Platform Money*

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Abstract

Do private profit motives inevitably lead to the overissuance of currency? We show that when the issuer is a two-sided platform, its core matching business disciplines its monetary policy. Unlike standalone issuers, a platform avoids overissuance because destroying trade surplus is costlier than gaining seigniorage. While private money generally creates congestion externalities, it improves social welfare if the platform possesses superior matching technology. Finally, we rationalize the "Closed Policy" of rejecting outside money as strategic segmentation. By ceding buyers biased toward outside money to the legacy market, the platform induces its competitor to raise fees, creating a high-cost environment that allows the platform to extract higher seigniorage from its own users.

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1 Introduction

The rise of the platform economy is evidenced by the growing popularity of super-apps such as Amazon, Facebook (Meta), WeChat, and Alibaba. Empowered by advanced data processing and machine learning, these applications deliver superior matching capabilities between buyers and sellers, offering a user experience that traditional brick-and-mortar marketplaces cannot match. Increasingly, these platforms are embedding proprietary payment systems (such as WeChat Pay, AliPay, or Diem) directly into their commercial ecosystems. This development raises a fundamental question: How does the ability to issue private money affect the competitive dynamics between digital platforms and traditional legacy markets? Do these platforms face the same temptation to overissue currency that (Friedman, 1960) warned against?

To address these questions we use a baseline New Monetarist model (Lagos and Wright, 2005) of a platform competing with a legacy market that uses government fiat. Both platform and legacy operate as two-sided marketplaces. In these two-sided marketplaces, the entry of an additional buyer reduces the matching probability for other buyers (congestion externality) while increasing it for sellers (cross-group positive externality) - or vice versa in the competing market.

We show that by leveraging its private money and advanced matching technology, the platform can lower the cost of attracting buyers while generating cross-group network externalities. Importantly, the effectiveness of using private money to attract buyers depends on several factors including the inflation regime in the legacy marketplace, the relative bargaining power of buyers and sellers regarding consumption goods, and other market-specific parameters affecting the choice of trading venue.

Moreover, the platform's superior matching capabilities further intensify the reinforcing network effects between buyers and sellers. In equilibrium, our analysis reveals that the platform attracts more buyers, imposes higher fees on sellers (yet still attracts more sellers overall), and earns higher profit compared to the legacy market. Market tightness (seller to buyer ratio) on the platform is lower than the legacy market when the two have similar matching technologies but can be higher if the platform has substantial superior matching technology. We derive closed-form results characterising these equilibrium properties and offer additional insights through numerical examples.

We find that the platform's monetary policy exhibits a form of *strategic dampening*. As central bank inflation rises, the legacy market is forced to cut fees to prevent user exit. In response, the platform raises its own inflation by *less* than one-for-one, strategically moderating its rate to compete against the legacy market's lower fees and maintain an "inflation shelter" for buyers.

To add richness to our model, we also allow for the possibility that consumers are "inattentive," perceiving inflation costs as less salient than explicit fees. This leads us to uncover a crucial interaction between choices of private versus public money and behavioral bias. This additional friction creates a distinct advantage for the platform: it can replace salient transaction fees with less salient inflation charges, effectively implementing a form of obfuscated pricing. However, we show this salience is double-edged. When the legacy inflation is high, low salience reduces the platform's ability to attract inattentive buyers from the legacy market, as these buyers fail to appreciate the platform's lower inflation rate.

We also evaluate the welfare implications of platform money. We find that private money is socially inefficient in the baseline model because the platform uses the inflation subsidy to attract "too many" buyers relative to sellers compared to the planner's solution, creating congestion. However, if the platform possesses a sufficiently superior matching technology, this result is overturned. In this case, the platform's ability to use money to subsidize buyer entry corrects the search frictions, moving the equilibrium closer to the social optimum than a fee-only model could. This suggests that the social value of private money depends critically on the technological advantage of the issuer.

Finally, we characterize when a platform should accept outside money. In the baseline model, the platform strictly prefers to use platform money and has no incentive to accept outside fiat money. In practice, platforms face a choice: enforce a "Closed Policy" (accepting only their own money) or an "Open Policy" (accepting outside fiat). To capture this, we extend the baseline model by introducing a fraction of buyers who prefer holding outside money over platform money. We show that platforms might strictly prefer the Closed Policy, even when a significant fraction of buyers are biased against using platform money. The mechanism is driven by *strategic complementarity*. By rejecting outside money, the platform effectively cedes the "biased" buyers to the legacy market, turning the legacy market into a monopoly over that segment. Secure in its monopoly, the legacy market raises its fees. Crucially, this high-fee legacy environment relaxes the competitive constraint on the platform, allowing it to charge higher inflation to its own "flexible" users without losing them. Thus, the platform strategically segments the market by sacrificing market share for higher margins, and exploits the legacy market's response to maximize seigniorage.

Regulators often view "walled gardens" (refusal to accept outside payments) as simple foreclosure. We show it is a monetary strategy: by segmenting the market, the platform induces the incumbent (legacy system) to raise fees, creating a "high-cost" equilibrium that benefits the platform. Mandating interoperability (forcing the Open Policy) would destroy this rent-extraction mechanism, potentially lowering fees for everyone.

A classic debate in monetary economics is whether private profit motives lead to the overissue of currency (Friedman, 1960) or whether market forces discipline issuers (Klein, 1974; Hayek, 1976). We argue that two-sided Platforms are fundamentally different from the standalone issuers envisioned by Hayek. For a traditional issuer, money is the product; for a platform, money is a *complement* to its core product which is matching. This complementarity creates a stricter disciplinary mechanism. Overissuing currency does not just devalue the token; it degrades the trade surplus by driving away buyers, which via cross-group network effects disincentivizes the entry of sellers. Thus, the platform's incentive to maximize the value of the matching ecosystem naturally curbs the temptation to currency overissuance.

Literature Our work bridges three distinct strands of literature: two-sided platform economics, the emerging theory of digital tokens, and classical monetary competition.

Platform Competition and Search Frictions. Our model builds on the foundational literature of two-sided markets. Early work focused on platforms with exogenous network effects, analyzing pricing structures where one side is subsidized by the other (Rochet and Tirole, 2003; Caillaud and Jullien, 2003; Armstrong, 2006; Weyl, 2010). A second stream of literature endogenizes these network effects using search and matching frictions (Chen and Huang, 2012; Goos et al., 2014; Gautier et al., 2023). We contribute to this tradition by introducing a new instrument: *private money.* Unlike standard models where platforms compete solely via fees, our platform manages market tightness and participation through its monetary policy (inflation and seigniorage), effectively bundling the matching service with a medium of exchange.

The Economics of Platform Tokens. We diverge from the growing literature that views platform tokens primarily as financing tools or loyalty points. For instance, Rogoff and You (2023) model tokens as non-tradable loyalty rewards, while Sockin and Xiong (2023) and Goldstein et al. (2024) analyze tokens as commitment devices or financing vehicles for platforms with weak fundamentals. In contrast, we model the token as money – a circulating medium of exchange that generates seigniorage. Furthermore, we enrich this analysis by incorporating behavioral insights motivated by evidence that consumers underweight non-salient costs (Bordalo et al., 2022; Blake et al., 2021) and exhibit money illusion (Shafir et al., 1997; Brunnermeier and Julliard, 2008).

Currency Competition and Monetary Theory. We are related to the literature on currency competition (Schilling and Uhlig, 2019; Fernández-Villaverde and Sanches, 2019; Skeie, 2019; Benigno et al., 2022; Guennewig, 2024). A central finding in this literature is an "impossibility result": private currency competition is typically unstable or fails to achieve stability because issuers face break-even constraints that prevent optimal deflation (Fernández-Villaverde and Sanches, 2019). We show that this logic is overturned when the issuer is a two-sided platform. Our framework fundamentally departs from the "money-as-product" view, where issuers maximize seigniorage in isolation. Instead, we treat money as a *complement* to the platform's core matching service. This complementarity creates a disciplinary force: the platform cannot over-issue currency without degrading the trade surplus and destroying its cross-side network effects. Thus, we identify a market-based discipline that allows platforms to sustain viable private money where standalone issuers cannot. Our result also offers a resolution to the classic indeterminacy puzzle of Kareken and Wallace (1981). They show that if private currencies are perfect substitutes, the exchange rate between them is indeterminate and volatile. In contrast, our model generates a determinate exchange rate because we treat platform money and outside money as *imperfect substitutes*. The demand for platform money is anchored by the platform's superior matching technology. By bundling the currency with a unique matching service, the platform creates a specific transactional demand that pins down the price level and the exchange rate, thereby eliminating the indeterminacy found in standard currency competition models.

The rest of the paper is organized as follows. Section 2 sets up the model and provides the equilibrium definition. Section 3 analyses equilibrium properties. Section 4 presents comparative statics and numerical exercises. Section 5 solves the planner's problem. Section 6 characterizes when a platform should accept outside money. Section 7 concludes.

2 The Model

2.1 The Environment

Time is discrete, lasts forever, and is indexed by $t \in \{0, 1, ...\}$. There are two marketplaces: a private platform P and a legacy market L. There are three types of agents: a measure $\bar{N}_b$ of buyers, a measure $\bar{N}_s$ of sellers, and the owners of the two trading marketplaces. Decentralized search and matching between buyers and sellers occurs in the two marketplaces.

The discount factor between periods is $\beta \in (0, 1)$ and each period is divided into two stages. In the first stage, the decentralized marketplaces (DM) are open to trade a perishable consumption good y that only sellers can produce at zero marginal cost. Buyers obtain u from consuming one unit and do not value more units. In each period, buyers and sellers are able to participate in one and only one of the trading marketplaces. When a buyer and a seller match, the transaction is executed using money: on platform P, trades are conducted using platform money, whereas on legacy market L, outside money is used. We also refer to this DM stage as the consumption goods market.

In the second stage, a centralized, frictionless settlement market (CM) is established. In this market, the owners of the trading platforms set buyer and seller fees, and agents decide which future DM consumption good marketplace to join, pay the corresponding fee, and rebalance their portfolios of platform and outside money. Consequently, the platform money as a medium of exchange is priced. We assume that all types of agents consume the perishable CM good, x, and can supply labor, h, to produce the good x via a linear production technology with a 1:1 ratio. All agents obtain utility $U(x,h)$ by consuming x of the CM good but incur dis-utility from labor. To simplify the exposition, we assume that $U(x,h) = x - h$. We also refer to this CM stage as money market.

Both market owners impose fees denoted by k_t^j to sellers and f_t^j to buyers, where $j \in \{P, L\}$ (with the fees measured in units of x) while the owner of the platform (P) also chooses the amount of additional platform money to issue. Sellers and buyers observe these fees, pay the fee associated with the market they choose to enter and adjust their money portfolios accordingly.

Under the assumption that consumption goods x and y are perishable, the only forms of money in this economy are platform money and outside money. We denote the money supply and the corresponding money price in each market by M_t^j and ϕ_t^j, respectively, where $j \in \{P, L\}$. In the legacy market, a central bank sets the money growth rate (that is, inflation) to achieve macroeconomic objectives. In contrast, the platform determines the growth rate of its own money to maximize profit.[1] In practice, the platform sets interest rates on its digital wallet, which effectively expands its money supply, and occasionally issues coupons or vouchers, which are equivalent to helicopter money.

In what follows, we specify the law of motion for the money supply in each marketplace:

$$M_{t+1}^j = \mu^j M_t^j. \tag{1}$$

We assume that $\mu^j > \beta$ is set in such a way that the money depreciation rate exceeds the discount factor; otherwise, agents' demand for money would be infinite. Hence, a seller doesn't carry any money across periods. A buyer doesn't carry the money of the market where she doesn't trade and carries the optimal amount of money necessary to trade in DM. Following the new monetarist literature, we focus on a stationary equilibrium where in steady state $M_t^j \phi_t^j$ is constant. Figure 2.1 summarizes the aforementioned events in this economic environment.

[1] Since fewer people hold cash and use bank accounts for digital payments, this means that central bank influences aggregate money supply indirectly via affecting banks' deposit rates.

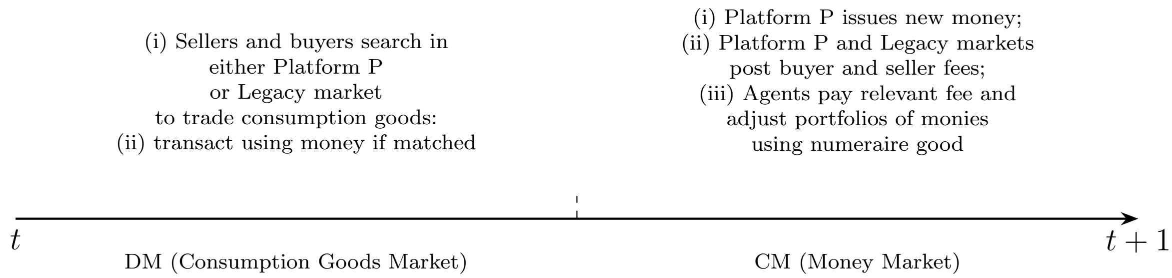


Figure 1: Timeline

2.2 Centralized Market

We denote an agent's value function in the CM by W_t and in the DM by V_t. The CM value function of a buyer denoted by subscript b is given by:

$$W_{b,t}(m^P_{b,t}, m^L_{b,t}) = \hat{\Pi}^P_{b,t} W^P_{b,t}(m^P_{b,t}, m^L_{b,t}) + \hat{\Pi}^L_{b,t} W^L_{b,t}(m^P_{b,t}, m^L_{b,t}), \tag{2}$$

where $\hat{\Pi}^j_{b,t}$ is the buyer's endogenously determined probability of choosing market j in the CM. This formulation allows buyers to bring both types of money from CM to DM but in fact buyers will bring at most one type of money (or none) since holding money is costly if not for transaction purposes. That is, buyers pay only the entry fee and purchase the relevant money of the chosen marketplace j. The continuation value of going to market j is:

$$W^j_{b,t}(m^P_{b,t}, m^L_{b,t}) = \max_{x_t, h_t, m^j_{b,t+1} \geq 0} x_t - h_t + \beta V^j_{b,t+1}(m^j_{b,t+1}, 0) \tag{3}$$

$$\text{s.t.} \quad x_t + f_j + \phi^j_t m^j_{b,t+1} \leq h_t + \phi^P_t m^P_{b,t} + \phi^L_t m^L_{b,t}, \tag{4}$$

where buyers choose consumption, labour, and money holding amount optimally. By substituting for $x_t - h_t$, we obtain

$$W^j_{b,t}(m^P_{b,t}, m^L_{b,t}) = \phi^P_t m^P_{b,t} + \phi^L_t m^L_{b,t} + W^j_{b,t}(0, 0),$$

where

$$W^j_{b,t}(0, 0) = \max_{m^j_{b,t+1} \geq 0} -\phi^j_t m^j_{b,t+1} - f_j + \beta V^j_{b,t+1}(m^j_{b,t+1}, 0) \tag{5}$$

Thus, a buyer's optimization problem in CM is not history dependent and her value function in CM can be written as

$$W_{b,t}(m_{b,t}^P, m_{b,t}^L) = \phi_t^P m_{b,t}^P + \phi_t^L m_{b,t}^L + \left[\hat{\Pi}_{b,t}^P W_{b,t}^P(0,0) + \hat{\Pi}_{b,t}^L W_{b,t}^L(0,0)\right]. \tag{6}$$

Similarly, a seller's value function denoted by subscript s can be written as:

$$W_{s,t}(m_{s,t}^P, m_{s,t}^L) = \phi_t^P m_{s,t}^P + \phi_t^L m_{s,t}^L + \left[\hat{\Pi}_{s,t}^P W_{s,t}^P(0,0) + \hat{\Pi}_{s,t}^L W_{s,t}^L(0,0)\right]. \tag{7}$$

where k is the seller entry fee and

$$W_{s,t}^j(0,0) = \max_{m_{s,t+1}^j \geq 0} -\phi_t^j m_{s,t+1}^j - k_j + \beta V_{s,t+1}^j(m_{s,t+1}^j, 0). \tag{8}$$

2.3 Decentralized Market

Trading in a decentralized marketplace is subject to search frictions that we capture with a matching function. For any market j with N_s sellers and N_b buyers, the matching function $Q^j(N_s, N_b)$ represents the total number of successful matches in j. We assume that the function $Q^j(\cdot)$ exhibits the constant-return-to-scale property. We also assume that Q^j is concave in both variables. It is also useful to define the market tightness in market j, denoted by n_j, as the ratio of sellers to buyers in this market, i.e., $n_j \equiv N_s/N_b$. Using this definition, the probability that a *buyer* successfully finds a match in market j is related to the market tightness in the following manner:

$$a_{jb}(n_j) \equiv \frac{Q^j(N_s, N_b)}{N_b} = Q^j(n_j, 1) \tag{9}$$

Similarly, the probability that a *seller* finds a match in market j is:

$$a_{js}(n_j) \equiv \frac{Q^j(N_s, N_b)}{N_s} = \frac{1}{n_j} Q^j(n_j, 1) \tag{10}$$

We assume that the platform has (weakly) better matching technology so that for any market tightness n buyers and sellers are (weakly) more likely to find a match on the platform: $a_{Pb}(n) \geq a_{Lb}(n)$ and $a_{Ps}(n) \geq a_{Ls}(n)$. Furthermore, the marginal increase (decrease) in matching probability is weakly larger for buyers (sellers) on the platform if market tightness increases: $a'_{Pb}(n) \geq a'_{Lb}(n)$ ($a'_{Ps}(n) \leq a'_{Ls}(n)$).

