the absence of changes. First, consumers are not concerned about the firm's bankruptcy that much. Airline bankruptcies are common in history. Big bankrupt airlines usually emerged from Chapter 11 and kept their LPs intact so that LP members would like to continue flying with the new firm. Second, consumers regard the adverse state as temporary and expect their demand for flying to bounce back.

6.2 The impact of LP financing

We show that LP financing makes the firm's cash flows less volatile and thus makes the firm more financially resilient. The formal result is stated as follows.

Proposition 9. *If adopting an LP leads to weakly higher cash flows than adopting no LP in the normal state, it also leads to a strictly lower bankruptcy probability than adopting no LP in the adverse state.*

We first analyze the case where the firm adopts no LP. The firm borrows $I_0 + L_0$ debt, and the debt coupon is

$$r_0 (I_0 + L_0).$$

In the adverse state, the firm's cash flows from sales dwindle to

$$(1 - \sigma) p_0 q_0.$$

The firm's negative cash flows translate into decreases in the liquidity buffer. Denote the liquidity buffers at t by $L_0(t;\sigma)$. Then its dynamics are characterized by the following differential equation:

$$\frac{dL_0(t;\sigma)}{dt} = (1-\sigma) p_0 q_0 - O_0 - r_0 (I_0 + L_0) + r_0 L_0 (t;\sigma),$$

where $r_0 L_0(t;\sigma)$ is the interest income on the liquidity buffers at t. With the boundary condition $L_0(0;\sigma) = L_0$, we obtain the liquidity buffers at t as follows:

$$L_0(t;\sigma) = L_0 e^{r_0 t} + \frac{e^{r_0 t} - 1}{r_0} \left[(1-\sigma) p_0 q_0 - O_0 - r_0 (I_0 + L_0) \right].$$

We then analyze the case where the firm adopts a LP. To incorporate all cases in one framework, we consider that the firm adopts a CLP and a PLP simultaneously. Their issuance rates are A_c and A_p respectively, the unit price of rewards is θ, and the combined reward balance is B. When either of A_c and A_p is zero, the LP degenerates to a pure CLP or a pure PLP.

The firm receives $B\theta$ through the LP and borrows $I_0 + L_0 - B\theta$ debt, so the debt coupon is

$$r_0 (I_0 + L_0 - B\theta).$$

In the adverse state, the firm's cash flows from sales dwindle to

$$(1-\sigma)\left[p_0 q_0 - (A_c + A_p)(1-\theta)\right] + \sigma\delta A_c\theta.$$

Only the consumers not affected by the shock buy services, which generates the cash flows equal to the first term. The consumers affected by the shock still use the CCC and earn rewards at the rate of δA_c, which generates cash flows equal to the second term. Denote the liquidity buffers at t by $L(t;\sigma)$. Then its dynamics are characterized by the following differential equation:

$$\begin{aligned}\frac{dL(t;\sigma)}{dt} = &(1-\sigma)\left[p_0 q_0 - (A_c + A_p)(1-\theta)\right] + \sigma\delta A_c\theta \\ &- O_0 - r_0 (I_0 + L_0 - B\theta) + r_0 L(t;\sigma),\end{aligned}$$

We then obtain the liquidity buffers at t as follows:

$$L(t;\sigma) = L_0(t;\sigma) + \frac{e^{r_0 t} - 1}{r_0}\left[r_0 B\theta - (1-\sigma)(A_c + A_p)(1-\theta) + \sigma\delta A_c\theta\right].$$

Note that the difference in the cash flows between adopting the LP and not in the normal state is

$$CF - CF_0 = r_0 B\theta - (A_c + A_p)(1-\theta).$$

We can write the liquidity buffers at t as

$$\begin{aligned}L(t;\sigma) = &\frac{e^{r_0 t} - 1}{r_0}\sigma (A_c + A_p)(1-\theta) + \frac{e^{r_0 t} - 1}{r_0}\sigma\delta A_c\theta \\ &+ \frac{e^{r_0 t} - 1}{r_0}(CF - CF_0) + L_0(t;\sigma).\end{aligned} \tag{13}$$

The first two terms in the right-hand side of equation (13) reflect the two mechanisms affecting the cyclical nature of LP financing. The first term stems from that the cost of LP financing is lower in the adverse state. LP financing is costly because consumers use rewards to buy services instead of cash, and rewards are issued at a discount. Given the unit price of rewards, the total discount on rewards that consumers actually enjoy is proportional to the redemption rate of rewards, which is lower in the adverse state due to the lower demand for the service. An alternative way to view this is that a LP is de facto a revenue-sharing arrangement—a fixed fraction of revenue is shared with consumers in the form of discounts on rewards. When revenue drops, so does the shared revenue.

