Mackay Radio & Telegraph Co. v. Radio Corp. of America, 306 U.S. 86 (1939)

Syllabus | Case

U.S. Supreme Court

Mackay Radio & Telegraph Co. v. Radio Corp. of America, 306 U.S. 86 (1939)

Mackay Radio & Telegraph Co. v. Radio Corporation of America

No. 127

Argued December 14, 15, 1938

Decided January 30, 1939

306 U.S. 86

CERTIORARI TO THE CIRCUIT COURT OF APPEALS

FOR THE SECOND CIRCUIT

Syllabus

1. Carter patent, No. 1,974,387, for a directive antenna system for use in radio communication, *held* not infringed by petitioner's antenna structures. Pp. 306 U. S. 88, 306 U. S. 101.

Claims 15 and 16 are invalid so far as they claim antennae of wire lengths intermediate of multiples of half-wavelengths. And, so far as the patent discloses and claims invention of a V antenna structure made in conformity to the Abraham formula, petitioner's structures do not infringe, for none of them conforms to the Abraham formula. One has wires which are an integral number of half-wavelengths long, but uses an angle 10% smaller than that derivable from the Abraham formula; all of the others have wires which are not multiples of half-wavelengths long and angles not derivable from the formula.

2. The disclosure of the Carter patent was that the best directional radio propagation by the V type antenna is obtained in the direction of its bisector, with a structure in which the angle of the wires, their length, and the length of wave propagated are in a definite mathematical relationship expressed by a formula disclosed in the specifications. The formula shows that the appropriate angle between each of the antenna wires and their bisector depends upon the wavelength to be propagated and the wire length, which is a multiple of half-wavelengths. The formula had been developed and published by Abraham thirty years previously. Lindenblad had taught that, with an arrangement of antenna wires at an angle, radiation will occur in the direction of the bisector of the angle, and that the preferred angle was dependent upon an indicated relationship between wire length and wavelength. Carter's invention therefore, if it was invention, consisted in taking the angle of the Abraham formula as the angle between each wire of the V antenna and its bisector, and thus establishing along the bisector the greatest directional radio activity. The empirical formula of the specifications and Claims 15 and

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16, derived graphically from the Abraham formula, disclosed no invention other than the application of the Abraham formula to the V antenna when wavelength and wire length are known.

Held, assuming that it was more than the skill of the art to combine the teaching of Abraham with that of Lindenblad and others who had pointed out that the arrangement of the wires at an angle enhanced directional radio activity along their bisector, then the invention was a narrow one, consisting of a structure conforming to the teachings of the Abraham formula as to angle and wire length relative to wavelength, and is to be strictly construed with regard both to prior art and to alleged infringing devices. Carter, avoiding prior art by defining his angle for antennae with wires of particular wavelengths with mathematical precision, cannot discard that precision to establish infringement. Pp. 306 U. S. 94, 306 U. S. 102.

3. The application of Carter, at least before the amendments introduced subsequently to the commencement of the present litigation, cannot be construed as embracing structures not conforming to the Abraham formula. And the attempt by amendment to extend the claims, based on the application of the empirical formula (derived from the Abraham formula) to wire lengths not multiples of half-wavelengths, must fail because it involved a departure from what Carter's application had described as his invention, and a contradiction of it. P. 306 U. S. 98.

4. It is unnecessary in this case to decide the further question whether petitioner's structures avoid infringement because the direction of their principal radioactivity is not in the plane of the wires. P. 306 U. S. 102.

96 F.2d 587 reversed.

Certiorari, 305 U.S. 582, to review a decree which modified and reversed a decree of the District Court, 16 F.Supp. 610, dismissing the bill in a suit to enjoin infringements of patents.

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MR. JUSTICE STONE delivered the opinion of the Court.

The questions presented for decision by the petition for certiorari are whether the Carter patent, No. 1,974,387, of September 18, 1934, for a directive antenna system for use in radio communication, is valid and is infringed by antennae structures used by petitioner in such communication.

Respondent brought the present suit in the District Court for eastern New York to enjoin infringements of four patents relating to radio antennae or their operation. Two were those of Carter and two those of Lindenblad. Of these, only the second Lindenblad patent, No. 1,927,522, of September 19, 1933, for an antenna for radio communication, is of present importance. When the suit was begun, the application for the third Carter patent, with which we are presently concerned, was pending. After petitioner had answered, and respondent, as a result of the litigation, had acquired knowledge of the particulars of the structure and operation of petitioner's antennae, Carter, respondent's assignor, amended the statement of his invention in his application so as, in terms, to embrace a differentiating feature of petitioner's structures. After this patent was issued, respondent was permitted to file a supplemental bill charging infringement of it. The suits were consolidated, and the parties proceeded to trial on the issues of the validity and infringement of all five patents.