Conditional on a successful match, we assume that the buyer and the seller bargain over the terms of trade. The literature uses two solution concepts to obtain a bargaining outcome. These concepts are the generalized Nash bargaining (Nash (1950, 1953)) and Kalai's proportional bargaining (Kalai (1977)). When the buyers are not liquidity constrained, the two approaches yield the same solution but become distinct when buyers are liquidity constrained which is critical in applications to money (Hu and Rocheteau (2020a)). We adopt the proportional bargaining approach for three reasons. First, Kalai's proportional bargaining is shown to be more empirically relevant over the generalized Nash solution when the buyers are liquidity constrained (See the experimental evidence in Duffy et al. (2021)). Second, under proportional bargaining buyers use money even when the sellers do not have cost of production which we find reasonable. And third, proportional bargaining provides expositional clarity. The solution to the proportional bargaining problem under liquidity constraints is well-known (e.g, Hu and Rocheteau (2020b)), the buyer brings real balance just sufficient to purchase the optimal quantity given the split surplus. Thus, the resulting real price for the consumption good y is: $p^j\phi^j = u(1-\gamma)$.[2]

2.4 Buyers in the DM

Given the matching probabilities, we obtain the DM value function for each individual buyer who chooses to trade on market $j \in \{P, L\}$ as

$$V^j_{b,t}(m^j_{b,t}, 0) = a_{jb}(n_{jt})[u + W_{b,t}(m^j_{b,t} - p^j_t, 0)] + (1 - a_{jb}(n_{jt}))W_{b,t}(m^j_{b,t}, 0). \tag{11}$$

The first term states that, conditional on being matched, the buyer gains utility u and carries the after-trade money balance $m^j_{b,t} - p^j_t$ into the CM. The second term captures the case where the buyer simply carries over their money to the CM if not matched. By plugging for

[2]We provide the generalized Nash bargaining and Kalai proportional bargaining solutions in the appendix and show our main results are robust to the bargaining approaches.

$W_{b,t}(\cdot)$, we simplify the value function as follows:

$$V_{b,t}^j(m_{b,t}^j, 0) = a_{jb}(n_{jt})[u - \phi_t^j p_t^j] + \phi_t^j m_{b,t}^j + W_{b,t}(0,0). \tag{12}$$

In the steady state, the real price of the DM good is set by the bargaining rule:

$$\phi_t^j m_t^j = \phi_t^j p_t^j = u(1-\gamma) \tag{13}$$

since buyers bring the exact amount of money to pay for the DM good. Plugging the real price, the money holding, and (12) into (5), we obtain:

$$W_b^j(0,0) = -\underbrace{f_j}_{\text{Fee}} + \underbrace{\beta a_{jb}(n_j)\gamma u}_{\text{Utility from trade}} + \underbrace{(\beta - \mu_j)(1-\gamma)u}_{\text{Cost of holding money}} + \underbrace{\beta W_b(0,0)}_{\text{Continuation value}} \tag{14}$$

2.5 Buyers' marketplace choice

Next, we formalize the buyer's marketplace selection as a random discrete choice problem. In this framework, the buyer's decision is influenced not only by the anticipated value each marketplace offers but also by an idiosyncratic choice shock and a behavioral bias toward inflation. In our interpretation, a buyer's actual experienced payoff is $W_b^j(0,0)$ but at the choice stage each buyer l uses their perceived payoffs $\hat{W}_{l,b}^j$ plus a white noise η_{jl} to choose between the two marketplaces. The idiosyncratic noise term η_{jl} captures the randomness of the choice stage. Formally, the perceived payoff of buyer l is given by:

$$\hat{W}_{l,b}^j(\xi) = -f_j + \beta a_{jb}(n_j)\gamma u + (\beta - \xi\mu_j)(1-\gamma)u + \beta W(0,0) \tag{15}$$

where the parameter $\xi \in [0,1]$ captures the salience of inflation (from either outside money or platform money) to the buyer. That is, the buyer does not fully account for inflation costs when choosing between marketplaces. Thus, the perceived advantage of platform (P) over legacy (L) marketplace is

$$\Delta_b \equiv \underbrace{\beta\left(a_{Pb}(n_P) - a_{Lb}(n_L)\right)\gamma u}_{\text{Utility from trade difference}} + \underbrace{\xi(\mu_L - \mu_P)(1-\gamma)\mu}_{\text{Inflation cost difference}} + \underbrace{(f_L - f_P)}_{\text{Fee difference}}. \tag{16}$$

It is important to note that the salience of inflation can either help or hinder platform P's ability to attract buyers. When the legacy marketplace faces higher inflation set by the central bank, any inflation cost savings offered by platform P become less compelling to buyers, who tend to discount these savings. Conversely, if the legacy marketplace experiences lower inflation, platform P can afford to impose a higher inflation rate, and buyers may not fully account for this increased inflation in their decision-making.

Hence, the probability of a buyer choosing market P in the CM can be determined by the following attraction function $\Pi_b(\cdot)$:

$$\Pi_b(\Delta_b) = \Pr\left\{l : \Delta_b \geq \eta_{Ll} - \eta_{Pl}\right\}. \tag{17}$$

This attraction function also yields the fraction of buyers choosing platform P.

2.5.1 Sellers

Sellers do not want to hold any additional money in the CM and would convert all the money they have received (if matched) to CM goods immediately to avoid facing inflation cost in the next period. A seller's value function in steady state if he chooses to enter market j to trade is then:

$$W_s^j(0,0) = -\underbrace{k_j}_{\text{Fee}} + \underbrace{\beta a_{js}(n_j)(1-\gamma)u}_{\text{Utility from trade}} + \underbrace{\beta W_s(0,0)}_{\text{Continuation value}} \tag{18}$$

The attraction function of a seller for platform P over legacy L can be similarly defined as $\Pi_s(\Delta_s)$ where

$$\Delta_s = \beta\left(a_{Ps}(n_P) - a_{Ls}(n_L)\right)(1-\gamma)u + (k_L - k_P), \tag{19}$$

and a seller also faces an idiosyncratic choice shock. The attraction function $\Pi_s(\Delta_s)$ yields the fraction of sellers who trade on P.

2.5.2 Legacy and Platform Owners

Marketplaces often use multi-channel marketing and set fees independently to customers who come to the marketplace through different channels. Although buyers and sellers enter the marketplace through various channels, they all face search frictions and find trading partners using the same matching technology. As a result, we assume that marketplace owners take matching probabilities (or equivalently equilibrium market tightness) as given and maximize their revenue by optimally setting the entry fees for sellers and buyers ($k_j \geq 0$ and $f_j \geq 0$ where $j \in \{P, L\}$) and choosing the rate of money growth in the case of platform P.[3]

We first study the platform P owner's optimization problem who chooses the sellers' entry fee k_P, money growth rate μ_P, and the buyers' entry fee f_P to maximize:

$$\underbrace{\bar{N}_s \Pi_s(\Delta_s) k_P}_{\text{Fee revenue from sellers}} + \underbrace{\bar{N}_b \Pi_b(\Delta_b) f_P}_{\text{Fee revenue from buyers}} + \underbrace{\left(M_{t+1}^P - M_t^P\right) \phi_t^P}_{\text{seigniorage}}. \tag{20}$$

where M_t^P is the supply of platform money at t, and Δ_b and Δ_s are given by (16) and (19).

Using the market clearing condition for platform money and (13), we obtain

$$M_t^P = \bar{N}_b \Pi_b(\Delta_b) m_{b,t}^P = \bar{N}_b \Pi_b(\Delta_b) \frac{u(1-\gamma)}{\phi_t^P}. \tag{21}$$

Combining this expression with (20), the objective function of the platform P owner becomes:

$$\bar{N}_s \Pi_s(\Delta_s) k_P + \bar{N}_b \Pi_b(\Delta_b) \left[u(1-\gamma)(\mu_P - 1) + f_P\right]. \tag{22}$$

We are now ready to take first order conditions with respect to fees and rate of money growth. The first order condition with regard to the seller fee k_P gives:

$$k_P = \frac{\Pi_s(\Delta_s)}{\Pi_s'(\Delta_s)}. \tag{23}$$

That is, the fee is set so that the marginal increase in the seller fee revenue from the fee levied on all sellers who choose to enter is equal to the marginal loss from those choose not

[3] Thinking of a marketplace as a multi-channel market that pools its revenues is reminiscent of the "Lucas family" (Lucas (1990)) and simplifies the equilibrium construction.

to enter.

The first order condition with regards to the buyer fee f_P gives:

$$\Pi_b(\Delta_b) - \Pi_b'(\Delta_b)\left[u(1-\gamma)(\mu_P - 1) + f_P\right] \leq 0, \text{ with equality if } f_P > 0. \tag{24}$$

The first order condition with regards to the rate of money growth μ_P gives:

$$\Pi_b(\Delta_b) - \Pi_b'(\Delta_b)\left[u(1-\gamma)(\mu_P - 1) + f_P\right]\xi \leq 0, \text{with equality if } \mu_P > 1. \tag{25}$$

Notice that for $\xi < 1$, we have the optimal solution as:

$$f_P = 0 \tag{26}$$

$$\mu_P = 1 + \frac{1}{\xi(1-\gamma)u}\frac{\Pi_b(\Delta_b)}{\Pi_b'(\Delta_b)} \tag{27}$$

This result implies that the platform exploits the behavioral bias ($\xi < 1$) to implement a form of obfuscated pricing. By substituting salient entry fees (f_P) with less salient inflation charges (μ_P), the platform effectively lowers the perceived price of participation to attract buyers, while maintaining or increasing real revenue.

That is, when buyers do not fully account for inflation costs, the owner of platform P prefers to charge buyers via an inflation mechanism rather than by imposing a direct fee. Conversely, if buyers fully internalize the inflation cost, the owner becomes indifferent between the two methods. In this case, his primary concern is to extract an optimal combined charge from buyers, which is given by $u(1-\gamma)(\mu_P - 1) + f_P = \Pi_b(\Delta_b)/\Pi_b'(\Delta_b)$, where μ_P represents the inflation rate and f_P is the buyer fee.

When the buyers are fully attentive ($\xi = 1$), the platform is indifferent between the two tools in collecting revenue – fees and inflation, but it strictly prefers to use its own money in a positive legacy inflation environment ($\mu_L > 1$). It is a subtle but important difference. Using platform money instead of outside money gives the platform two advantages. First, the platform can set a different inflation rate than the central bank to control the cost of inflation experienced by the buyers on the platform. Hence, the platform strictly prefers to use platform money as long as $\xi < 1$ since it is more effective to charge buyers through inflation than fees. This advantage is demonstrated by the perceived advantage of platform over the legacy marketplace for buyers expressed in (16). Second, it earns seigniorage from issuing new money when $\mu_P > 1$. When $\mu_L > 1$, adopting outside money would mean losing this seigniorage income and the platform strictly prefers adopting its own money on its marketplace. Only when both $\mu_L = 1$ and $\xi = 1$ – a rare edge case – is the platform indifferent between using outside money and platform money, since both advantages of issuing its own money disappear. The salience parameter (ξ) mainly serves to add richness to the model and is not essential to money choice, as the next proposition shows.

Proposition 1 *Platform P strictly prefers to adopt its own money if either $\mu_L > 1$, or $\xi < 1$, or both.*

It is important, however, to distinguish the result in Proposition 1 from the "double-edged" effect of inflation salience shown in (16). While Proposition 1 shows that the platform benefits from low salience ($\xi < 1$) because it allows for more effective revenue collection from its own users, the same lack of attention may make it harder to compete with the legacy market. If buyers do not pay attention to inflation costs, they will not fully appreciate the savings offered by a low-inflation platform. Therefore, in a high legacy inflation environment, a low ξ actually reduces the platform's ability to attract new buyers from the legacy market.

We next study legacy market (L) owner's optimization problem which is simpler since legacy market's owner does not have control over the outside money supply. The optimization problem is to chooses f_L and k_L to maximize:

$$\bar{N}_s \left(1 - \Pi_s \left(\Delta_s\right)\right) k_L + \bar{N}_b \left(1 - \Pi_b \left(\Delta_b\right)\right) f_L. \tag{28}$$

The two first order conditions are:

$$k_L = \frac{1 - \Pi_s \left(\Delta_s\right)}{\Pi_s' \left(\Delta_s\right)}, \tag{29}$$

$$f_L = \frac{1 - \Pi_b \left(\Delta_b\right)}{\Pi_b' \left(\Delta_b\right),} \tag{30}$$

where Δ_b and Δ_s are given by (16) and (19).

2.6 Equilibrium Definition

Definition 1 *A stationary equilibrium consists of market tightness measures on the two platforms* (n_P^*, n_L^*)*, platform entry fees for the buyers and the sellers* (f_P^*, k_P^*)*, platform's money growth policy* μ_P^**, and legacy market entry fees for the buyers and the sellers* (f_L^*, k_L^*) *such that*

1. *In the CM buyers and sellers optimally choose which market to enter (and hold money of that market for trade).*

2. *Given* $(n_P^*, n_L^*, f_L^*, k_L^*)$ *and buyers' and sellers' entry decisions, platform's profit maximizing fees are* $f_P = f_P^*$ *and* $k_P = k_P^*$ *and its optimal money growth policy is* $\frac{M_{t+1}^P}{M_t^P} = \mu_P^*$*.*

3. *Given* $(n_P^*, n_L^*, f_P^*, k_P^*, \mu_P^*)$ *and buyers' and sellers' entry decisions, legacy market's profit maximizing fees are* $f_L = f_L^*$ *and* $k_L = k_L^*$*.*

4. *Market tightness on the two markets are given by*

$$\begin{aligned} \frac{\overline{N}_s \Pi_s(\Delta_s)}{\overline{N}_b \Pi_b(\Delta_b)} &= n_P^*, \\ \frac{\overline{N}_s (1 - \Pi_s(\Delta_s))}{\overline{N}_b (1 - \Pi_b(\Delta_b))} &= n_L^*. \end{aligned}$$

3 Equilibrium Properties

In the remainder of the paper we assume that the attraction functions take the following form: $\Pi_b(\Delta_b) = \left[1 + \exp\left(-\frac{\Delta_b}{\sigma_b}\right)\right]^{-1}$ and $\Pi_s(\Delta_s) = \left[1 + \exp\left(-\frac{\Delta_s}{\sigma_s}\right)\right]^{-1}$. These functional forms are standard in discrete choice where shocks to payoffs follow Gumbel distribution with scale parameters σ_b for buyers and σ_s for sellers.[4]

We next establish a lemma that links the comparison of market tightness on the platform versus the legacy market to the scale-normalized attractiveness of the platform to the sellers versus buyers.