The second term stems from that the firm raises more funds from consumers in the adverse state. Under the assumption that consumers do not change their behavior, reward issuance is always linked to consumers' aggregate consumption while reward redemption is linked to their demand for the service. Then for firms with volatile business analyzed here, reward issuance is less volatile than reward redemption, and the former becomes higher than the latter in the adverse state. Such divergence in the two rates results in the firm raising more funds from consumers, which is reflected by the reward balance increasing at the rate of $\sigma\delta A_c$. This mechanism is consistent with Fact 3 in Section 2 that LP financing helps smooth firms' cash flows.

If the difference in the cash flows in the normal state is nonnegative, i.e., $CF - CF_0 \geq 0$, the liquidity buffers under the LP are strictly higher than those under no LP. This implies that adopting the LP leads to a strictly lower bankruptcy probability, so we obtain Proposition 9. This cyclical nature makes LP financing attractive to firms with highly volatile business such as airlines, hotels, upscale department stores, and durable goods retailers. Specially, airlines feature heavy assets, high financial expenses, and low profit margins. Highly volatile business plus financial weakness makes airlines particularly eager to utilize LP financing.

Proposition 10. *Given the total issuance rate $A_c + A_p$, the higher the issuance rate under the CLP is, the less sensitive the liquidity buffers are to the magnitude of the shock; i.e.,*

$$\frac{\partial^2 L(t;\sigma)}{\partial\sigma\partial A_c} > \frac{\partial^2 L(t;\sigma)}{\partial\sigma\partial A_p}$$

Proposition 10 suggests that given the total issuance rate, more issuance through the CLP strengthens the cyclical nature. The reason is that the second mechanism is unique to the CLP. The issuance under the CLP is not linked to purchases, while the issuance under the PLP is. As a result, the former does not decrease with the demand for the service, while the latter does. A more general implication is that to better leverage the cyclical nature of LP financing, firms should weaken the correlation between reward issuance and their business. This implication aligns well with Fact 2 in Section 2 that in practice, rewards are also issued through broad consumption, including that unrelated to firms' business.

7 Conclusion

This paper demonstrates that loyalty programs can be used and are de facto used by some firms to raise funds from consumers. With this starting point, the paper mainly addresses two questions. First, what firms should do LP financing? They are firms supplying high-value, low-frequency services and firms with highly volatile business. Second, How can firms leverage LP financing more effectively? They should have more diverse issuance channels and weaken the linkage of reward issuance to the firm's business, and they should not give consumers too much freedom in purchasing rewards from the firm or from other consumers. These theoretical predictions are consistent with observations in practice.

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Appendix

A Additional Tables & Figures

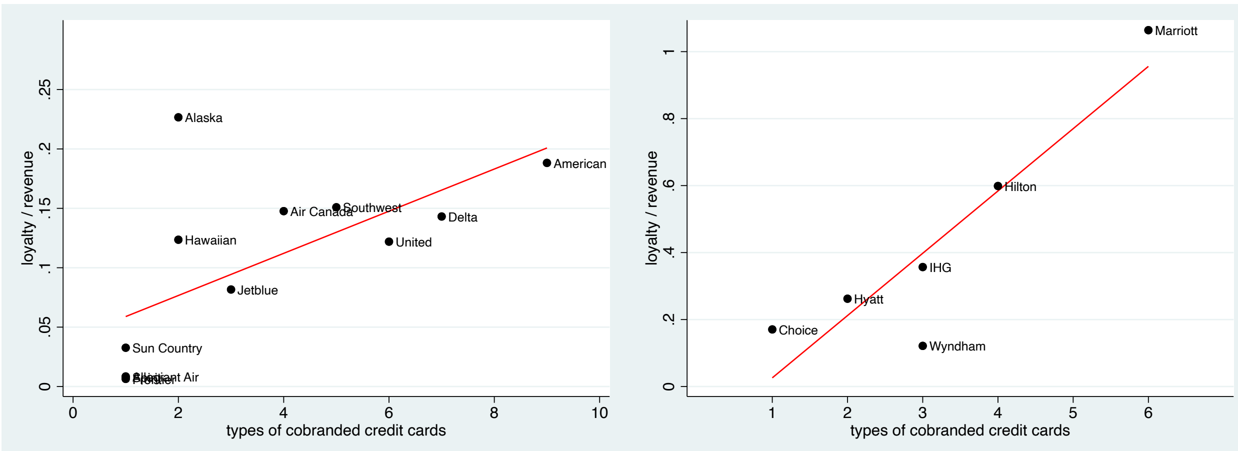

Figure 9: LP liabilities and varieties of CCCs

Table 5: LP liabilities and varieties of CCCs

	loyalty / revenue	
	(1)	(2)
	Airlines	Hotels
Varieties of CCCs	0.018**	0.186**
	(0.007)	(0.044)
Constant	0.041	-0.160
	(0.029)	(0.155)
Observations	12	6
R-squared	0.411	0.819