After taking the voluminous testimony of numerous witnesses, the trial court found that none of the patents in suit was infringed, and decreed that the suits be dismissed. 16 F.Supp. 610. It held that none of them was a pioneer patent; that none had been employed by anyone; that respondent's commercial structures did

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not follow the teachings of any of them, and that consequently they were not entitled to a broad construction. With respect to the Carter patent in suit it said: "The disclosure and the claims were broadened not only contrary to their original terminology, but to their spirit as well." And,

". . . by those amendments, the plaintiff attempted to mold the third Carter patent, both as to disclosure and claims, to cover defendant's antenna systems. This could not lawfully be done."

On appeal from so much of the decree as related to the second Lindenblad patent and the third Carter patent, the Court of Appeals for the Second Circuit affirmed as to the Lindenblad patent, but reversed as to the Carter patent, holding Claims 15 and 16 valid and infringed. 96 F.2d 587. We granted certiorari, 305 U.S. 582, because of the nature and importance of the case, on a petition which urged as grounds for its allowance that validity and infringement of the Carter patent were in doubt, and that, as petitioner is the only competitor of respondent in the business of worldwide public radio communication, further litigation, resulting in conflict of decision among circuits, was improbable.

In ordinary broadcasting, radio waves are projected in all directions from the sending station. In radio communication, it is advantageous, and has long been the practice, to use a directive antenna by which the waves of radio activity emanating from it are projected as a beam in the direction of the receiving station. In practice, the beam is directed at an angle from the earth's surface toward the ionized layer of the stratosphere, or Heaviside layer, from which the beam is deflected toward the earth's surface in the compass direction of the receiving station. In more recent years, it has been the practice to use relatively short wavelengths for radio communication.

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The radio waves are generated at the sending station by feeding an oscillating electric current of appropriate character into the wire or wires of the antenna. The electric waves in the wires energize radio activity, which the antenna projects as radio waves toward the receiving station. By modulating or interrupting the current, corresponding modulation or interruption of the radio waves is effected, which may be used as a means of transmitting any desired signal. The waves, as modulated, impinge on the antenna of the receiving station devised to receive and utilize them as a means of controlling, with corresponding modulation, an electric current which, in turn, actuates a mechanism contrived to give audible or visual expression to the transmitted signal.

The effective part of the antenna is a copper wire from which the radio waves are radiated, supported on towers or poles at a height above the ground depending on the wavelength used. The wire may be parallel with the earth, or vertical, or arranged at an angle, depending upon the function to be performed. Before Carter, antennae of two or more wires in varying arrangement had been used. The second Lindenblad patent showed an antenna of two wires arranged at an angle in the form of a V or an X, and it pointed out that, in such an arrangement, radiation will take place in the direction of the axis or bisector of the angle of the diverging wires, and that

"the spacing at the open end [of the wires], while variable over a great range, should be in the neighborhood of a fifth of the length, and the length of each antenna section should be of the order of magnitude of five to ten waves long."

While such an arrangement projects the radio waves principally in two directions along the bisector of the angle of the antenna wires, the prior art had made use of an arrangement of wires, parallel to the wires of the angular

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antenna, as a "reflector" by which the radiation was projected as a beam in one direction away from the reflector and along the bisector of the angle of the wires.

The present Carter patent is for an "antenna system utilizing standing wave phenomena." Like the second Lindenblad patent, it is concerned with a V antenna by which the principal radiation is directed in the plane of the wires along the bisector of their angle. The disclosure of the patent, in which the court below found invention, was that the best directional radio propagation by the V type antenna is obtained with a structure in which the angle of the wires, their length, and the length of wave propagated are in a definite mathematical relationship expressed by a formula disclosed in the specifications.

In explaining his invention, Carter pointed out that

"It is known that, when a wire having a length greater than the operating wavelength is excited in such manner that standing waves are produced thereon, radiation will occur principally in the direction of symmetrical cones having their apices at the center of the wire. Such is the case with a wire having a length equal to a plurality of one-half-wavelengths at the operating frequency. The radiation pattern produced in such instance appears, in cross-section, in the form of symmetrical cones about the wire. The present invention, which makes use of these phenomena, in its most simple aspect, employs a pair of open-ended wires energized in phase opposition to have standing waves throughout the length of the wires, the wires having such angular relation with respect to each other as to obtain a highly directional, efficient, and simple antenna system. [Footnote 1] "

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The patent states the mathematical formula by which the desired relationship is secured, which shows that the appropriate angle between each of the antenna wires and their bisector depends upon the wavelength to be propagated and the use of antenna wires of a length which is a multiple of half-wavelengths. [Footnote 2]

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The significance of the formula lies in the fact that the angle between the wire and the direction of greatest radio activity is a trigonometrical function of two variables, the wavelength used and the "number of half-wavelengths contained in the wire," and that, as the application stated, the use of the formula, in practice, presupposes the use of a wire whose length is a multiple of half-wavelengths. The patent then explains that the angle � of the formula is the angle between each wire of the V antenna and its bisector -- in other words, that the angle of the wires of the antenna is twice �, and hence, like the angle of the formula, is a function of the wavelength and the length of the wires, which are each a multiple of half-wavelengths long.