[4]It becomes difficult to attract buyers (sellers) as the scale parameter increases. When σ_b (σ_s) approaches ∞, the likelihood of buyers (sellers) to enter the platform versus the legacy market approaches 0.5 regardless of Δ_b (Δ_s). At the other extreme, when σ_b (σ_s) approaches 0, all buyers (sellers) go to the platform if $\Delta_b > 0$ ($\Delta_s > 0$) and to the legacy if $\Delta_b < 0$ ($\Delta_s < 0$).

Lemma 1 *The platform has lower market tightnesses than the legacy market (i.e. seller buyer ratio is lower on the platform) iff its scale-normalized attractiveness is lower for sellers than for buyers, i.e.,*

$$n_P \leq n_L \Leftrightarrow \frac{\Delta_s}{\sigma_s} \leq \frac{\Delta_b}{\sigma_b}. \tag{31}$$

This lemma allows us to show that platform P has a unique advantage in attracting buyers by controlling its own money supply and hence offers sellers a higher matching probability.

Proposition 2 *In equilibrium, sellers are more likely to be matched on the platform than on legacy market, i.e.* $a_{Ps}(n_P) > a_{Ls}(n_L)$.

A corollary of this result is that if two marketplaces have similar matching technologies, platform P has lower seller to buyer ratio than the legacy marketplace.

Corollary 1 *If* $a_{Ps}(n) - a_{Ls}(n) \geq 0$ *is small enough for all* n *then* $n_P < n_L$.

This follows directly from the previous proposition: if $a_{Ps} = a_{Ls}$ then $a_{Ps}(n_P) > a_{Ls}(n_L) \Rightarrow n_P < n_L$. By continuity this must also hold if a_{Ps} is close to a_{Ls}.

In search and matching models, cross-group positive network externalities are common: an increase in the number of buyers improves the matching probabilities for sellers, and vice versa. Since platform P has an advantage in attracting buyers from legacy market, it initiates the positive externalities from the buyer side that improves the seller matching probability. This dynamic enables platform P to leverage its advantage by drawing more sellers into its market while also charging sellers a higher fee. This finding is summarized in the next proposition.

Proposition 3 *Platform* P *charges a higher seller fee than the legacy market, i.e.,* $k_P > k_L$, *and attracts more sellers than the legacy market, i.e.* $\Delta_s > 0$.

The following proposition demonstrates the effect of the feedback loop of cross-group positive externalities in search and matching models. As more sellers are drawn in by the better matching probability on platform P, the buyer's matching probability is improved and more buyers choose to move from legacy market to platform P, creating a reinforced feedback loop.

Proposition 4 *There are more buyers on the platform.*

Taken together, Propositions 2 through 4 illustrate a reinforcing network effects loop. The platform uses its monetary control (or lower perceived cost) to attract a critical mass of buyers (Proposition 4). This higher buyer volume increases the matching probability for sellers (Proposition 2), making the platform more valuable to them. Consequently, the platform can extract this surplus by charging sellers higher fees than the legacy market (Proposition 3), without inducing them to leave.

The final proposition in this section compares platform inflation and outside money inflation and shows that when outside inflation is below a threshold platform inflation is below the outside inflation and otherwise it is above. Hence, all else equal, in environments where inflation rate of central bank money is low (high), we would expect the inflation rate on platform money to be high (low).

Proposition 5 *Suppose $\xi < 1$. There is a threshold value $\hat{\mu}_L > 1$ such that if $\mu_L \lesseqgtr \hat{\mu}_L$ then $\mu_P \gtreqless \mu_L$.*

It is clear that if $\mu_L = 1$, platform sets $\mu_P > 1$ to generate seigniorage. This proposition follows because as legacy inflation μ_L goes up, platform inflation μ_P increases at a rate less than one and eventually falls below the legacy inflation. To see why this strategic dampening occurs, note that as legacy inflation rises, the legacy market lowers its buyer fee to retain customers. Consequently, the platform owner must take into account both the higher legacy inflation and the reduced legacy buyer fee, resulting in less than a one-for-one increase in its inflation.

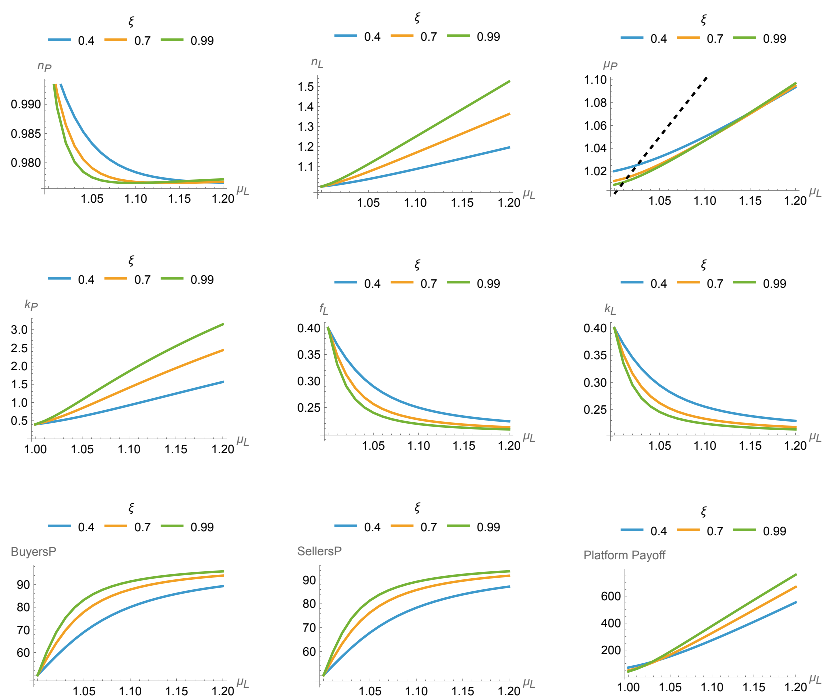
4 Comparative Statics: Numerical Exercises

4.1 Identical Matching Technology

In our initial set of numerical analysis, we assume that the platform and legacy market share the same matching technology. Hence, the numerical findings in this subsection focus exclusively on the platform's unique advantage over the legacy market - its ability to control its money supply and collect seigniorage.

Figure 2 summarizes our key findings regarding buyers' inflation salience by illustrating how variations in inflation salience affect equilibrium outcomes. Each graph plots legacy inflation on the x-axis against one outcome variable on the y-axis (e.g. market tightness, inflation rates, seller fees, the number of buyers and sellers, the platform owner's payoff, and buyer and seller fees in the legacy market). In every graph, three lines represent different levels of inflation salience (e.g., 0.4, 0.7, and 0.99).

Figure 2: Legacy Inflation: Salience



The graphs have μ_L on x-axis and an outcome variable (n_P, n_L, μ_P, k_P, f_L, k_l, buyer number on platform (BuyersP), seller number on platform (SellersP), platform owner's payoff) on y-axis. $\xi = 0.4$ in the blue line, $\xi = 0.7$ in the orange line, and $\xi = 0.99$ in the green line. Legacy inflation μ_L in the black dashed line. Parameters: $\alpha_P = 0.1$; $\alpha_L = 0.1$; $\rho = 0.5$; $\beta = 0.9$; $u = 100$; $\sigma_b = 0.2$; $\gamma = 0.5$; $\sigma_s = 0.2$; $\bar{N}_s = 100$; $\bar{N}_b = 100$.

In the third graph on the top row, we observe that the platform's inflation policy exhibits strategic dampening. As legacy inflation rises, the platform raises its own inflation to capture seigniorage, but does so less than one-for-one. It strategically moderates its inflation increase to maintain a relative price advantage, preventing buyers from switching back to the legacy market. In other words, the platform provides buyers "a shelter" from rising legacy inflation.

Proposition 5 has explained that this subdued response occurs because, as fiat inflation rises, the legacy marketplace lowers its buyer fee to retain customers (see the middle graph in the middle row). Consequently, the platform owner must raise its own inflation in reaction to both the higher legacy inflation and the reduced buyer fee but not on a one-for-one basis since the platform does not charge a buyer fee. Therefore, for low legacy inflation, the platform inflation exceeds the legacy inflation, and for high legacy inflation, the opposite holds.

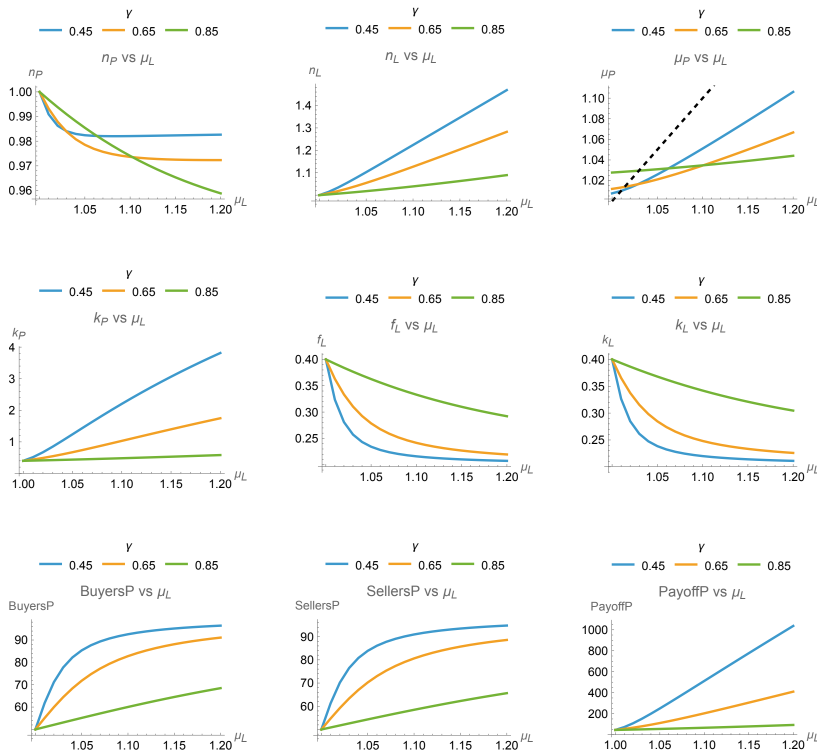
The first and the second graphs on the bottom row demonstrate that, as legacy inflation rises, the platform attracts more buyers and sellers. This attraction is the strongest when buyers fully account for inflation costs (i.e., with a higher ξ). In other words, if buyers are less sensitive to inflation, it becomes more challenging for the platform to attract them (as evidenced by the green lines ($\xi = 0.99$) lying above the blue lines ($\xi = 0.4$) in these graphs). Nonetheless, the platform's ability to control its own inflation and collect seigniorage income enables it to reduce buyer's participation costs, thereby drawing a large fraction of buyers and sellers away from the legacy market.

This advantage, in turn, allows the platform to charge a higher seller fee - since its increased buyer base improves matching probabilities for sellers (see k_P in the first graph, middle row) - while the legacy market is forced to lower its seller fee (k_L in the third graph, middle row). As a result, the market tightness (measured by the seller-buyer ratio) is lower on the platform (n_P, first graph, top row) and higher in the legacy market (n_L in the second graph, top row) as legacy inflation increases.

Finally, the third graph on the bottom row illustrates the double-edged effect of inflation salience. On the one hand, low inflation salience allows the platform to set higher inflation and generate more seigniorage. On the other hand, it makes the legacy inflation appear to be less costly to buyers. When legacy inflation is low, the first effect dominates and the platform's payoff is higher under low inflation salience (the blue line lies above the green line). As the legacy inflation rises, eventually the second effect dominates and the platform earns more under high inflation salience (the green line then lies above the blue).

Using the same set of graphs and parameters (with $\xi = 0.8$), Figure 3 summarizes how variations in buyer's bargaining power affect equilibrium outcomes. In each graph, three lines represent different levels of buyer's bargaining power (for example, 0.45, 0.65, and 0.85). The

Figure 3: Legacy Inflation: Bargaining Power



The graphs have μ_L on x-axis and an outcome variable (n_P, n_L, μ_P, k_P, f_L, k_l, buyer number on platform (BuyersP), seller number on platform (SellersP), platform owner's payoff) on y-axis. $\gamma = 0.45$ in the blue line, $\gamma = 0.65$ in the orange line, and $\gamma = 0.85$ in the green line. Legacy inflation μ_L in the black dashed line. Parameters: $\alpha_P = 0.1$; $\alpha_L = 0.1$; $\xi = 0.8$; $\rho = 0.5$; $\beta = 0.9$; $u = 120$; $\sigma_b = 0.2$; $\sigma_s = 0.2$; $\bar{N}_s = 100$; $\bar{N}_b = 100$.

figure shows that as buyers bargaining power decreases, the platform's profit increases, for instance, in the third graph (bottom row), the blue line (indicating the lowest bargaining power) lies above the others.

Intuitively, when buyers have low bargaining power, they must pay higher prices for DM goods from sellers. As a result, they are required to hold more platform money for their transactions, which in turn gives the platform a greater advantage over the legacy market. We see this advantage in the third graph on the third row where platform's payoff is higher when the buyers' bargaining power is lower. However, as the third graph on the first row show bargaining power has a subtle impact on platform inflation. When legacy inflation is low, platform faces stronger competition and sets lower inflation when buyers hold more money (i.e., the blue line is below all other lines). As legacy inflation increases, this competitive pressure eases and the platform charges higher inflation when buyers hold more money to generate more seigniorage revenue (i.e., the blue line is above all other lines).

Platform's enhanced competitive advantage under low buyer bargaining power means that the platform is able to attract more buyers and, in turn, more sellers (see the first and the second graph on the third row). This advantage also allows the platform to charge sellers higher fees. As a result, the legacy market faces more competition for both buyers and sellers and must set lower fees for buyers and, surprisingly also for sellers (In both the second and third graphs on the second row, the blue line is the lowest and the green line is the highest). Finally, the market tightness is higher for legacy under low bargaining power but the effect on the market tightness is more complex, as the platform balances the influx of new buyers with the need to extract higher fees from sellers.

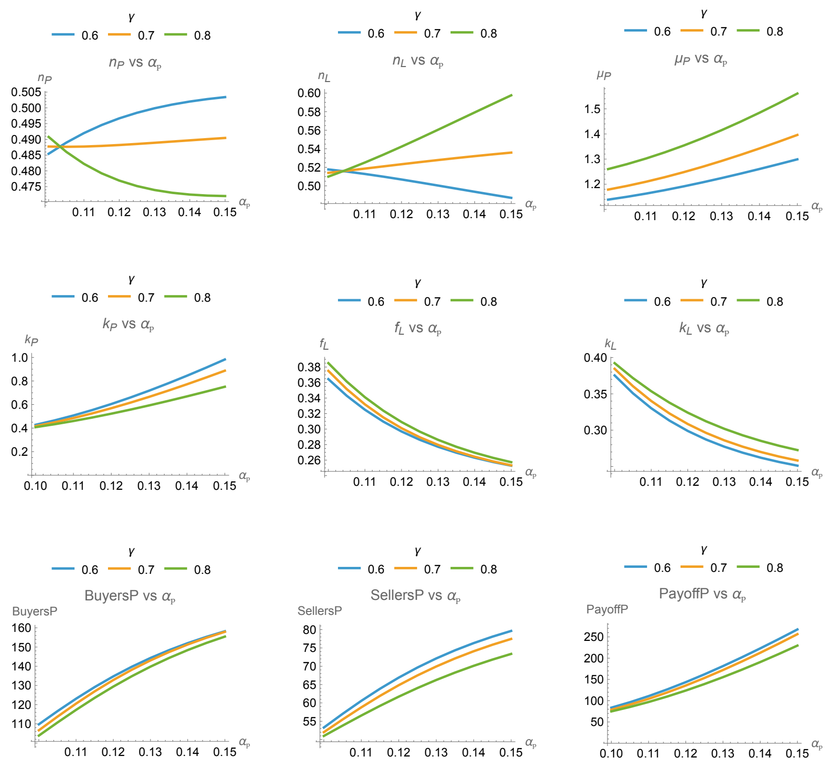
4.2 Better Platform Matching Technology

Next, we study the impact of better matching technology. As the platform matching technology improves, the likelihood of matches and the expected trade surplus generated on the platform both increase, introducing additional tradeoffs relative to the identical technology case. We use either buyer bargaining power γ or inflation salience ξ to measure the extent of the advantage of platform over legacy by having private money and α_P to measure the superiority of platform matching technology.