Table 6: The change in LP liabilities during Covid

	Change in loyalty / revenue from 2019 to 2020			
	(1)	(2)	(3)	(4)
	Airlines	Airlines	Hotels	Hotels
Large_LP	0.021***	0.014***	0.111***	0.096**
	(0.006)	(0.004)	(0.014)	(0.022)
Change in loyalty / revenue		1.727***		0.590
from 2018 to 2019		(0.315)		(0.669)
Constant	0.003	-0.002	0.002	-0.006
	(0.005)	(0.003)	(0.008)	(0.012)
Observations	12	11	6	6
R-squared	0.546	0.896	0.942	0.954

B Proofs

Proof of Lemma 1

Suppose that a consumer does not follow the strategy in Lemma 1 and use a combination of cash and rewards with positive probability. That implies $y > 0$ and $x < p_0$. If $x \le \varepsilon$, then $y(x-\varepsilon) - z\theta < 0$, so the consumer is better off if he does not join the LP and buy a service with only cash. If $x > \varepsilon$, consider an alternative strategy that he still receives rewards at the rate of z, and when needing a service, he buys it with only rewards if he has at least p_0 rewards and with only cash otherwise. Under this strategy, he redeems p_0 rewards for a service at the rate of z/p_0. Then his average payoff flow is

$$\frac{z}{p_0}(p_0 - \varepsilon) - z\theta.$$

By $\varepsilon < x < p_0$ and $xy \le z$,

$$\frac{z}{p_0}(p_0 - \varepsilon) - z\theta > \frac{z}{x}(x - \varepsilon) - z\theta \ge y(x - \varepsilon) - z\theta.$$

Hence, the consumer is better off by using the alternative strategy under which he buys a service with either only cash or only rewards.

Proof of Lemma 3

Denote the probability that a consumer needs exactly one service over t units of time as $P_1(t)$ and the probability that a consumer needs at least one service over t units of time as $P_0(t)$. We first show that $P_1(t) = q_0 t e^{-q_0 t}$, $P_0(t) = 1 - e^{-q_0 t}$, and $P_0(t) - P_1(t) = o(t)$.

$P_0(t)$ satisfies

$$\begin{aligned} P_0(t+dt) &= P_0(t) + (1 - P_0(t)) P_0(dt) \\ \Rightarrow -\frac{d \ln(1 - P_0(t))}{dt} &= \frac{P_0(dt)}{dt} = P_0'(0) = q_0. \end{aligned}$$

Combining with the boundary condition $P_0(0) = 0$, we obtain $P_0(t) = 1 - e^{-q_0 t}$. $P_1(t)$ satisfies

$$\begin{aligned} P_1(t+dt) &= P_1(t)(1 - P_0(dt)) + (1 - P_0(t)) P_1(dt) \\ \Rightarrow P_1(t+dt) - P_1(t) &= -P_1(t) q_0 dt + e^{-q_0 t} P_1(dt) \\ \Rightarrow \frac{dP_1(t)}{dt} &= -P_1(t) q_0 + e^{-q_0 t} q_0 \\ \Rightarrow \frac{d e^{q_0 t} P_1(t)}{dt} &= q_0 \end{aligned}$$

Combining with the boundary condition $P_1(0) = 0$, we obtain $P_1(t) = q_0 t e^{-q_0 t}$. $P_0(t) - P_1(t)$ satisfies

$$\begin{aligned}
P_0(t) - P_1(t) &= 1 - e^{-q_0 t} - q_0 t e^{-q_0 t} \\
&= 1 - (1 + q_0 t)\left[1 - q_0 t + \sum_{k=2}^{+\infty} \frac{1}{k!}(-q_0 t)^k\right] \\
&= (q_0 t)^2 - (1 + q_0 t)\sum_{k=2}^{+\infty} \frac{1}{k!}(-q_0 t)^k.
\end{aligned}$$

Next, we prove the equation system. For a small τ, let $\Delta\Gamma(N + A\tau) \triangleq \Gamma(N + A\tau) - \Gamma(N)$ be the mass of consumers holding rewards between N units and $N + A\tau$ units at t. These consumers consist of three groups as follows.