Carter did not invent the formula. It had been developed by Abraham and published in a scientific journal thirty years before. Annalen der Physik, 1898, Physikalische Zeitschrift, March 2, 1901. Abraham's formula expressed the scientific truth that, when radio activity is projected from a charged wire of finite length -- *i.e.,* one having standing waves, and having a length of a multiple of half-wavelengths, the angle between the direction of the principal radio activity and the wire is dependent on

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wavelength and wire length, which is a multiple of half-wavelengths. Lindenblad had described his antenna as using either standing or traveling waves and, as we have seen, had taught that, with an arrangement of antenna wires at an angle, radiation will occur substantially in the direction of the bisector of the angle, and that the preferred angle was dependent upon an indicated relationship between wire length and wavelength.

It is plain, therefore, that the Carter invention, if it was invention, consisted in taking the angle of the Abraham formula as the angle between each wire of the V antenna and its bisector. By so doing, he brought the cones of principal radio activity, each having one of the wires of the antenna as its axis, into conjunction at their periphery and along the bisector of the angle between the wires, and thus established there the greatest directional radio activity.

While a scientific truth, or the mathematical expression of it, is not patentable invention, a novel and useful structure created with the aid of knowledge of scientific truth may be. But we do not stop to solve the problem whether it was more than the skill of the art to combine the teaching of Abraham with that of Lindenblad and others who had pointed out that the arrangement of the wires at an angle enhanced directional radio activity along their bisector. We assume, without deciding the point, that this advance was invention, even though it was achieved by the logical application of a known scientific law to a familiar type of antenna. But it is apparent that, if this assumption is correct, the invention was a narrow one, consisting of a structure conforming to the teachings of the Abraham formula as to angle and wire length relative to wavelength, and is to be strictly construed with regard both to prior art and to alleged infringing devices. *Kokomo Fence Machine Co. v. Kitselman,*

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189 U. S. 8; *Cimiotti Unhairing Co. v. American Fur Refining Co.,* 198 U. S. 399. Carter's structure was a V having an angle double the Abraham angle and wires containing a multiple of half-wavelengths.

Carter, using the Abraham formula, calculated the value of the angle � in that formula for wires up to fourteen wavelengths long. He plotted the result, which he expressed graphically in figure 12 of the patent by drawing a smooth curve connecting the discrete points on the graph which indicated the results of his computation by use of the Abraham formula. From this calculation, he derived a formula in empirical form [Footnote 3] for determining the desired angle when wavelength and length of wire are known, in which the angle between the wires is described as twice *a*, which is the equivalent of the angle � of the Abraham formula.

Petitioner uses antennae with wires in V arrangement, but their wires are not an integral number of half-wavelengths long, with the exception of one antenna, No. 8, which is four wavelengths long and uses an angle 10% smaller than that prescribed by the Abraham formula for that length of wire. The others are of lengths which are approximately multiples of quarter wavelengths, and their angles differ from the angles of the formula. The crucial question in the case is whether a V antenna structure, having a wire length to which the Abraham formula does not apply and using an angle not to be derived from that formula, which is the basis of the patent, infringes Carter's patent. Respondent insists that it does, because, as it argues, the invention disclosed by Carter's

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application, elaborated by amendment and broad claims, embraces all V antennae arranged at an angle double the angle of the empirical formula, even though the length of the wires is not an exact multiple of half-wavelengths, as prescribed by the Abraham formula. This is the invention of Claim 15, [Footnote 4] and it is urged that the claim is amply supported by the statement in the specifications appearing in the original application that the empirical formula represented by the plotted curve of figure 12 of the patent "will be found accurate for all practical purposes where the length of wire dealt with does not correspond to a whole number of half-wavelengths."

The trial court, analyzing Carter's application and taking into account the essentials of the Abraham formula and the statement in the application that the "object of the present invention is to disclose the proper angle for conductors or radiators" measured in multiples of half-wavelengths, evidently thought, as petitioner argues, that the references in the application to "wires of finite" [Footnote 5]

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length and to wires "of any length whatsoever" [Footnote 6] were intended only to refer to wires of electrically finite, as distinguished from electrically infinite, length, capable of producing standing waves utilized by the antenna of the patent, and of any length conforming to the requirements of the basic formula. [Footnote 7] It concluded that the correct construction of the application was that the invention described did not go beyond the scope of the Abraham formula, and so extended only to the angles calculable by that formula for standing wave wires measured by multiples of half-wavelengths. Support is given to this conclusion by the statement in the application that "[t]he law, giving the correct angle for lengths between odd and even number of half-wavelengths, is not given due to its complexity. . . ."