Figure 4 examines how variations in buyer bargaining power affect equilibrium outcomes. In these graphs, α_P is plotted on the x-axis while the y-axis represents a specific outcome variable. Each graph includes three lines corresponding to different levels of γ (e.g., 0.6, 0.7, and 0.8).

Our findings indicate that as the platform's matching technology improves, several key variables increase, including the fraction of buyers on the platform, the number of sellers attracted, the seller fee, platform inflation, and the platform owner's payoff. At the same time, the seller-to-buyer ratio (market tightness) might increase or decreases with further technological improvements. This pattern suggests that while the platform earns additional seigniorage income by attracting more buyers through its private money system, superior

Figure 4: Bargaining Power with Better Platform Technology



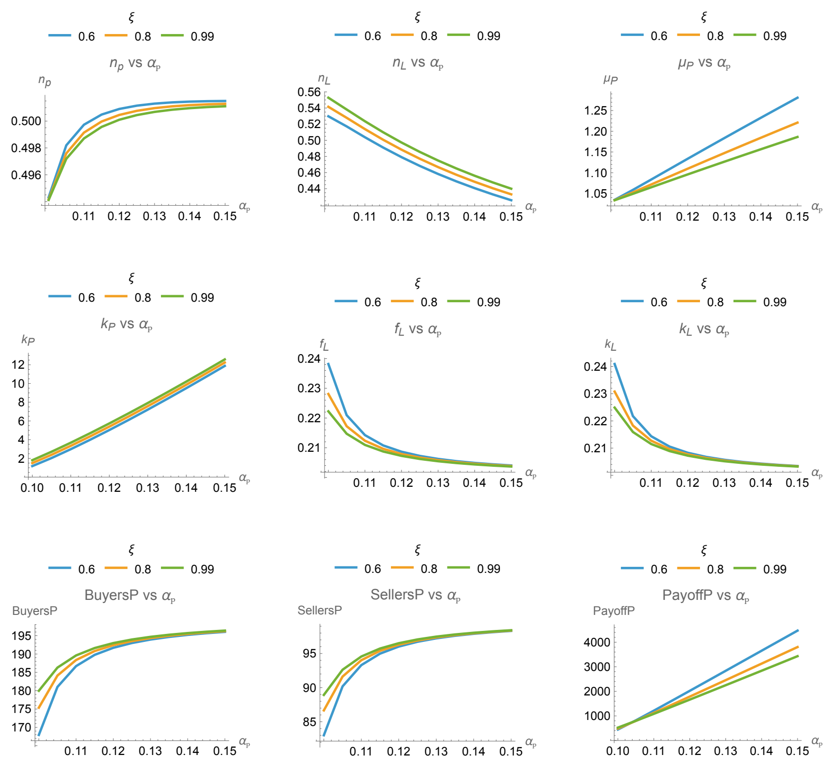
The graphs have α_P on x-axis and an outcome variable (n_P, n_L, μ_P, k_P, f_L, k_l, buyer number on platform (BuyersP), seller number on platform (SellersP), platform owner's payoff) on y-axis. $\gamma = 0.6$ in the blue line, $\gamma = 0.7$ in the orange line, and $\gamma = 0.8$ in the green line. Parameters: $\alpha_L = 0.1$; $\rho = 0.5$; $\beta = 0.9$; $u = 10$; $\sigma_b = 0.2$; $\xi = 0.8$; $\mu_L = 1.05$, $\sigma_s = 0.2$; $\bar{N}_s = 100$; $\bar{N}_b = 200$.

technology also boosts the number of matches and enables higher seller fee extraction. The combination of these effects makes it more profitable for the platform to adjust the balance between buyers and sellers.

Moreover, buyer bargaining power plays a crucial role in these dynamics. We see in the third graph on the first row that the sensitivity of platform inflation to improvements in technology is lower when buyer bargaining power is lower. This is because as the matching technology improves, most of the increase in expected gains from trade accrue to the sellers when the buyers bargaining power is lower. As a result, the platform raises its inflation at a lower rate.

Next, we examine how variations in buyer's inflation salience affect equilibrium outcomes

Figure 5: Inflation Salience with Better Platform Technology



The graphs have α_P on x-axis and an outcome variable (n_P, n_L, μ_P, k_P, f_L, k_l, buyer number on platform (BuyersP), seller number on platform (SellersP), platform owner's payoff) on y-axis. $\xi = 0.6$ in the blue line, $\xi = 0.8$ in the orange line, and $\xi = 0.99$ in the green line. Parameters: $\alpha_L = 0.1$; $\rho = 0.5$; $\beta = 0.9$; $u = 120$; $\sigma_b = 0.2$; $\gamma = 0.5$; $\mu_L = 1.05$, $\sigma_s = 0.2$; $\bar{N}_s = 100$; $\bar{N}_b = 200$.

in Figure 5. Each graph plots bargaining power on the x-axis against one outcome variable on the y-axis. In every graph, three lines represent different levels of ξ (e.g., 0.6, 0.8, and 0.99).

Figure 5 offers an additional insight: improved matching technology amplifies the platform's advantage in controlling its own money supply. This effect is most evident in the third graph on the first row, which shows that as matching technology improves, the platform's inflation rate rises more rapidly when inflation salience is low. Consequently, even when legacy inflation is high, the platform earns greater revenue under low inflation salience if it has superior matching technology (see the third graph on the last row).

This result contrasts with the case of identical matching technology (refer to the third graph on the bottom row in Figure 2). In that scenario, high legacy inflation combined with low inflation salience disadvantages the platform, because legacy buyers do not fully internalize the true cost of inflation. In the case of superior matching technology, however, the platform can offset this drawback by increasing its inflation rate more aggressively, thereby collecting more seigniorage income.

5 The Planner's Problem

In this section, we analyze social welfare by studying the planner's solution. The planner's objective is to maximize the total utility for all buyers, sellers, and the owners of marketplaces in this economy. Given that all the transactions in the CM as well as the payments from buyers to sellers are transfers between agents, maximization of total utility is equivalent to maximization of the total surplus from trade in DM.[5] Hence planner's problem can be stated as:

$$\begin{aligned} \max_{\Pi_b,\Pi_s} \quad & \overline{N}_b \gamma u \left[\Pi_b a_{Pb}(n_P) + (1-\Pi_b) a_{bL}(n_L)\right] \\ & + \overline{N}_s \left(1-\gamma\right) u \left[\Pi_s a_{Ps}(n_P) + (1-\Pi_s) a_{Ls}(n_L)\right] \end{aligned} \tag{32}$$

and subject to the market clearing conditions:

$$n_P = \frac{\overline{N}_s \Pi_s}{\overline{N}_b \Pi_b} \text{ and } n_L = \frac{\overline{N}_s \left(1-\Pi_s\right)}{\overline{N}_b \left(1-\Pi_b\right)}. \tag{33}$$

Note that we allow the planner to allocate the shares of buyers (Π_b) and of sellers (Π_s) to each marketplace directly. Clearly, any allocation that the planner can achieve by choosing fees and money growth rates, she can also achieve by directly allocating buyers and sellers. In fact, the opposite is also true. The planner can achieve any allocation of buyers and sellers by choosing the fees to buyers and sellers appropriately.

We can simplify the above objective function using $a_{js}(n_j) = a_{jb}(n_j)/n_j$ and plugging in for the expressions of n_P and n_L. The objective becomes:

$$\max_{\Pi_b,\Pi_s} \Pi_b a_{Pb}(n_P) + (1-\Pi_b) a_{Lb}(n_L). \tag{34}$$

[5] See the appendix for a formal derivation.

That is, maximizing the trading surplus is equivalent to maximizing the combined matching probabilities for buyers on the two marketplaces.

Proposition 6 *When the matching technology is symmetric across marketplaces, the planner's solution is* $n_P = n_L = \overline{N}_s/\overline{N}_b$*. When platform* P *has superior matching technology, the planner's solution is* $\Pi_s = \Pi_b = 1$*.*

Due to the concavity of the matching function, in any marketplace where there is trade, it is optimal to set market tightness equal to $\overline{N}_s/\overline{N}_b$. With symmetric technology any allocation of buyers and sellers to the two marketplaces that preserves the optimal tightness is socially optimal. When platform P has superior matching technology, it is optimal to have all sellers and buyers on the platform which automatically preserves the optimal tightness.

Recall from Corollary 1 that $n_P < n_L$. This leads to the next corollary.

Corollary 2 *When the matching technology is the same cross two marketplaces, competitive equilibrium with private money does not achieve the social optimum because the seller-buyer ratio on platform* P *is too low.*

The inefficiency arises because private money acts as a subsidy for buyers. This subsidy leads to excessive buyer entry, creating a congestion effect where the seller-to-buyer ratio (n_P) drops below the socially optimal level ($\overline{N}_s/\overline{N}_b$). The platform's private incentive to maximize seigniorage thus misaligns with the social goal of maximizing matching efficiency, unless the platform's matching technology is sufficiently superior to compensate for this distortion. Thus, the welfare loss stems not just from search frictions, but from the platform's strategic use of money as a tool for rent extraction rather than efficiency.

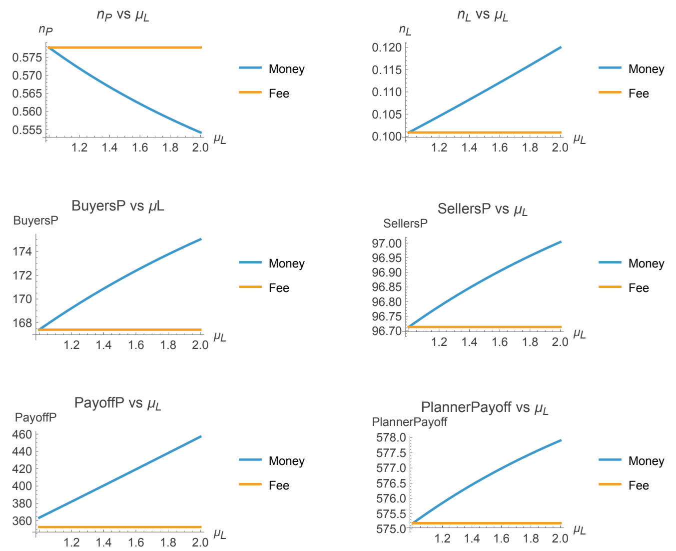
However, this monetary wedge between private and social incentives can act as a *second-best instrument* to correct search frictions when technologies are asymmetric. If the platform possesses a superior matching technology, the social optimum requires shifting a large mass of buyers to the platform. By issuing private money, the platform effectively subsidizes buyer entry to capture seigniorage. While this instrument is profit driven, it coincidentally aligns with the social goal of relocating agents to the more efficient marketplace. Thus, platform money can improve welfare not by eliminating the congestion externality, but by counteracting the friction that prevents agents from adopting superior technology. The following proposition states this result formally.

Proposition 7 *When platform P has strictly better matching technology, allowing private money in the decentralized equilibrium can achieve a better outcome from the planner's perspective than the case where private money is not allowed.*

The proof of Proposition 7 follows the following logic. When platform P has superior matching technology, the social optimum is for all buyers and sellers to be on the platform. There are two important outcome differences between the competitive equilibrium and the social optimum: the market tightness is generically not equal to $\overline{N}_s/\overline{N}_b$ and the number of buyers or sellers are too low on the platform. Controlling platform money helps platform P to attract buyers, moving the competitive equilibrium towards the social optimum. However it also has a cost, which is that as more buyers come to platform, there are not enough sellers to enter the platform due to the high entry cost set by the platform's owner. Because of the concavity of the matching function, the probability of buyers being matched is not increasing fast enough, causing the competitive outcome to deviate from the social optimum.

We also illustrate the result in Proposition 7 in a numerical example shown in Figure 6. In this example, we compare how the planner's payoff and each of the other equilibrium outcomes (n_P, n_L, number of buyers (BuyerP), number of sellers (SellerP), and platform's payoff) varies with the legacy inflation for the following two cases: the case when the platform is allowed to use private money (labeled as money in the blue line) and the case where the private platform money is not allowed (labeled as fee in the orange line). In these graphs, the legacy inflation is plotted on the x-axis while the y-axis represents a specific outcome variable. We observe that when the platform is not allowed to use private money and charges a fee denominated in outside money, the equilibrium outcomes do not vary with the legacy inflation. This is because in this case legacy inflation does not affect how platform competes with the legacy marketplace as both experience the same inflation rate. The second graph on the bottom row shows that allowing private money in the equilibrium the planner achieves a higher payoff than the fee only outcome. In the equilibrium where the platform money is allowed, the market tightness on the platform is lower (further away from the social optimum), but the number of buyers on the platform is larger (closer to the social optimum),

Figure 6: Money vs Fiat-Fee



The graphs have μ_L on x-axis and an outcome variable (n_p, n_L, buyer number on platform (BuyersP), seller number on platform (SellersP), platform owner's payoff, planner's payoff) on y-axis. Platform money in the blue line and platform fee (denominated in outside money) only in the orange line. Parameters: $\alpha_P = 1$, $\alpha_L = 0.1$; $\rho = 0.5$; $\beta = 0.9$; $u = 1$; $\sigma_b = 0.2$; $\sigma_s = 0.05$; $\gamma = 0.5$; $\bar{N}_s = 100$; $\bar{N}_b = 200$; $\xi = 0.95$.

than the case when platform is only allowed to charge a fixed fee.

6 Should the platform accept outside money?

In the baseline model, the platform strictly prefers to use platform money and has no incentive to accept outside fiat money. In practice, however, many platforms accept both platform money and outside payment options. To capture this, we extend the model by distinguishing between flexible buyers, who are indifferent to the payment medium, and biased buyers, who prefer outside money. We show that as long as flexible buyers exist, the platform always optimally issues its own currency to capture seigniorage. Consequently, the strategic decision reduces to whether to additionally accept outside money (Open Policy) or to enforce exclusivity (Closed Policy).

We uncover a trade-off between market share and strategic segmentation. By accepting outside money (Open policy) the platform competes directly for all buyers, maximizing its potential user base. However, by rejecting outside money (Closed Policy), the platform effectively cedes the biased buyers to the legacy market. This strategic segmentation can soften competition: it turns the legacy market into a monopoly over the biased segment, encouraging it to raise fees. This high-fee environment, in turn, allows the platform to extract higher seigniorage from its own flexible users.

To simplify the analysis we fix the number of sellers on each market and focus only on the buyer side of the market. Specifically, we assume that the number of sellers on the platform is $\bar{N}_s \Pi_s$ and on the legacy market is $\bar{N}_s(1-\Pi_s)$. We also assume that $\mu_L > 1$.

Buyers are heterogeneous in their payment preferences. A fraction ϕ are flexible (indifferent to the medium of exchange), while $1-\phi$ are biased: they incur a utility cost κ when using platform money. In this environment, we compare the following two policies available to the platform:

(i) Closed policy: accept only platform money, as in the baseline model; or

(ii) Open policy: accept both platform and outside money.