- The first group is the consumers who do not redeem rewards for any service during the past τ units of time. Since they are always accumulating rewards at the rate of A, they must hold rewards between $N - A\tau$ units and N units at $t - \tau$.
- The second group is the consumers who redeem rewards for one service during the past τ units of time. They must hold rewards between $N + p_0 - A\tau$ units and $N + p_0$ units at $t - \tau$.
- The third group is the consumers who redeem rewards for more than one service during the past τ units of time and end up holding rewards between N units and $N + A\tau$ units at t.

For $N \in [0, p_0 - A\tau)$, consumers holding rewards between $N - A\tau$ units and N units at $t - \tau$ do not have sufficient rewards to redeem for one service by t, so all of them belong to the first group. On the other hand, consumers holding rewards between $N + p_0 - A\tau$ units and $N + p_0$ units at $t - \tau$ have needs for services with probability $1 - e^{-q_0\tau}$. Since $N + p_0 + A\tau < 2p_0$, these consumers have sufficient rewards to redeem for only one service, so all the fraction $1 - e^{-q_0\tau}$ of them belong to the second group. The mass of the third group is of a higher order than τ. Hence, we obtain the following equation:

$$\Delta\Gamma(N + A\tau) = \Delta\Gamma(N) + \Delta\Gamma(N + p_0)\left(1 - e^{-q_0\tau}\right) + o(\tau).$$

For $N \in [p_0 + A\tau, +\infty)$, consumers holding rewards between $N - A\tau$ units and N units at $t - \tau$ have sufficient rewards to redeem for one service, so only those of them having no needs for services, which make up the fraction $e^{-q_0\tau}$, belong to the first group. On the other hand, consumers holding rewards between $N + p_0 - A\tau$ units and $N + p_0$ units at $t - \tau$ have sufficient rewards to redeem for more than one service, so only those of them having needs for exactly one service by t, which make up the fraction $q_0 \tau e^{-q_0 \tau}$, belong to the second group. The mass of the third group is of a higher order than τ. Hence, we obtain the following equation:

$$\Delta\Gamma(N + A\tau) = \Delta\Gamma(N)\, e^{-q_0\tau} + \Delta\Gamma(N + p_0)\, q_0 \tau e^{-q_0\tau} + o(\tau).$$

Taking τ to 0, we obtain equation (5). The boundary conditions (6), (7) and (8) stem from that the distribution is continuous everywhere, no consumer holds a negative number of rewards, and the total mass of consumers is 1, respectively.

Proof of Proposition 3

The equation system in Lemma 3 is an advance differential equation system. We conjecture

$$\gamma(N) = e^{-N/\lambda}\gamma_0 \tag{14}$$

for $N \in (p_0, +\infty)$, where $\lambda > 0$. Then according to equation (5), we have

$$\begin{aligned} -\frac{1}{\lambda} e^{-N/\lambda}\gamma_0 + e^{-N/\lambda}\gamma_0 \frac{q_0}{A} - e^{-N/\lambda} e^{-p_0/\lambda}\gamma_0 \frac{q_0}{A} = 0 \\ \Leftrightarrow \left(1 - e^{-p_0/\lambda}\right) \cdot \lambda = \frac{A}{q_0}. \end{aligned}$$

Plugging the conjectured distribution (14) into equation (5), we obtain that for $N \in [0, p_0)$,

$$\begin{aligned} \gamma(N) &= \int_0^N \gamma(x + p_0)\frac{A}{q_0} dx + \gamma(0) = \int_0^N e^{-x/\lambda} e^{-p_0/\lambda} \cdot \gamma_0 \frac{q_0}{A} dx \\ &= \lambda e^{-p_0/\lambda} \cdot \gamma_0 \frac{q_0}{A} \cdot \left(1 - e^{-N/\lambda}\right). \end{aligned}$$

Note that the boundary condition (6) naturally holds:

$$\lim_{N \uparrow p_0} \gamma(N) = \lambda e^{-p_0/\lambda} \cdot \gamma_0 \frac{q_0}{A} \cdot \left(1 - e^{-p_0/\lambda}\right) = e^{-p_0/\lambda} \cdot \gamma_0 = \gamma(p_0).$$

The boundary conditions (7) and (8) require

$$\int_0^{+\infty} \gamma(N)\, dN = 1.$$

Since

$$
\begin{aligned}
\int_0^{+\infty} \gamma(N)\, dN =& \lambda e^{-p_0/\lambda} \cdot \gamma_0 \frac{q_0}{A} \cdot \int_0^{p_0} \left(1 - e^{-N/\lambda}\right) dN + \gamma_0 \int_{p_0}^{+\infty} e^{-N/\lambda} dN \\
=& \lambda e^{-p_0/\lambda} \cdot \gamma_0 \frac{q_0}{A} \cdot \left(p_0 + \lambda e^{-p_0/\lambda} - \lambda\right) + \gamma_0 \lambda e^{-p_0/\lambda} \\
=& \lambda e^{-p_0/\lambda} \cdot \gamma_0 \frac{q_0}{A} \cdot \left(p_0 - \frac{A}{q_0}\right) + \gamma_0 \lambda e^{-p_0/\lambda} \\
=& \gamma_0 \lambda e^{-p_0/\lambda} \frac{p_0 q_0}{A} = \gamma_0 p_0 \left(\lambda \frac{q_0}{A} - 1\right)
\end{aligned}
$$

So,

$$
\gamma_0 = \frac{1}{p_0 \left(\lambda \frac{q_0}{A} - 1\right)}.
$$

And for $N \in [0, p_0)$,

$$
\begin{aligned}
\gamma(N) &= \lambda e^{-p_0/\lambda} \cdot \gamma_0 \frac{q_0}{A} \cdot \left(1 - e^{-N/\lambda}\right) \\
&= \left(\lambda \frac{q_0}{A} - 1\right) \cdot \frac{1}{p_0 \left(\lambda \frac{q_0}{A} - 1\right)} \left(1 - e^{-N/\lambda}\right) \\
&= \frac{1}{p_0} \left(1 - e^{-N/\lambda}\right).
\end{aligned}
$$

Next, we show that the equation

$$
\left(1 - e^{-p_0/\lambda}\right) \cdot \lambda = \frac{A}{q_0}
$$

has a unique positive root λ. First, since

$$
\lim_{\lambda \to 0} \left(1 - e^{-p_0/\lambda}\right) \cdot \lambda = 0 < \frac{A}{q_0}
$$

and

$$
\lim_{\lambda \to +\infty} \left(1 - e^{-p_0/\lambda}\right) \cdot \lambda = \lim_{\lambda \to +\infty} \frac{1 - e^{-p_0/\lambda}}{p_0/\lambda} \cdot p_0 = \lim_{x \to 0} e^{-x} \cdot p_0 = p_0 > \frac{A}{q_0},
$$

the equation must have positive roots. Second, since

$$
\frac{d\left[1 - e^{-x} - e^{-x} x\right]}{dx} = e^{-x} x > 0,
$$

we have for $\lambda > 0$,

$$
\frac{d\left[\left(1 - e^{-p_0/\lambda}\right) \cdot \lambda\right]}{d\lambda} = 1 - e^{-p_0/\lambda} - e^{-p_0/\lambda} \frac{p_0}{\lambda} > 1 - e^{-0} - e^{-0} \cdot 0 = 0.
$$

This implies that $\left(1-e^{-p_0/\lambda}\right)\cdot\lambda$ is strictly increasing in λ for $\lambda>0$. Therefore, the equation has a unique positive root.

Finally, we derive the reward balance.

$$
\begin{aligned}
B &= \int_0^{+\infty}\gamma(N)NdN \\
&= \frac{1}{p_0}\cdot\int_0^{p_0}\left(1-e^{-N/\lambda}\right)NdN+\frac{1}{p_0\left(\lambda\frac{q_0}{A}-1\right)}\cdot\int_{p_0}^{+\infty}e^{-N/\lambda}NdN \\
&= \frac{1}{p_0}\cdot\frac{1}{2}p_0^2-\frac{1}{p_0}\cdot\int_0^{p_0}e^{-N/\lambda}NdN+\frac{1}{p_0\left(\lambda\frac{q_0}{A}-1\right)}\cdot\int_{p_0}^{+\infty}e^{-N/\lambda}NdN \\
&= \frac{p_0}{2}-\frac{1}{p_0}\frac{\lambda\frac{q_0}{A}}{\lambda\frac{q_0}{A}-1}\cdot\int_0^{p_0}e^{-N/\lambda}NdN+\frac{1}{p_0\left(\lambda\frac{q_0}{A}-1\right)}\cdot\int_0^{+\infty}e^{-N/\lambda}NdN \\
&= \frac{p_0}{2}-\frac{1}{p_0}\frac{\lambda\frac{q_0}{A}}{\lambda\frac{q_0}{A}-1}\cdot\lambda^2\left[e^{-p_0/\lambda}\left(-\frac{p_0}{\lambda}-1\right)+1\right]+\frac{1}{p_0\left(\lambda\frac{q_0}{A}-1\right)}\cdot\lambda^2 \\
&= \frac{p_0}{2}+\frac{1}{p_0}\frac{1}{\lambda\frac{q_0}{A}-1}\cdot\lambda^2\left[e^{-p_0/\lambda}\left(\frac{p_0}{\lambda}+1\right)\lambda\frac{q_0}{A}-\lambda\frac{q_0}{A}+1\right].
\end{aligned}
$$