We denote the variables under the closed policy with the superscript c and write the platform's and the legacy market's objective functions respectively as:

$$\bar{N}_b \left(\phi \Pi_b \left(\Delta_b^c\right) + (1-\phi)\, \Pi_b \left(\Delta_b^c - \kappa\right)\right) \left[u\, (1-\gamma)\, (\mu_P^c - 1)\right] \tag{35}$$

and

$$\bar{N}_b \left(1 - \phi \Pi_b \left(\Delta_b^c\right) - (1-\phi)\, \Pi_b \left(\Delta_b^c - \kappa\right)\right) f_L^c \tag{36}$$

where the perceived advantage of the platform over the legacy market for the flexible buyers, Δ_b^c, is defined analogously to (16):

$$\Delta_b^c = \beta \left(a_{Pb}(n_P^c) - a_{Lb}(n_L^c)\right) \gamma u + \xi \left(\mu_L - \mu_P^c\right) (1-\gamma)\, u + f_L^c, \tag{37}$$

and market tightnesses are given by:

$$n_P^c = \frac{\bar{N}_s \Pi_s}{\left(\phi \Pi_b \left(\Delta_b^c\right) + (1-\phi)\, \Pi_b \left(\Delta_b^c - \kappa\right)\right)}, \tag{38}$$

$$n_L^c = \frac{\bar{N}_s (1 - \Pi_s)}{(1 - \phi \Pi_b (\Delta_b^c) - (1 - \phi) \Pi_b (\Delta_b^c - \kappa))}. \tag{39}$$

Note that under the closed policy, the perceived advantage of the platform for the biased buyers is reduced by κ – the additional cost that these buyers experience when they hold platform money.[6]

Under the open policy we denote the perceived advantage of the platform over the legacy market for the flexible (the biased) buyers by Δ_b^{of} (Δ_b^{ob}) and the other variables with the superscript o. We characterize the equilibrium where flexible buyers use platform money and biased buyers use outside money. We write the platform's and the legacy market's objective functions respectively as:

$$\bar{N}_b \left(\phi \Pi_b \left(\Delta_b^{of} \right) [u (1 - \gamma) (\mu_P^o - 1)] + (1 - \phi) \Pi_b \left(\Delta_b^{ob} \right) f_P^o \right) \tag{40}$$

and

$$\bar{N}_b \left(1 - \phi \Pi_b \left(\Delta_b^{of} \right) - (1 - \phi) \Pi_b \left(\Delta_b^{ob} \right) \right) f_L^o \tag{41}$$

where

$$\Delta_b^{of} = \beta \left(a_{Pb}(n_P^o) - a_{Lb}(n_L^o) \right) \gamma u + \xi \left(\mu_L - \mu_P^o \right) (1 - \gamma) u + f_L^o, \tag{42}$$

$$\Delta_b^{ob} = \beta \left(a_{Pb}(n_P^o) - a_{Lb}(n_L^o) \right) \gamma u + \left(f_L^o - f_P^o \right), \tag{43}$$

$$n_P^o = \frac{\bar{N}_s \Pi_s}{\left(\phi \Pi_b \left(\Delta_b^{of} \right) + (1 - \phi) \Pi_b \left(\Delta_b^{ob} \right) \right)}, \tag{44}$$

$$n_L^o = \frac{\bar{N}_s (1 - \Pi_s)}{\left(1 - \phi \Pi_b \left(\Delta_b^{of} \right) - (1 - \phi) \Pi_b \left(\Delta_b^{ob} \right) \right)}. \tag{45}$$

Under the open policy, flexible buyers use platform money and the biased buyers use outside

[6] We assume that this cost is fixed and does not depend on the amount of money that the biased buyers hold. It is easy to show that all the results in this section go through even if the cost is proportional to the amount of money that the biased buyers hold.

money which introduces two incentive compatibility constraints. The first constraint, $\Delta_b^{of} \geq \Delta_b^{ob}$, requires that the flexible buyers use platform money and the second constraint, $\Delta_b^{of} - \kappa \leq \Delta_b^{ob}$, requires that the biased buyers use outside money. We next show that if the second policy is optimal then the first constraint cannot be binding and flexible buyers strictly prefer using platform money.

Lemma 2 *If the platform prefers the open policy and accepts both types of money then the flexible consumers strictly prefer to use platform money, i.e.,* $\Delta_b^{of} > \Delta_b^{ob}$.

Intuitively, the proof is based on the platform's unique competitive advantage to attract flexible consumers by controlling its own money supply – ie, setting a lower inflation while collecting the seignorage income.

So far, we have not considered the possibility that the platform accepts only outside money. Next, we show that as long as there are some flexible buyers, accepting only outside money cannot be optimal for the platform.[7] This result holds even if most buyers are biased and they have strong preference for outside money.

Proposition 8 *As long as* $\phi > 0$*, platform either uses only platform money or accepts both types of money but never uses only outside money.*

The proof requires a detailed comparison of the platform?s payoff under two cases: the case where the platform only allows the use of outside money and charges buyers a fee, versus the case where it allows both types of money, charging flexible buyers platform money inflation and biased buyers a fee. There are two effects that go in the same direction.

First, we show that, in absence of the strategic response from the legacy, that is, when the legacy marketplace fee is fixed, platform can always achieve a higher payoff by allowing both types of money. This is again due to the unique advantage of the platform to control its own money supply and collect seignorage.

Second, we show that, using both platform and outside money as opposed to only outside money elicits strategic response from the legacy marketplace leading to a higher platform payoff. When the platform allows both types of money, since flexible buyers prefer to use

[7]This result is different from Proposition 1 which shows that when *all* buyers are flexible, platform strictly prefers using platform money over outside money.

platform money (as shown in Lemma 2) and are attracted to the platform, legacy mainly compete with the platform over biased buyers rather than a mixed group of biased and flexible buyers. The means that legacy face a reduced competition since they only need to attract biased buyers now, and hence can raise its fees. In response to the higher fee in legacy, the platform can raise its charge to its buyers via both inflation and fee and obtain higher payoffs consequently. It is then easy to show that the combined effects always favour accepting both types of money as opposed to only outside money.

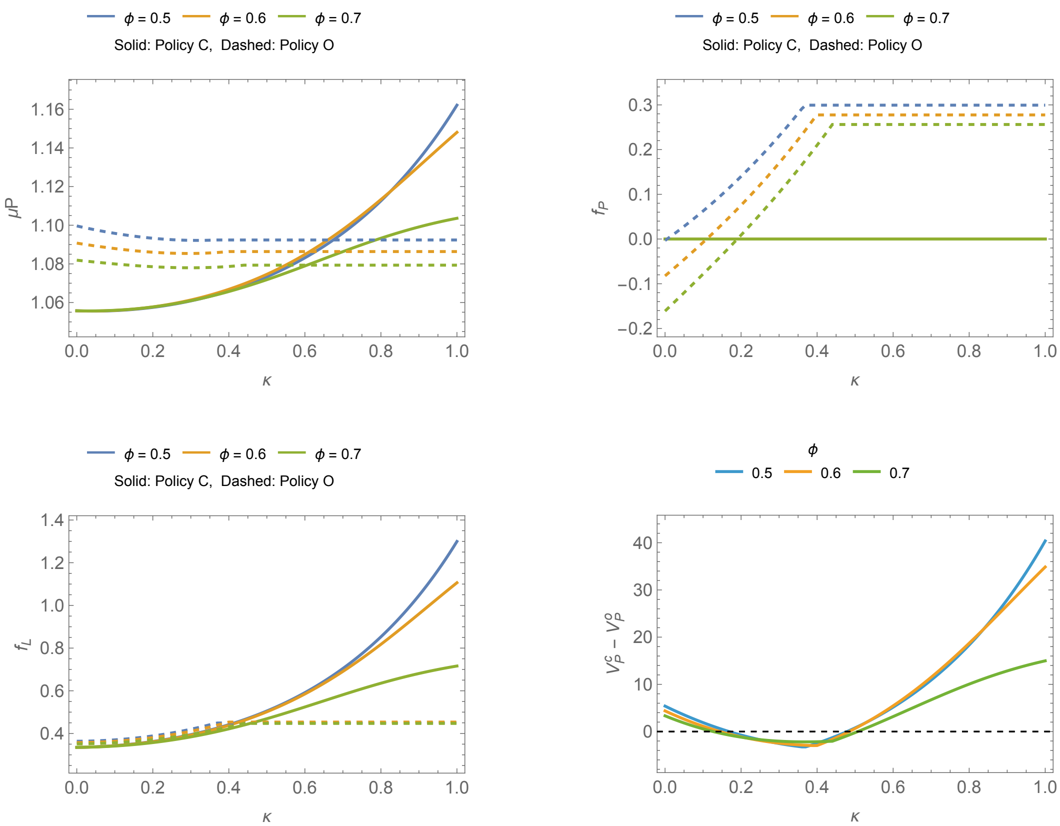
Next, we consider two edge cases to characterize platform's choice of allowing the use of outside money to highlight the underlying economic tradeoff. In the first edge case, the disutility of using platform money for biased buyers, κ, is small. In the second edge case, $\kappa = \infty$ so that biased buyers never hold platform money.

In the first edge case, if the platform follows the closed policy and accepts only platform money, it loses a very small fraction of biased buyers to the legacy market. When $\mu_L > 1$, the platform's profit from the biased buyers that it captures is higher when it charges them through inflating the platform money as opposed to a fee that is paid in outside money. Hence, when the bias is small enough, the platform strictly prefers the closed policy. The result in the following proposition is immediate.

Proposition 9 *There exists $\bar{\kappa} > 0$ such that when $\kappa < \bar{\kappa}$, the platform adopts the closed policy (accepts only platform money).*

In the second edge case, suppose $\kappa = \infty$ so that biased buyers never hold platform money. In this case there are thresholds $\underline{\phi} \leq \overline{\phi}$ such that if $\phi < \underline{\phi}$ then the platform follows the open policy and if $\phi > \overline{\phi}$ then the platform follows the closed policy. Underlying this result again are two effects. The first is a direct effect: with the closed policy the platform loses the biased buyers to the legacy market. The second is a strategic effect: with the closed policy the legacy market effectively becomes a monopolist for the biased buyers, raises its buyer fee, in turn allowing the platform to raise its seignorage income. When ϕ is low, the direct effect dominates and the open policy is optimal. When ϕ is high, the strategic effect dominates and the closed policy is optimal.

Figure 7: Comparison of the Closed and Open Policies



The graphs have κ on the x-axis and μ_p, f_P, f_L, and the platform's payoff under the closed (accept only platform money) versus the open policy (accept both forms of money) on the y-axis. Parameters: $\alpha_P = 0.1$, $\alpha_L = 0.1$; $\rho = 0.5$; $\beta = 0.9$; $u = 18$; $\gamma = 0.5$; $\sigma_b = 0.2$; $\sigma_s = 0.2$; $\bar{N}_b = 100$; $\xi = 0.99$.

Proposition 10 *Suppose $\kappa = \infty$. There exists thresholds $\underline{\phi}$ and $\overline{\phi}$ such that if $\phi < \underline{\phi}$ then the platform follows the open policy (accepts both outside money and platform money) and if $\phi > \overline{\phi}$ then the platform follows the closed policy (accepts only platform money).*

While it may seem intuitive that under high buyer bias κ the platform would accept outside money, Proposition 10 states that this is not necessarily the case. The deciding factor is the fraction of flexible buyers ϕ. If the fraction of flexible buyers is small, this intuition is correct and, the platform has to allow outside money in order to attract the large population of extremely biased buyers. However, if the fraction of flexible buyers is large enough, this intuition does not apply and platform chooses not to accept outside money at all.

This result is driven by strategic complementarity. As κ increases, the Closed Policy effectively segments the market: the legacy market captures the biased buyers, while the platform serves the flexible ones. Secure in its monopoly over the biased 'captive' segment, the legacy market optimally raises its buyer fees (f_L). Crucially, this higher legacy fee reduces the attractiveness of the outside option for flexible buyers. This relaxes the competitive constraint on the platform, allowing it to charge a higher inflation (μ_P) to its own flexible users without losing them. Thus, by refusing to compete for biased buyers, the platform induces a high-fee equilibrium that maximises its seigniorage revenue from the flexible majority.

These theoretical findings can also be demonstrated numerically for the intermediate cases where the disutility of using platform money for biased buyers, κ, is moderate, and the fraction of flexible buyers, $\phi \in (0, 1)$. We next conduct a numerical exercise, and in Figure 7 graph the equilibrium outcome variables: platform inflation, platform buyer fee, legacy buyer fee, and difference in platform payoff between the closed versus the open policy when κ varies from 0 to 1 and ϕ takes values 0.5, 0.6, and 0.7.

The bottom right panel of Figure 7 shows that platform follows the closed policy when κ is small or large, echoing the theoretical findings in Proposition 9 and Proposition 10. The numerical exercise illustrates that for the intermediate range of κ, open policy can become optimal. This result follows from our earlier discussion. As κ increases the direct effect can dominate the strategic effect favoring the open policy, but eventually the strategic effect overtakes the direct effect and the platform optimally follows the closed policy.

The discussion in this section does not involve inflation salience at all. It is important to note that again that the result of the paper is driven by structural market power rather than by this behaviour bias. It is a parameter that affects the degree of seignorage benefit enjoyed by the platform and potentially amplify or dampen the main result depending on the magnitude of the outside inflation.

7 Conclusion

Our analysis demonstrates that when a platform possesses the ability to issue its own money, it can strategically control its money supply to attract buyers. This increased buyer participation subsequently draws more sellers, thereby launching and amplifying network externalities between buyers and sellers - especially when combined with enhanced matching technologies. Moreover, our results indicate that such platforms exercise considerable market power over seller entry by imposing relatively high seller fees. While the platform's monetary policy creates inefficient congestion when technologies are similar, it can become socially desirable when the platform possesses superior matching technology, as the inflation subsidy effectively corrects search frictions that a fee-only model cannot.

Furthermore, our analysis explains why platforms often resist interoperability. We show that a "Closed Policy" (i.e., rejecting outside money) is not merely about forcing adoption, but about strategic segmentation. By ceding biased buyers to the legacy market, the platform induces its competitor to raise fees, creating a high-cost environment that paradoxically allows the platform to extract higher seigniorage from its own users. This suggests that "walled gardens" may be sustained not just by technological lock-in, but by an endogenous monetary strategy that softens price competition.

This study raises a critical policy question regarding the social welfare consequences of allowing platforms to maintain private payment systems. Currently, regulated financial institutions, regarded as trustworthy third parties, dominate payment systems. However, digital platforms are increasingly equipped with advanced data processing and machine learning capabilities that not only improve buyer-seller matching but also secure transactions. With the growing prevalence of platform-based economies, it is essential to examine whether these digital marketplaces, through their intrinsic economic synergy with payment systems, should be entitled to the seigniorage income traditionally captured by financial institutions.[8]

Ultimately, our findings offer a modern resolution to the classic debate on private money. While Friedman (1960) warned that private issuers would succumb to the temptation of inflation, and Hayek (1976) argued that competition would discipline them, we show that the truth lies in the business model of the issuer. When the issuer is a platform, the value

[8] Empirical policy experiences further underscore the relevance of this inquiry. For instance, following the easing of COVID-19 restrictions, cities and regional governments in China deployed e-coupons and e-voucher disbursed directly to resident's WeChat or Alipay wallet – i.e., platform money – to boost consumption. In contrast, the U.S. response to the pandemic involved dispersing Economic Impact Payments (or Stimulus checks) via direct deposits and bank-issued cards – i,e., bank money – under the $1.9 billion American Rescue Plan Act, without spending restrictions.

of "match-making" enterprise disciplines the currency competition. In the digital age, the viability of private money may depend less on the issuer's reputation and more on the value of the ecosystem it serves.