By $\left(1-e^{-p_0/\lambda}\right)\cdot\lambda=\frac{A}{q_0}$, we have

$$
\begin{aligned}
B &= \frac{p_0}{2}+\frac{1}{p_0}\frac{1}{\lambda\frac{q_0}{A}-1}\cdot\lambda^2\left[\left(\lambda\frac{q_0}{A}-1\right)\left(\frac{p_0}{\lambda}+1\right)-\lambda\frac{q_0}{A}+1\right] \\
&= \frac{p_0}{2}+\frac{1}{p_0}\frac{1}{\lambda\frac{q_0}{A}-1}\cdot\lambda^2\left(\lambda\frac{q_0}{A}-1\right)\frac{p_0}{\lambda} \\
&= \frac{p_0}{2}+\lambda.
\end{aligned}
$$

Proof of Proposition 4

When $1/\pi$ is a rational number, there exists a unique pair of positive coprime integers μ and n such that $\mu/n=(1+\pi)/\pi$. We then obtain

$$
\frac{h_i}{(p_0+p_0\pi)/\mu}=i\cdot n \pmod{\mu}.
$$

Since μ and n are coprime, $\frac{h_i}{(p_0+p_0\pi)/\mu}$ follows a cycle, and in one cycle, $\frac{h_i}{(p_0+p_0\pi)/\mu}$ hits each element in $\{0,1,2,\ldots,\mu-2,\mu-1\}$ once. That means, consumers are uniformly distributed over the set $\left\{\frac{i}{\mu}(p_0+p_0\pi)\right\}_{i=0}^{\mu-1}$ in terms of reward holdings. Further, the reward balance is

$$
B=\frac{1}{\mu}(p_0+p_0\pi)\sum_{i=0}^{\mu-1}i\cdot\frac{1}{\mu}=\frac{\mu-1}{2\mu}(1+\pi)p_0.
$$

When $1/\pi$ is an irrational number, the economic forces are basically the same. Conceptually, we can construct an infinite sequence of rational numbers that converges to $1/\pi$. During this process, μ will go to infinity, and the set $\left\{\frac{i}{\mu}(p_0+p_0\pi)\right\}_{i=0}^{\mu-1}$ will get closer to the interval $[0, p_0+p_0\pi)$. Formally, by Weyl's equidistribution theorem, if $\frac{\pi}{1+\pi}$ is an irrational number and

$$\frac{h_i}{p_0+p_0\pi} = i \cdot \frac{\pi}{1+\pi} \pmod 1,$$

h_i is uniformly distributed over the interval $[0, p_0+p_0\pi)$. Hence, the reward balance is $(p_0+p_0\pi)/2$.

The reward balance is generally characterized by equation (12). Since n is a positive integer and must be at least 1, μ is greater than or equal to $(1+\pi)/\pi$. That means

$$\frac{1}{2}(1+\pi)p_0 \geq B \geq \frac{\frac{1+\pi}{\pi}-1}{2\frac{1+\pi}{\pi}}(1+\pi)p_0 = \frac{p_0}{2}.$$

The left equality is attained when $1/\pi$ is an irrational number, and the right equality is attained when $1/\pi$ is an integer.

Proof of Proposition 5

For any CLP, let $\hat{\lambda} \triangleq \lambda/p_0$. Then by equation (10), $\hat{\lambda}$ satisfies

$$\left(1-e^{-1/\hat{\lambda}}\right)\cdot\hat{\lambda} = \frac{A}{p_0 q_0},$$

and is thus fixed given $p_0 q_0$ and A. The reward balance is

$$B = \left(\frac{1}{2}+\hat{\lambda}\right)p_0,$$

which is greater if p_0 is higher and q_0 is lower.

Under any PLP, by $A = \frac{p_0 q_0}{1/\pi+1}$, π is fixed given $p_0 q_0$ and A. So is μ, as it is completely determined by π. The reward balance is

$$B = \frac{\mu-1}{2\mu}(1+\pi)p_0$$

which is greater if p_0 is higher and q_0 is lower.

Proof of Proposition 6

Under a PLP with π, the issuance rate is

$$A = \frac{p_0 q_0}{1/\pi + 1},$$

and the reward balance is no greater than

$$\frac{p_0}{2} + \frac{p_0 \pi}{2}.$$

On the other hand, for the same issuance rate, the reward balance under a CLP is

$$\frac{p_0}{2} + \lambda,$$

where

$$\left(1 - e^{-p_0/\lambda}\right) \cdot \lambda = \frac{A}{q_0}.$$

It is easy to see

$$\lambda > \frac{A}{q_0} = \frac{p_0 \pi}{1 + \pi} \geq \frac{p_0}{2} \pi.$$

The last inequality follows $\pi \leq 1$.

Proof of Proposition 7

For convenience of illustration, we stipulate that consumers do free purchase or free transfer, as long as it is at least weakly beneficial. Following the argument of Lemma 1, each redemption must involve p_0 rewards.

Case I: no LP is adopted.

The average benefit of a reward obtained through free purchase is $1 - \frac{\varepsilon}{p_0}$. If $\hat{\theta} > 1 - \frac{\varepsilon}{p_0}$, a consumer never obtains rewards through free purchase, so free purchase makes no difference. If $\hat{\theta} \leq 1 - \frac{\varepsilon}{p_0}$, a consumer purchases p_0 rewards whenever he needs a service, and he redeems them immediately. Hence, consumers enjoy positive discounts of rewards, and the reward balance is 0. The firm's cash flows become

$$p_0 q_0 \hat{\theta} - OE_0,$$

which is lower than CF_0.

Case II: a PLP is adopted.

Exercising the option to buy rewards in a cash purchase, a consumer spends $p_0 + p_0\pi\theta_p$ cash and receives a payoff of p_0 and $p_0\pi$ rewards. Instead, if a consumer obtains $p_0 + p_0\pi$ rewards through free purchase and redeem p_0 rewards for a service, he receives a payoff of $p_0 - \varepsilon$ and $p_0\pi$ rewards. A consumer prefers the latter if and only if

$$\begin{aligned} p_0 - \varepsilon - (p_0 + p_0\pi)\,\hat{\theta} &\geq p_0 - (p_0 + p_0\pi\theta_p) \\ \Leftrightarrow \hat{\theta} &\leq \frac{\pi}{1+\pi}\theta_p + \frac{1}{1+\pi}\left(1 - \frac{\varepsilon}{p_0}\right). \end{aligned}$$

It is easy to see that this inequality implies that a cash purchase without exercising the option is also dominated. Therefore, if $\hat{\theta} \leq \frac{\pi}{1+\pi}\theta_p + \frac{1}{1+\pi}\left(1 - \frac{\varepsilon}{p_0}\right)$, a consumer does not use cash purchases: he purchases p_0 rewards whenever he needs a service, and he redeems them immediately. Similar to the case when no LP is adopted, the firm's cash flows become

$$p_0 q_0 \hat{\theta} - OE_0.$$

Without free purchase, the firm's cash flows under a PLP are

$$\begin{aligned} & r_0 B\theta_p - \frac{p_0 q_0}{1/\pi + 1}(1 - \theta_p) + p_0 q_0 - OE_0 \\ >& -\frac{p_0 q_0}{1/\pi + 1} + \frac{p_0 q_0}{1/\pi + 1}\theta_p + p_0 q_0 - OE_0 \\ \geq& -\frac{p_0 q_0}{1/\pi + 1} + p_0 q_0 \left[\hat{\theta} - \frac{1}{1+\pi}\left(1 - \frac{\varepsilon}{p_0}\right)\right] + p_0 q_0 - OE_0 \\ =& q_0 \frac{1}{1+\pi}\varepsilon + p_0 q_0 \hat{\theta} - OE_0 \\ >& p_0 q_0 \hat{\theta} - OE_0. \end{aligned}$$

Therefore, free purchase with $\hat{\theta} \leq \frac{\pi}{1+\pi}\theta_p + \frac{1}{1+\pi}\left(1 - \frac{\varepsilon}{p_0}\right)$ reduces the firm's cash flows.

If $\hat{\theta} > \frac{\pi}{1+\pi}\theta_p + \frac{1}{1+\pi}\left(1 - \frac{\varepsilon}{p_0}\right)$, a consumer prefers using cash to buy the bundle of a service and rewards to free purchase. A consumer never obtains rewards through free purchase, so free purchase makes no difference in this case.

Case III: a CLP is adopted.