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A Appendix

A.1 Proof of Lemma 1

By the equilibrium condition:

$$\frac{\overline{N}_s \Pi_s(\Delta_s)}{\overline{N}_b \Pi_b(\Delta_b)} = n_P, \text{ and} \tag{A.1}$$

$$\frac{\overline{N}_s(1-\Pi_s(\Delta_s))}{\overline{N}_b(1-\Pi_b(\Delta_b))} = n_L. \tag{A.2}$$

Hence:

$$n_p \leq n_L \Leftrightarrow \Pi_s(\Delta_s) \leq \Pi_b(\Delta_b) \Leftrightarrow \frac{1}{1+\exp\left(-\frac{\Delta_s}{\sigma_s}\right)} \leq \frac{1}{1+\exp\left(-\frac{\Delta_b}{\sigma_b}\right)} \Leftrightarrow \frac{\Delta_s}{\sigma_s} \leq \frac{\Delta_b}{\sigma_b}.$$

A.2 Proof of Proposition 2

Suppose towards a contradiction that $a_{Ps}(n_P) \leq a_{Ls}(n_L)$. Then $n_P \geq n_L$ and by Lemma 1 $\frac{\Delta_s}{\sigma_s} \geq \frac{\Delta_b}{\sigma_b}$. Unpacking the expression of Δ_b using (16) and Δ_s using (19) and plugging optimal $f_L, f_P, k_P, k_L, \mu_P$ as in (23), (29), (30), (26), and (27), we obtain

$$\frac{1}{\sigma_s}\beta\left(a_{Ps}(n_P) - a_{Ls}(n_L)\right)(1-\gamma)\,u + \frac{1}{\sigma_s}\left(\frac{1-2\Pi_s\left(\Delta_s\right)}{\Pi_s'\left(\Delta_s\right)}\right) \geq$$

$$\frac{1}{\sigma_b}\left[\beta\left(\underbrace{a_{Pb}(n_P) - a_{Lb}(n_L)}_{\text{due to } n_P \geq n_L}\right)\gamma u + \underbrace{\xi\left(1-\gamma\right)u\left(\mu_L - 1\right)}_{\text{Control money supply}}\right] + \frac{1}{\sigma_b}\left(\frac{1-2\Pi_b\left(\Delta_b\right)}{\Pi_b'\left(\Delta_b\right)}\right). \quad \text{(A.3)}$$

Since $\mu_L > 1$, $a_{Ps}(n_P) \leq a_{Ps}(n_L)$, and $a_{Pb}(n_P) \geq a_{Lb}(n_L)$, (A.3) implies:

$$\frac{1}{\sigma_s}\left[\left(\frac{1-2\Pi_s\left(\Delta_s\right)}{\Pi_s'\left(\Delta_s\right)}\right)\right] \geq \frac{1}{\sigma_b}\left[\left(\frac{1-2\Pi_b\left(\Delta_b\right)}{\Pi_b'\left(\Delta_b\right)}\right)\right]. \quad \text{(A.4)}$$

By Gumble distribution, we know that:

$$\Pi_i\left(x\right) = \left[1+\exp\left(-\frac{\Delta_i}{\sigma_i}\right)\right]^{-1} \text{ and} \quad \text{(A.5)}$$

$$\Pi_i'\left(x\right) = \frac{1}{\sigma_i}\left[1+\exp\left(-\frac{\Delta_i}{\sigma_i}\right)\right]^{-2}\exp\left(-\frac{\Delta_i}{\sigma_i}\right), i \in \{b, s\}. \quad \text{(A.6)}$$

Plugging in for $\Pi_i(\cdot)$ and $\Pi_i'(\cdot)$, (A.4) implies:

$$\exp\left(-\frac{\Delta_b}{\sigma_b}\right)\exp\left(-\frac{\Delta_s}{\sigma_s}\right)\left[\left(\exp\left(-\frac{\Delta_s}{\sigma_s}\right)\right) - \left(\exp\left(-\frac{\Delta_b}{\sigma_b}\right)\right)\right] > \exp\left(-\frac{\Delta_b}{\sigma_b}\right) - \exp\left(-\frac{\Delta_s}{\sigma_s}\right).$$

Note that for this inequality to hold we must have:

$$\frac{\Delta_s}{\sigma_s} < \frac{\Delta_b}{\sigma_b}, \quad \text{(A.7)}$$

which is a contradiction.

A.3 Proof of Proposition 3

Using FOCs for k_P and k_L, we have $k_P > k_L$ if and only if $\frac{\Pi_s(\Delta_s)}{\Pi_s'(\Delta_s)} > \frac{1-\Pi_s(\Delta_s)}{\Pi_s'(\Delta_s)}$. This is true if and only if $\Pi_s\left(\Delta_s\right) > \frac{1}{2}$ if and only if $\Delta_s > 0$. Note $\Delta_s = \beta\left(a_{Ps}(n_P) - a_{Ls}(n_L)\right)(1-\gamma)\,u+$

$(k_L - k_P)$. The first term is strictly positive. Suppose that $k_P \leq k_L$, the second term is also positive and hence, $\Delta_s > 0$, which implies $k_P > k_L$. Thus, we must have $k_P > k_L$. We must also have $\Delta_s > 0$.

A.4 Proof of Proposition 4

More buyers on platform P if

$$\Delta_b = \beta \left(a_{Pb}(n_P) - a_{Lb}(n_L)\right) \gamma u + \xi \left(1-\gamma\right) u \left(\mu_L - 1\right) + \left(\frac{1 - 2\Pi_b\left(\Delta_b\right)}{\Pi_b'\left(\Delta_b\right)}\right) > 0. \tag{A.8}$$

Towards a contradiction, let us suppose that $\Delta_b < 0$ or equivalently $\Pi_b\left(\Delta_b\right) < 0.5$. In this case, it must be that $a_{Pb}(n_P) < a_{Lb}(n_L)$ since otherwise all the terms on the right side of the above equation are positive.

However, if $a_{Pb}(n_P) < a_{Lb}(n_L)$ then $n_P < n_L$, which by Lemma 1 implies $\frac{\Delta_s}{\sigma_s} < \frac{\Delta_b}{\sigma_b} < 0$.

But then $\Delta_s < 0$, which is a contradiction since we already established $\Delta_s > 0$ in Proposition 2. Then, we must have $\Delta_b > 0$.

A.5 Proof of Proposition 5

Taking total derivatives of n_P and n_L, we obtain:

$$\begin{aligned}
\frac{\partial n_P}{\partial \mu_L} &= n_P \left(\frac{\Pi_s'\left(\Delta_s\right)}{\Pi_s\left(\Delta_s\right)} \frac{\partial \Delta_s}{\partial \mu_L} - \frac{\Pi_b'\left(\Delta_b\right)}{\Pi_b\left(\Delta_b\right)} \frac{\partial \Delta_b}{\partial \mu_L}\right) \\
\frac{\partial n_L}{\partial \mu_L} &= n_L \left(\frac{\Pi_b'\left(\Delta_b\right)}{\left(1 - \Pi_b\left(\Delta_b\right)\right)} \frac{\partial \Delta_b}{\partial \mu_L} - \frac{\Pi_s'\left(\Delta_s\right)}{\left(1 - \Pi_s\left(\Delta_s\right)\right)} \frac{\partial \Delta_s}{\partial \mu_L}\right).
\end{aligned}$$

Taking total derivatives of Δ_b and Δ_s, we obtain:

$$\begin{aligned}
\frac{\partial \Delta_b}{\partial \mu_L} &= \beta \left(a_{Pb}'(n_P) \frac{\partial n_P}{\partial \mu_L} - a_{Lb}'(n_L) \frac{\partial n_L}{\partial \mu_L}\right) \gamma u + \xi \left(1-\gamma\right) u - \xi \frac{\partial \mu_P}{\partial \mu_L} \left(1-\gamma\right) u \\
&\quad - \left(1 + \frac{\Pi_b''\left(\Delta_b\right)\left(1 - \Pi_b\left(\Delta_b\right)\right)}{\left(\Pi_b'\left(\Delta_b\right)\right)^2}\right) \frac{\partial \Delta_b}{\partial \mu_L}, \text{ and} \\
\frac{\partial \Delta_s}{\partial \mu_L} &= \beta \left(a_{Ps}'(n_P) \frac{\partial n_P}{\partial \mu_L} - a_{Ls}'(n_L) \frac{\partial n_L}{\partial \mu_L}\right) \left(1-\gamma\right) u \\
&\quad - \left(2 + \frac{\Pi_s''\left(\Delta_s\right)\left(1 - 2\Pi_s\left(\Delta_s\right)\right)}{\left(\Pi_s'\left(\Delta_s\right)\right)^2}\right) \frac{\partial \Delta_s}{\partial \mu_L}.
\end{aligned}$$

Substituting for $\partial n_P/\partial \mu_L$ and $\partial n_L/\partial \mu_L$, we can rewrite these expressions as:

$$\frac{\partial \Delta_b}{\partial \mu_L}\left(2+\frac{\Pi_b''(\Delta_b)(1-\Pi_b(\Delta_b))}{(\Pi_b'(\Delta_b))^2}+\beta\left(a_{Pb}'(n_P)n_P\frac{\Pi_b'(\Delta_b)}{\Pi_b(\Delta_b)}+a_{Lb}'(n_L)n_L\frac{\Pi_b'(\Delta_b)}{(1-\Pi_b(\Delta_b))}\right)\gamma u\right)=$$
$$\beta\left(a_{Pb}'(n_P)n_P\frac{\Pi_s'(\Delta_s)}{\Pi_s(\Delta_s)}+a_{Lb}'(n_L)n_L\frac{\Pi_s'(\Delta_s)}{(1-\Pi_s(\Delta_s))}\right)\frac{\partial \Delta_s}{\partial \mu_L}\gamma u+\xi(1-\gamma)u\left(1-\frac{\partial \mu_P}{\partial \mu_L}\right),\text{ and}$$
$$\frac{\partial \Delta_s}{\partial \mu_L}\left(3+\frac{\Pi_s''(\Delta_s)(1-2\Pi_s(\Delta_s))}{(\Pi_s'(\Delta_s))^2}-\beta\left(a_{Ps}'(n_P)n_P\frac{\Pi_s'(\Delta_s)}{\Pi_s(\Delta_s)}+a_{Ls}'(n_L)n_L\frac{\Pi_s'(\Delta_s)}{(1-\Pi_s(\Delta_s))}\right)(1-\gamma)u\right)$$
$$=-\beta\left(a_{Ps}'(n_P)n_P\frac{\Pi_b'(\Delta_b)}{\Pi_b(\Delta_b)}+a_{Ls}'(n_L)n_L\frac{\Pi_b'(\Delta_b)}{(1-\Pi_b(\Delta_b))}\right)\frac{\partial \Delta_b}{\partial \mu_L}(1-\gamma)u.$$

From these two equations we obtain:

$$\frac{\partial \Delta_b}{\partial \mu_L}\left(2+\frac{\Pi_b''(\Delta_b)(1-\Pi_b(\Delta_b))}{(\Pi_b'(\Delta_b))^2}+\frac{\beta\left(a_{Pb}'(n_P)n_P\frac{\Pi_b'(\Delta_b)}{\Pi_b(\Delta_b)}+a_{Lb}'(n_L)n_L\frac{\Pi_b'(\Delta_b)}{(1-\Pi_b(\Delta_b))}\right)\left(3+\frac{\Pi_s''(\Delta_s)(1-2\Pi_s(\Delta_s))}{(\Pi_s'(\Delta_s))^2}\right)\gamma u}{\left(3+\frac{\Pi_s''(\Delta_s)(1-2\Pi_s(\Delta_s))}{(\Pi_s'(\Delta_s))^2}-\beta\left(a_{Ps}'(n_P)n_P\frac{\Pi_s'(\Delta_s)}{\Pi_s(\Delta_s)}+a_{Ls}'(n_L)n_L\frac{\Pi_s'(\Delta_s)}{(1-\Pi_s(\Delta_s))}\right)(1-\gamma)u\right)}\right)$$
$$=\xi(1-\gamma)u\left(1-\frac{\partial \mu_P}{\partial \mu_L}\right).$$

Taking total derivative of $\partial \mu_P/\partial \mu_L$, we get:

$$\frac{\partial \mu_P}{\partial \mu_L}=\frac{1}{\xi(1-\gamma)u}\left(1-\frac{\Pi_b''(\Delta_b)\Pi_b(\Delta_b)}{(\Pi_b'(\Delta_b))^2}\right)\frac{\partial \Delta_b}{\partial \mu_L}.$$

Solving these equations gives us:

$$\frac{\partial \mu_P}{\partial \mu_L}=\frac{\left(1-\frac{\Pi_b''(\Delta_b)\Pi_b(\Delta_b)}{(\Pi_b'(\Delta_b))^2}\right)}{\left(1-\frac{\Pi_b''(\Delta_b)\Pi_b(\Delta_b)}{(\Pi_b'(\Delta_b))^2}\right)+2+\frac{\Pi_b''(\Delta_b)(1-\Pi_b(\Delta_b))}{(\Pi_b'(\Delta_b))^2}+\frac{\beta\left(a_{Pb}'(n_P)n_P\frac{\Pi_b'(\Delta_b)}{\Pi_b(\Delta_b)}+a_{Lb}'(n_L)n_L\frac{\Pi_b'(\Delta_b)}{(1-\Pi_b(\Delta_b))}\right)\left(3+\frac{\Pi_s''(\Delta_s)(1-2\Pi_s(\Delta_s))}{(\Pi_s'(\Delta_s))^2}\right)\gamma}{\left(3+\frac{\Pi_s''(\Delta_s)(1-2\Pi_s(\Delta_s))}{(\Pi_s'(\Delta_s))^2}-\beta\left(a_{Ps}'(n_P)n_P\frac{\Pi_s'(\Delta_s)}{\Pi_s(\Delta_s)}+a_{Ls}'(n_L)n_L\frac{\Pi_s'(\Delta_s)}{(1-\Pi_s(\Delta_s))}\right)(1-\gamma)\right)}}. \tag{A.9}$$

Observe that the numerator and the first term of the denominator of Equation (A.9) are positive since

$$1-\frac{\Pi_b''(\Delta_b)\Pi_b(\Delta_b)}{(\Pi_b'(\Delta_b))^2}=-1+\frac{1}{(\Pi_b(x))(\exp(-x))}>0.$$

Moreover,

$$
\begin{aligned}
2+\frac{\Pi_b''(\Delta_b)(1-\Pi_b(\Delta_b))}{(\Pi_b'(\Delta_b))^2} &= 2+\frac{\left(2(\Pi_b(x))^3(\exp(-x))^2-(\Pi_b(x))^2\exp(-x)\right)(1-\Pi_b(\Delta_b))}{(\Pi_b(x))^4\exp(-x)^2} \\
&= \frac{2}{\Pi_b(x)}-\frac{1}{(\Pi_b(x))^2\exp(-x)}+\frac{1}{\Pi_b(x)\exp(-x)}>0.
\end{aligned}
$$

Finally, both the numerator and the denominator of the third term in the denominator of Equation (A.9) are positive. Hence, $\frac{\partial \mu_P}{\partial \mu_L}>1$.

A.6 Proof of Proposition 6

The first order condition with respect to $\Pi_b<1$ is:

$$
a_{Pb}(n_P)-a_{Pb}'(n_P)n_P \;\geq\; a_{Lb}(n_L)-a_{Lb}'(n_L)n_L, \tag{A.10}
$$

where the above condition holds with equality if $\Pi_b<1$.

The first order condition with respect to $\Pi_s<1$ is:

$$
a_{Pb}'(n_P) \;\geq\; a_{Lb}'(n_L), \tag{A.11}
$$

where the above condition holds with equality if $\Pi_s<1$.

We observe $a_{Pb}(0)=a_{Lb}(0)=0$. We assume that $a_{Pb}'(n)>a_{Lb}'(n), \forall n>0$. That is, the concave matching function for for the platform has a steeper slope for the same tightness than the legacy marketplace.

Let us suppose that $\Pi_s<1$. Then FOC gives $a_{Pb}'(n_P)=a_{Lb}'(n_L)$. By concavity of a_{Pb} and a_{Lb}, $n_P>n_L$. However, we find that

$$
\begin{aligned}
a_{Pb}(n_P)-a_{Pb}'(n_P)n_P &= \\
a_{Pb}(n_L)-a_{Pb}'(n_P)n_L+\int_{n_L}^{n_P}a_{Pb}'(n)dn-a_{Pb}'(n_P)(n_P-n_L) &> \\
a_{Pb}(n_L)-a_{Pb}'(n_P)n_L+\int_{n_L}^{n_P}a_{Pb}'(n_P)dn-a_{Pb}'(n_P)(n_P-n_L) &= \\
a_{Pb}(n_L)-a_{Pb}'(n_P)n_L &> \\
a_{Lb}(n_L)-a_{Lb}'(n_L)n_L.
\end{aligned}
$$

The second inequality holds because $a'_{Pb}(n) > a'_{Pb}(n_P)$ for all $n \in (n_L, n_P)$. Since the first order condition with respect to Π_b holds in strict inequality, therefore $\Pi_b = 1$. Plugging $\Pi_b = 1$ into the objective function, we get:

$$\max_{\Pi_s,\Pi_b} \Pi_b a_{Pb}\left(\frac{\overline{N}_s\Pi_s}{\overline{N}_b\Pi_b}\right) + (1-\Pi_b)a_{Lb}\left(\frac{\overline{N}_s(1-\Pi_s)}{\overline{N}_b(1-\Pi_b)}\right) = \max_{\Pi_s,\Pi_b} a_{Pb}\left(\frac{\overline{N}_s\Pi_s}{\overline{N}_b}\right).$$

This maximization problem implies that $\Pi_s = 1$ since a_{Pb} is increasing. Hence we must have $\Pi_s = 1$.

After proving that $\Pi_s = 1$, we now turn to show that $\Pi_b = 1$. Plugging $\Pi_s = 1$ into the planner's objective function, we obtain

$$\begin{aligned}\max_{\Pi_s,\Pi_b} \Pi_b a_{Pb}\left(\frac{\overline{N}_s\Pi_s}{\overline{N}_b\Pi_b}\right) + (1-\Pi_b)a_{Lb}\left(\frac{\overline{N}_s(1-\Pi_s)}{\overline{N}_b(1-\Pi_b)}\right) &= \\ \max_{\Pi_s,\Pi_b} \Pi_b a_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right) + (1-\Pi_b)a_{Lb}(0) &= \max_{\Pi_b} \Pi_b a_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right).\end{aligned}$$

The first order condition with respect to Π_b becomes:

$$a_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right) - a'_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right)\frac{\overline{N}_s}{\overline{N}_b\Pi_b} \geq a_{Lb}(0) - a'_{Lb}(0)0 = 0. \tag{A.12}$$

Or

$$\frac{a_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right)}{\frac{\overline{N}_s}{\overline{N}_b\Pi_b}} \geq a'_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right). \tag{A.13}$$

Since a_{Pb} is concave, the above cannot hold with equality. So we must have $\Pi_b = 1$.

A.7 Proofs of the Results in Section 6

In this section we prove Lemma 2 and Propositions 8 and 10.

A.7.1 Proof of Lemma 2

Suppose the open policy is optimal. Suppose also that flexible buyers who are on the platform are indifferent between platform money and outside money and biased buyers strictly prefer outside money. That is, $\Delta_b^{of} = \Delta_b^{ob}$ and $\Delta_b^{of} - \kappa < \Delta_b^{ob}$. From the binding constraint we obtain:

$$\mu_P^o = \frac{f_P^o}{\xi\left(1-\gamma\right)u} + \mu_L. \tag{A.14}$$

Plugging μ_P^o into the platform's objective function we obtain:

$$\Pi_b\left(\Delta_b^{ob}\right)\left(\left(\frac{\phi}{\xi} + (1-\phi)\right) f_P^o + \phi\left(\mu_L - 1\right)u\left(1-\gamma\right)\right) \tag{A.15}$$

and platform's fee for biased buyers is:

$$f_P^o = \frac{\Pi_b\left(\Delta_b^{ob}\right)}{\Pi_b'\left(\Delta_b^{ob}\right)} - \frac{\phi\left(\mu_L - 1\right)u\left(1-\gamma\right)}{\left(\frac{\phi}{\xi} + (1-\phi)\right)} \tag{A.16}$$

and platform money inflation is:

$$\mu_P^o = 1 + \frac{1}{\xi\left(1-\gamma\right)u}\frac{\Pi_b\left(\Delta_b^{ob}\right)}{\Pi_b'\left(\Delta_b^{ob}\right)} + \frac{\xi\left(1-\phi\right)\left(\mu_L - 1\right)}{\phi + \xi\left(1-\phi\right)}. \tag{A.17}$$

Substituting back into platform's objective we obtain:

$$\left(\frac{\phi}{\xi} + (1-\phi)\right)\frac{\left(\Pi_b\left(\Delta_b^{ob}\right)\right)^2}{\Pi_b'\left(\Delta_b^{ob}\right)}$$

where $\Delta_b^{ob} = \beta\left(a_{Pb}(n_P^o) - a_{Lb}(n_L^o)\right)\gamma u + (f_L^o - f_P^o)$. If the platform is not constrained it would choose $\mu_P^* = 1 + \frac{1}{\xi(1-\gamma)u}\frac{\Pi_b\left(\Delta_b^{ob}\right)}{\Pi_b'\left(\Delta_b^{ob}\right)}$, which is strictly lower than the above value. But then if the platform lowers μ_P^o slightly, its profit from flexible buyers increases. Moreover, Δ_b^{of} goes up and both constraints hold. Hence at an optimum flexible buyers must strictly prefer platform money.

A.7.2 Proof of Proposition 8

Suppose $\phi > 0$. We consider two cases:

Case i: The platform uses only outside money. In this case platform chooses f_P to maximize its payoff $\Pi_b(\Delta_b^{out})f_P$ and the legacy chooses f_L to maximize its payoff $(1-\Pi_b(\Delta_b^{out}))f_L$

where

$$\Delta_b^{out} = \beta(a_{Pb}(n_P^{out}) - a_{Lb}(n_L^{out}))\gamma u + (f_L - f_P)$$

and n_P^{out}, n_L^{out} are the seller to buyer ratios.

Case ii: The platform uses both platform and outside money. In this case platform chooses μ_P and f_P to maximize its payoff $\phi\Pi_b(\Delta_b^{of})u(1-\gamma)(\mu_P - 1) + (1-\phi)\Pi_b(\Delta_b^{ob})f_P$ and the legacy chooses f_L to maximize its payoff $(1 - \phi\Pi_b(\Delta_b^{of}) - (1-\phi)\Pi_b(\Delta_b^{ob}))f_L$ where

$$\Delta_b^{of} = \beta(a_{Pb}(n_P^o) - a_{Lb}(n_L^o))\gamma u + \xi(\mu_L - \mu_P)(1-\gamma)u + f_L$$

$$\Delta_b^{ob} = \beta(a_{Pb}(n_P^o) - a_{Lb}(n_L^o))\gamma u + (f_L - f_P)$$

and n_P^o, n_L^o are the seller to buyer ratios. We also impose the constraints $\Delta_b^{of} \geq \Delta_b^{ob}$ and $\Delta_b^{of} - \kappa \leq \Delta_b^{ob}$ although these constraints do not play a role in the proof.[9]

We begin by proving three lemmas.

Lemma 3 *For a fixed f_L the platform can guarantee itself a higher payoff in case ii.*

Proof: Suppose that for a given f_L, platform's optimal fee in case i is f_P^{out}. Now suppose the platform accepts both types of money, and sets $f_P^o = f_P^{out}$, and sets $\mu_P = \mu_L + \frac{f_P^{out}}{\xi u(1-\gamma)}$. By construction $\Delta_b^{of} = \Delta_b^{ob} = \Delta_b^{out}$ and the constraints are automatically satisfied. And, the platform's payoff goes up since:

$$\phi\Pi_b(\Delta_b^{out})((\mu_L - 1)(1-\gamma)u + \frac{f_P}{\xi}) + (1-\phi)\Pi_b(\Delta_b^{out})f_P > \Pi_b(\Delta_b^{out})f_P.$$

Q.E.D.

Lemma 4 *Let $\bar{\Pi} = \phi\Pi_b(\Delta_b^{of}) + (1-\phi)\Pi_b(\Delta_b^{ob})$. In case ii, legacy's optimal fee satisfies $f_L^o \geq \frac{\sigma}{\Pi}$.*

[9] In the proof we use logistic functional forms for $\Pi_b(\cdot)$. The proof can be extended to the case where $\Pi_b'(\cdot)$ is concave.

Proof: We can write legacy's optimal fee in case ii as:

$$f_L^o = \sigma \frac{1-\bar{\Pi}}{\phi \Pi_b(\Delta_b^{of})(1-\Pi_b(\Delta_b^{ob})) + (1-\phi)\Pi_b(\Delta_b^{ob})(1-\Pi_b(\Delta_b^{ob}))}$$

Since $x(1-x)$ is strictly concave, by Jensen's Inequality,

$$\phi \Pi_b(\Delta_b^{of})(1-\Pi_b(\Delta_b^{of})) + (1-\phi)\Pi_b(\Delta_b^{ob})(1-\Pi_b(\Delta_b^{ob})) \leq \bar{\Pi}(1-\bar{\Pi})$$

which implies $f_L^o \geq \frac{\sigma}{\bar{\Pi}}$. Q.E.D.

Lemma 5 *In case i, the platform's payoff is increasing in f_L.*

Proof of lemma: Let $B(\Pi_b) = \beta(a_{Pb}(n_P^{out}(\Pi_b)) - a_{Lb}(n_L^{out}(\Pi_b)))\gamma u$. Note that n_P^{out} is decreasing and n_L^{out} is increasing in Π_b. Also a_{Pb} and a_{Pb} are increasing functions. Hence $\frac{\partial B(\Pi_b)}{\partial \Pi_b} < 0$. Thus $\frac{\partial \Delta_b^{out}}{\partial f_L} = \frac{1}{1-\frac{\partial B(\Pi_b)}{\partial \Pi_b)}\Pi_b'(\Delta_b^{out})} \in (0,1)$. Platform's payoff in case i is $V_P^{out} = \Pi_b(\Delta_b^{out})f_P$. By the envelope theorem $\frac{dV_P^{out}}{df_L} = \Pi_b'(\Delta_b^{out})\frac{\partial \Delta_b^{out}}{\partial f_L} f_P > 0$. Q.E.D.

To finish the proof suppose towards a contradiction that $V_p^o < V_p^{out}$. Since $\xi \leq 1$ we have

$$\left[\phi\left(\frac{\Pi_b(\Delta_b^{of})}{1-\Pi_b(\Delta_b^{ob})}\right) + (1-\phi)\left(\frac{\Pi_b(\Delta_b^{ob})}{1-\Pi_b(\Delta_b^{ob})}\right)\right] < \frac{\Pi_b(\Delta_b^{out})}{1-\Pi_b(\Delta_b^{out})}.$$

Since $\frac{x}{1-x}$ is an increasing and convex function Jensen's inequality gives:

$$\frac{\bar{\Pi}}{1-\bar{\Pi}} < \left[\phi\left(\frac{\Pi_b(\Delta_b^{of})}{1-\Pi_b(\Delta_b^{of})}\right) + (1-\phi)\left(\frac{\Pi_b(\Delta_b^{ob})}{1-\Pi_b(\Delta_b^{ob})}\right)\right] < \frac{\Pi_b(\Delta_b^{out})}{1-\Pi_b(\Delta_b^{out})}.$$

which implies $\bar{\Pi} < \Pi_b(\Delta_b^{out})$. From Lemma 4 we obtain:

$$f_L^o \geq \frac{\sigma}{\bar{\Pi}} > \frac{\sigma}{\Pi^b(\Delta_b^{out})} = f_L^{out}.$$

Combining Lemma 3 and Lemma 5 we obtain $V_P^b(f_L^b) \geq V_P^{out}(f_L^b) > V_P^{out}(f_L^{out})$ which is a contradiction.

A.7.3 Proof of Proposition 10

Let $\phi \in [0,1]$ be the fraction of flexible buyers and let $\epsilon = 1-\phi$ be the fraction of biased buyers. Let $V_j^c(\phi)$ and $V_j^o(\phi)$ be the payoff of market $j \in \{P, L\}$ under closed and open policies respectively as functions of ϕ. We can write these functions explicitly as:

$$\begin{aligned}
V_P^c(\epsilon) &= \bar{N}_b(1-\epsilon)\Pi_b\big(\Delta_b^c\big)\Big[u(1-\gamma)\big(\mu_P^c - 1\big)\Big],\\
V_L^c(\epsilon) &= \bar{N}_b\Big(1-(1-\epsilon)\Pi_b\big(\Delta_b^c\big)\Big)\, f_L^c,\\
V_P^o(\epsilon) &= \bar{N}_b\Big((1-\epsilon)\Pi_b\big(\Delta_b^{of}\big)\Big[u(1-\gamma)\big(\mu_P^o - 1\big)\Big] + \epsilon\Pi_b\big(\Delta_b^{ob}\big) f_P^o\Big),\\
V_L^o(\epsilon) &= \bar{N}_b\Big(1-(1-\epsilon)\Pi_b\big(\Delta_b^{of}\big) - \epsilon\Pi_b\big(\Delta_b^{ob}\big)\Big)\, f_L^o.
\end{aligned}$$

W.l.o.g we set $\bar{N}_b = 1$.

We define the profit difference of the platform under closed and open policies as $D(\phi) = V_P^c(\phi) - V_P^o(\phi)$.

Observe that:

1. At $\phi = 0$ ($\epsilon = 1$): Biased buyers cannot join the closed platform ($\kappa = \infty$), so $V_P^c(0) = 0$. Under the open policy, they generate fee revenue, so $V_P^o(0) > 0$. Thus $D(0) < 0$.

2. At $\phi = 1$ ($\epsilon = 0$): There are no biased buyers. The optimization problem for the flexible majority is identical under both policies. Thus $D(1) = 0$.

To prove the platform strictly prefers the closed policy for a large flexible share ($\epsilon \to 0$), we show that $D(1-\epsilon)$ is strictly increasing in ϵ at the limit $\epsilon = 0$. Equivalently, we show $\frac{dD}{d\epsilon} > 0$ at $\epsilon = 0$.

At the limit $\epsilon = 0$ define Δ^* and Π^* so that $\Delta_b^c = \Delta_b^{of} = \Delta^*$, $\Pi_b = \Pi^*$, and $\Pi_b' = \Pi^{*\prime}$.

Lemma 6 *The difference in the marginal response of the legacy fee to an increase in biased buyers between the closed (c) and open (o) policies is given by:*

$$\frac{df_L^c}{d\epsilon} - \frac{df_L^o}{d\epsilon} = \frac{\Pi^*}{\Pi^{*\prime}} \tag{A.18}$$

Proof:

Closed policy:

Define $G^c(\epsilon, f_L) = (1-(1-\epsilon)\Pi_b(\Delta_b^c)) - (1-\epsilon)\Pi_b'(\Delta_b^c)f_L$. Let $f_L^c(\epsilon)$ be the optimal legacy fee under the closed policy. The first-order condition implies $G^c(\epsilon, f_L^c(\epsilon)) = 0$. From the implicit function theorem $\frac{df_L^c}{d\epsilon} = -\frac{\partial G^c/\partial\epsilon}{\partial G^c/\partial f_L}$. Note that $\frac{\partial G^c}{\partial\epsilon} = \Pi_b(\Delta_b^c) + f_L^c(\epsilon)\Pi_b'(\Delta_b^c)$. Since $G^c(0, f_L^c(0)) = (1-\Pi_b(\Delta_b^c)) - \Pi_b'(\Delta_b^c)f_L^c(0) = 0$, we have $\frac{\partial G^c(0,f_L^c(0))}{\partial\epsilon} = 1$. Hence at $\epsilon = 0$, $\frac{df_L^c}{d\epsilon} = -\frac{1}{\partial G^c/\partial f_L}$.

Open policy:

Define $G^o(\epsilon, f_L) = \epsilon[(1-\Pi_b(\Delta_b^{ob})) - f_L\Pi_b'(\Delta_b^{ob})] + (1-\epsilon)[(1-\Pi_b(\Delta_b^{of})) - f_L\Pi_b'(\Delta_b^{of})]$. Let $f_L^o(\epsilon)$ be the optimal legacy fee under the open policy. The first-order condition implies $G^o(\epsilon, f_L^o(\epsilon)) = 0$. From the implicit function theorem and following steps as in the previous paragraph at $\epsilon = 0$, $\frac{df_L^o}{d\epsilon} = -\frac{(1-\Pi_b(\Delta_b^{of}))-f_L\Pi_b'(\Delta_b^{of})}{\partial G^o/\partial f_L}$. Since the numerator is 0 at $\epsilon = 0$ (by plugging in for the FOC for f_L), we have $\frac{df_L^o}{d\epsilon} = 0$.