If $\hat{\theta} > 1 - \frac{\varepsilon}{p_0}$, a consumer never obtains rewards through free purchase, so free purchase makes no difference. If $\hat{\theta} \leq \theta_c$, a consumer will not use the CCC to earn rewards or use cash purchases: he purchases p_0 rewards whenever he needs a service, and he redeems them immediately. Similar to the case when no LP is adopted, the firm's cash flows are

$$p_0 q_0 \hat{\theta} - OE_0.$$

Without free purchase, the firm's cash flows under a CLP are

$$\begin{aligned}
& r_0 B \theta_c - A(1-\theta_c) + p_0 q_0 - OE_0 \\
> & -A\left(1-\hat{\theta}\right) + p_0 q_0 - OE_0 \\
> & -p_0 q_0 \left(1-\hat{\theta}\right) + p_0 q_0 - OE_0 \\
= & p_0 q_0 \hat{\theta} - OE_0.
\end{aligned}$$

Therefore, free purchase with $\hat{\theta} \leq \theta_c$ reduces the firm's cash flows.

If $1 - \frac{\varepsilon}{p_0} \geq \hat{\theta} > \theta_c$, a consumer will fully use the CCC to earn rewards but do not use cash purchases: he earns rewards through the CCC at the rate of A, and if he needs a service but does not have p_0 rewards, he get the rest through free purchase and redeem all rewards immediately. Similar to Section 4.2, we derive the steady-state distribution of consumers' reward holdings. Note that holding fewer than p_0 rewards does not impede reward redemption. The steady-state distribution of consumers' reward holdings is

$$\gamma(N) = \begin{cases} 0, & \text{if } N \in (-\infty, 0), \\ \frac{1}{\lambda} e^{-N/\lambda}, & \text{if } N \in [0, +\infty), \end{cases} \tag{15}$$

where λ is still the one defined in Proposition 3 as follows:

$$\left(1 - e^{-p_0/\lambda}\right) \cdot \lambda = \frac{A}{q_0}.$$

We then obtain that the reward balance is

$$\int_0^{+\infty} N \cdot \frac{1}{\lambda} e^{-N/\lambda} dN = \lambda,$$

which is smaller than the reward balance without free purchase. Taken together, the firm's cash flows are

$$\begin{aligned}
& r_0 \lambda \theta_c + A \theta_c + (p_0 q_0 - A) \hat{\theta} - OE_0 \\
< & r_0 \left(\frac{p_0}{2} + \lambda\right) \theta_c - A(1-\theta_c) + p_0 q_0 - OE_0.
\end{aligned}$$

Therefore, free purchase with $1 - \frac{\varepsilon}{p_0} \geq \hat{\theta} > \theta_c$ reduces the firm's cash flows.

Proof of Proposition 8

Assume free transfer is allowed. A consumer who need a service would like to buy a total of p_0 rewards from consumers at a price no greater than $1-\frac{\varepsilon}{p_0}$. On the other hand, a consumer who has rewards and does not need a service would like to sell rewards at a price no smaller than $1-\frac{\varepsilon}{p_0}$, due to the preference for holding cash.

If the reward balance is positive, then each consumers who needs a service can get enough rewards to redeem for a service. Hence, rewards are redeemed at the rate of p_0q_0 in total. The maximum issuance rate under a CLP is $\gamma C_0 a_c$, and that under a PLP is $p_0q_0\pi$. Both are smaller than p_0q_0, so the reward balance must decrease. In the steady state, the reward balance must be 0. In the steady state, the rewards issued at each instant will be transferred to the consumers who need services and redeemed immediately.

Proof of Proposition 10

The liquidity buffer at t is decreasing in the magnitude of the shock σ:

$$\frac{\partial L(t;\sigma)}{\partial \sigma} = \frac{e^{r_0t}-1}{r_0}(A_c+A_p)(1-\theta) + \frac{e^{r_0t}-1}{r_0}\delta A_c\theta - \frac{e^{r_0t}-1}{r_0}p_0q_0 < 0.$$

Since

$$\frac{\partial^2 L(t;\sigma)}{\partial \sigma \partial A_c} = \frac{e^{r_0t}-1}{r_0}(1-\theta) + \frac{e^{r_0t}-1}{r_0}\delta\theta > \frac{\partial^2 L(t;\sigma)}{\partial \sigma \partial A_p} = \frac{e^{r_0t}-1}{r_0}(1-\theta),$$

an increase in A_c weakens the sensitivity of the liquidity buffer to the magnitude of the shock more than an increase in A_p. The difference $\frac{e^{r_0t}-1}{r_0}\delta\theta$ stems from the second mechanism.