Under closed policy: $\frac{\partial G^c}{\partial f_L} = (1-\epsilon)[-2\Pi_b'(\Delta_b^c) - f_L\Pi_b''(\Delta_b^c)]$. Under open policy: $\frac{\partial G^o}{\partial f_L} = (1-\epsilon)[-2\Pi_b'(\Delta_b^{of}) - f_L\Pi_b''(\Delta_b^{of})] + \epsilon[-2\Pi_b'(\Delta_b^{ob}) - f_L\Pi_b''(\Delta_b^{ob})]$.

We evaluate these at $\epsilon = 0$ using the properties of the logistic distribution where $\Pi_b' = \frac{1}{\sigma}\Pi_b(1-\Pi_b)$ and $\Pi_b'' = \frac{1}{\sigma}\Pi_b'(1-2\Pi_b)$. Substituting the first-order condition $f_L = \frac{1-\Pi_b}{\Pi_b'}$ into the derivative expression:

$$\frac{\partial G}{\partial f_L} = -2\Pi_b' - \left(\frac{1-\Pi_b}{\Pi_b'}\right)\Pi_b''$$

Substituting Π_b'':

$$\frac{\partial G}{\partial f_L} = -2\Pi_b' - \left(\frac{1-\Pi_b}{\Pi_b'}\right)\frac{1}{\sigma}\Pi_b'(1-2\Pi_b) = -2\Pi_b' - \frac{1}{\sigma}(1-\Pi_b)(1-2\Pi_b)$$

Using $\frac{1}{\sigma}(1-\Pi_b) = \frac{\Pi_b'}{\Pi_b}$:

$$\frac{\partial G}{\partial f_L} = -2\Pi_b' - \frac{\Pi_b'}{\Pi_b}(1-2\Pi_b) = -\frac{\Pi_b'}{\Pi_b}(2\Pi_b + 1 - 2\Pi_b) = -\frac{\Pi_b'}{\Pi_b}$$

At $\epsilon = 0$, this yields $\frac{\partial G^c}{\partial f_L} = \frac{\partial G^o}{\partial f_L} = -\frac{\Pi^{*\prime}}{\Pi^*}$.

Hence $\frac{df_L^c}{d\epsilon} - \frac{df_L^o}{d\epsilon} = \frac{1}{-(\partial G/\partial f_L)} - 0 = \frac{\Pi^*}{\Pi^{*\prime}}$. Q.E.D.

Lemma 7 *Assume $\xi \leq 1$ and legacy inflation $\mu_L > 1$. At the limit $\epsilon \to 0$:*

$$u(1-\gamma)(\mu_P - 1) > \frac{f_P}{\xi} \tag{A.19}$$

Proof: Consider the open policy at the limit $\epsilon = 0$. The platform chooses inflation μ_P and fee f_P to maximize profit from the flexible majority while retaining optimal conditions for the vanishing biased segment. The first-order conditions (FOCs) for μ_P and f_P are:

$$\xi u(1-\gamma)(\mu_P - 1) = \frac{\Pi_b(\Delta_b^{of})}{\Pi_b'(\Delta_b^{of})} \quad \text{and} \quad f_P = \frac{\Pi_b(\Delta_b^{ob})}{\Pi_b'(\Delta_b^{ob})} \tag{A.20}$$

Let $H(\Delta) \equiv \frac{\Pi_b(\Delta)}{\Pi_b'(\Delta)}$. For the logistic Π_b, $H'(\Delta) > 0$.

The perceived advantages for flexible buyers (Δ_b^{of}) and biased buyers (Δ_b^{ob}) are defined as:

$$\Delta_b^{of} = \beta \left(a_{Pb}(n_P^o) - a_{Lb}(n_L^o)\right) \gamma u + f_L + \xi(1-\gamma)u(\mu_L - \mu_P) \tag{A.21}$$

$$\Delta_b^{ob} = \beta \left(a_{Pb}(n_P^o) - a_{Lb}(n_L^o)\right) \gamma u + f_L - f_P \tag{A.22}$$

Subtracting these yields:

$$\Delta_b^{of} - \Delta_b^{ob} = \xi u(1-\gamma)(\mu_L - 1) - (\xi u(1-\gamma)(\mu_P - 1) - f_P) \tag{A.23}$$

Let $Y = \xi u(1-\gamma)(\mu_P - 1) - f_P$ and $K = \xi u(1-\gamma)(\mu_L - 1)$. Since $\mu_L > 1$, $K > 0$. The difference in advantages is $\Delta_b^{of} - \Delta_b^{ob} = K - Y$.

Substituting the FOCs back into the definition of Y:

$$Y = H(\Delta_b^{of}) - H(\Delta_b^{ob}) = H(\Delta_b^{ob} + K - Y) - H(\Delta_b^{ob}) \tag{A.24}$$

Define the function $\Phi(Y) = H(\Delta_b^{ob} + K - Y) - H(\Delta_b^{ob}) - Y$. We seek Y such that $\Phi(Y) = 0$.

- At $Y = 0$: $\Phi(0) = H(\Delta_b^{ob} + K) - H(\Delta_b^{ob})$. Since H is strictly increasing and $K > 0$, $\Phi(0) > 0$.

- Slope: $\Phi'(Y) = -H'(\cdot) - 1 < 0$.

Since $\Phi(0) > 0$ and $\Phi'(Y) < 0$, the unique solution Y^* must be strictly positive ($Y^* > 0$). Therefore, $\xi R_\mu - f_P > 0$, which implies $R_\mu > f_P/\xi$. Q.E.D.

To finish the proof of the proposition we use the envelope theorem and write the total derivatives of the platform payoffs with respect to ϵ at $\epsilon = 0$ as:

$$\begin{aligned} \frac{dV_P^c}{d\epsilon} &= \frac{\partial V_P^c}{\partial \epsilon} + \frac{\partial V_P^c}{\partial f_L}\frac{df_L^c}{d\epsilon} \\ &= -u(1-\gamma)(\mu_P^c - 1)\Pi_b(\Delta_b^c) + (1-\epsilon)u(1-\gamma)(\mu_P^c - 1)\Pi_b'(\Delta_b^c)\frac{df_L^c}{d\epsilon} \end{aligned} \tag{A.25}$$

$$\begin{aligned} \frac{dV_P^o}{d\epsilon} &= \frac{\partial V_P^o}{\partial \epsilon} + \frac{\partial V_P^o}{\partial f_L}\frac{df_L^o}{d\epsilon} \\ &= -u(1-\gamma)(\mu_P^o - 1)\Pi_b(\Delta_b^{of}) + \Pi_b(\Delta_b^{ob})f_P^o + (1-\epsilon)u(1-\gamma)(\mu_P^o - 1)\Pi_b'(\Delta_b^{of})\frac{df_L^o}{d\epsilon} \end{aligned} \tag{A.26}$$

$$\left.\frac{dD}{d\epsilon}\right|_{\epsilon=0} = u(1-\gamma)(\mu_P - 1)\Pi^{*\prime}\left[\frac{df_L^c}{d\epsilon} - \frac{df_L^o}{d\epsilon}\right] - f_P\Pi^*$$

Substituting the strategic gap $\frac{df_L^c}{d\epsilon} - \frac{df_L^o}{d\epsilon} = \frac{\Pi^*}{\Pi^{*\prime}}$:

$$\frac{dD}{d\epsilon} = u(1-\gamma)(\mu_P - 1)\Pi^{*\prime}\left(\frac{1}{\Pi^{*\prime}/\Pi^*}\right) - f_P\Pi^*$$

From Lemma 7, $u(1-\gamma)(\mu_P - 1) > \frac{f_P}{\xi}$ so:

$$\frac{dD}{d\epsilon} > \Pi^* f_P\left(\frac{1}{\xi} - 1\right) \geq 0$$

This proves that for a sufficiently small mass of biased buyers, the platform earns strictly higher profits under the closed policy, implying $D(1-\epsilon) > 0$. This completes the proof of the proposition.

A.8 Other proofs

Claim 1 *The social planner's objective function can be written as:*

$$\max \overline{N}_b \gamma u \left[\Pi_b a_{Pb}(n_P) + (1-\Pi_b) a_{Lb}(n_L)\right] + \overline{N}_s (1-\gamma) u \left[\Pi_s a_{Ps}(n_P) + (1-\Pi_s) a_{Ls}(n_L)\right] \tag{A.27}$$

By summing all agents' utility:

$$\begin{aligned}
\max &\underbrace{\overline{N}_b[\Pi_b(\Delta_b) W_b^P(0,0) + [1-\Pi_b(\Delta_b)] W_b^L(0,0)]}_{\text{Buyers' Utility}} \\
&+ \underbrace{\overline{N}_s[\Pi_s(\Delta_s) W_s^P(0,0) + (1-\Pi_s(\Delta_s)) W_s^L(0,0)]}_{\text{Sellers' Utility}} \\
&+ \underbrace{\overline{N}_s \Pi_s(\Delta_s) k_P + \overline{N}_b \Pi_b(\Delta_b) [f_P + (\mu_P - 1) u (1-\gamma)]}_{P \text{ Owner's Profit}} \\
&+ \underbrace{\overline{N}_s (1-\Pi_s(\Delta_s)) k_L + \overline{N}_b (1-\Pi_b(\Delta_b)) f_L}_{L \text{ Owner's Profit}} \\
&+ \underbrace{\overline{N}_b (1-\Pi_b(\Delta_b)) (\mu_L - 1) u (1-\gamma)}_{\text{Tax rebate for outside money seigniorage}},
\end{aligned}$$

which is subject to

$$\begin{aligned}
n_P[\bar{N}_b \Pi_b(\Delta_b)] &= \bar{N}_s \Pi_s(\Delta_s) \\
n_L[\bar{N}_b(1-\Pi_b(\Delta_b))] &= \bar{N}_s(1-\Pi_s(\Delta_s)).
\end{aligned}$$

Next, we plug in (5) and (18) and after some algebra we get to equation (A.27).

A.9 Nash Bargaining with Liquidity Constraints

We assume that the seller's cost of producing x units of the good is xc where $0 \leq c < u$. We also assume that a buyer obtains utility xu if the seller produces $0 \leq x \leq 1$ and does not value more than one unit. (Alternatively we can assume the seller has the production capacity of one unit instead.) To begin, suppose the buyer is not liquidity constrained. The (generalized) Nash bargaining problem is :

$$\max_{x,q} (xu - q)^{\gamma} (q - xc)^{1-\gamma} \tag{A.28}$$

where q is the terms of trade for exchanging the consumption goods. The solution is: $x^* = 1$ and $q^* = (1-\gamma)\, u + \gamma c$. Hence the buyer's surplus is: $\gamma\,(u-c)$ and the seller's surplus is $(1-\gamma)\,(u-c)$.

Now suppose the buyer is liquidity constrained so that they will not be able to bring enough money to pay for the consumption good price from the above Nash solution. That is, $q \leq q^*$. The liquidity constraint is binding so the Nash bargaining problem becomes:

$$\max_{x,q}\, (xu-q)^{\gamma}\, (y-xc)^{1-\gamma} \tag{A.29}$$

subject to $q \leq q^*$. The first order condition is

$$\frac{q-xc}{xu-q} = \frac{1-\gamma}{\gamma}\frac{c}{u} \Rightarrow x = \frac{(\gamma u + (1-\gamma)\, c)}{cu} q.$$

Therefore, the solution is

$$x^* = \begin{cases} 1 & \text{if } \frac{(\gamma u+(1-\gamma)c)}{cu} q \geq 1 \\ \frac{(\gamma u+(1-\gamma)c)}{cu} q & \text{otherwise} \end{cases}$$

Hence, the term of trade is:

$$q^{**} = \frac{cu}{\gamma u + (1-\gamma)\, c}. \tag{A.30}$$

. The seller's surplus is:

$$\frac{cu}{\gamma u + (1-\gamma)\, c} - c = \frac{(1-\gamma)\, c\, (u-c)}{\gamma u + (1-\gamma)\, c}. \tag{A.31}$$

The buyer's surplus is:

$$u - \frac{cu}{\gamma u + (1-\gamma)\, c} = \frac{\gamma u\, (u-c)}{\gamma u + (1-\gamma)\, c}. \tag{A.32}$$

The Nash bargaining solution indicates that the buyer when liquidity constrained chooses to bring q^{**} specified in eq. (A.30) from the CM to the DM in order to trade the consumption goods. That is, Thus, the resulting real price for the good y is: $p^j \phi^j = cu/(\gamma u + (1-\gamma)\, c)$. The welfare analysis will be based on the split of the total surplus $u - c$ specified in eq. (A.31) and eq. (A.32). The qualitative results on the platform use of platform money to gain competitive advantage remain the same but the quantitative implications are different – since the liquidity constrained buyer potentially would bring less money, platform money in this case give the platform owner less advantages than the Kalai bargaining solution.

However, note that there is one peculiarity in this Nash bargaining solution that we consider undesirable. The derivative of the seller's surplus with respect to c has same sign as: $\gamma u^2 - 2\gamma cu - (1-\gamma)\, c^2$. It is negative quadratic in c and at $c = 0$ this is $\gamma u^2 > 0$ and at $c = u$ this is $-u^2 < 0$. Hence, if the seller has a low cost, he has incentive to exaggerate cost or engage in wasteful expenditure.

A.10 Kalai Bargaining with and without Liquidity Constraints

Suppose that the seller's cost of producing x units of the good is xc where $0 \leq c < u$. The buyer obtains utility xu if the seller produces $0 \leq x \leq 1$ and does not value more than one unit.[10] We can also interpret x as probability of trade.

First, suppose the buyer is not liquidity constrained. Under Kalai's proportional bargaining with bargaining parameter γ, the optimization problem is to choose the terms of trade q and the amount to produce x to maximize:

$$\max_{x,q} \quad xu - q$$
$$\text{subject to} \quad \tfrac{xu-q}{q-xc} = \tfrac{\gamma}{1-\gamma} \text{ and } x \leq 1.$$

Solving for q from the constraint and plugging into the objective we see that objective is increasing in x. Hence, the solution without liquidity constraint is $x^* = 1$ and $q^* = (1-\gamma)\, u + \gamma c$.[11]

Now, suppose the real value of the money that the buyer brings to the DM is $\bar{q}$ and the

[10] Alternatively, we can assume that the seller has the production capacity of one unit.

[11] When buyers are not liquidity constrained, the Kalai and the Nash bargaining solutions are the same.

buyer is liquidity constrained so that $\bar{q} \leq q^*$. Now the problem becomes:

$$\max_{x,q} \quad xu - q \tag{A.33}$$

$$\text{subject to} \quad \frac{xu-q}{q-xc} = \frac{\gamma}{1-\gamma} \text{ and } q \leq \bar{q}. \tag{A.34}$$

Solving for q and plugging into the objective, we can rewrite this problem as:

$$\max_{x} \quad x\,(u - c) \tag{A.35}$$

$$\text{subject to} \quad x((1-\gamma)u + \gamma c) \leq \bar{q}. \tag{A.36}$$

The solution is

$$x^{**} = \frac{\bar{q}}{(1-\gamma)\,u + \gamma c} \text{ and } q^{**} = \bar{q} \tag{A.37}$$

The buyer's utility under this solution is $(\gamma\,(u-c)\,/((1-\gamma)\,u+\gamma c))\,\bar{q}$. Now, let's step back and ask how much liquidity the buyer brings to the DM. Buyer's utility is increasing in $\bar{q}$ (up to q^*). Hence, the buyer's optimal liquidity, or the price of the DM good, is $q^* = (1-\gamma)\,u + \gamma c$. Consequently, the buyer's utility is $\gamma\,(u-c)$ and the seller's utility is $(1-\gamma)\,(u-c)$. When $c = 0$, the resulting real price for the consumption good y is: $p^j \phi^j = u(1-\gamma)$.