The Court of Appeals placed emphasis on the reference to "wires of any finite length," and on the statement that

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"the empirical formula and the curve of figure 12 will be found accurate, for all practical purposes, where the length of wire dealt with does not correspond to a whole number of half-wavelengths."

It held that the invention disclosed was the application of the empirical formula to all lengths of antenna wires, and embraced all angles resulting from such calculation and that the invention was consequently infringed by petitioner's structures.

Whether or not it was the purpose of the patentee, by these references to wire lengths in his application to extend his patent to structures not conforming to the Abraham formula, we are not able to construe the application, before amendment, at least, as embracing such an extension. And we think that the attempt to extend the claims based on the application of the empirical formula to wire lengths not multiples of half-wavelengths must fail, because such structures are not within the invention described in the application.

The formula in Claims 15 and 16 is the empirical formula derived from the Abraham formula, which is, by its terms, applicable only to antenna wires which are multiples of half-wavelengths long. Carter's empirical formula, wholly derived from Abraham's formula, and taken together with it, therefore discloses no invention or discovery more than the application of the Abraham formula to the V antenna. It reveals no scientific law applicable to wire lengths which are intermediate of multiples of half-wavelengths, and the application explicitly states that "the law, giving the correct angle for lengths between odd and even number of half-wavelengths, is not given." The preparation, by methods familiar to engineers, of the graph in figure 12, which was but a pictorial representation of the Abraham formula applied to certain wire lengths specified in the formula, did not involve invention. Neither the empirical formula nor its graph gives any clue to the directional radio activity resulting

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from the use of wire lengths intermediate of multiples of half-wavelengths, which were excluded by the Abraham formula, and neither afforded any basis for a claim of invention not supported by the use of the Abraham formula itself.

The claimed use of the empirical formula for the calculation of the angle for wires which are not multiples of half-wavelengths long thus involved a departure from what Carter's application had described as his invention, and a contradiction of it. What Carter did was to describe his structure in terms of its dimensions, arrived at by the use of the Abraham formula, which was stated to embody the applicable scientific law. He then derived the empirical formula from Abraham. From the very method of derivation, the empirical formula meant nothing different from that of Abraham. He then declared the empirical formula to embody a method of arriving at the measurements of a structure different from the structure first described as the invention, and not capable of construction by the method of the Abraham formula. If, as a result of this legerdemain, a V antenna having wire lengths a multiple of any fractional wavelength is to be taken as the invention of Carter's application, then everything that it said of the Abraham formula and of wires "either an even number of half-wavelengths long or an odd number of half-wavelengths long" could be discarded without changing its meaning. [Footnote 8]

This attempt to broaden the only invention described in the application through a purely mechanical alteration of the meaning of the empirical formula, which had been devised as a shorthand expression of the scientific law on which the invention was declared to rest, cannot, we think, be taken to enlarge the description of the invention as measured by the Abraham formula, so as to include a structure to which that formula does not apply.

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This use of the empirical formula for a purpose for which it was not devised does not justify our construing the application as though all reference to the Abraham formula had been eliminated and a new and different one expressing a new and different scientific law had been substituted for it. The result of reading the application as respondent contends it should be construed is precisely the same as though full effect were given to a claim which goes beyond the invention described, and it is open to the same objection.

After the present suit was brought, the application was altered by amendment so as in effect to wipe out all reference to the scientific law by which Carter's invention was defined. This was accomplished by changes which implicitly assert that the letter *n* of the formula of the invention, the Abraham formula, meant something different from "the number of half-wavelengths contained in the wire" of a length of multiples of half-wavelengths as stated both in the application and in the patent.

The reference in the application to the purpose of the invention to disclose the "proper angle" for radiators of multiples of half-wavelengths long was altered by eliminating from it all mention of half-wavelengths. [Footnote 9] A sentence added after formal allowance of the patent states:

"By the term 'plurality of wavelengths' or 'plurality of half-wavelengths' or 'several half-wavelengths,' it is not intended that the wires so described shall necessarily be an exact or approximate integral number of such lengths, unless so specified, but rather that each of the wires so described shall be sufficiently long to include the lengths specified."

These amendments operated to modify the Abraham formula so as to cancel from the application the statement of the scientific law defining the invention. They left as its definition the modified

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Abraham formula and its counterpart, the empirical formula, stating a different law which their genesis did not authorize.

We think that these alterations were not permissible. *Schriber-Schroth Co. v. Cleveland Trust Co.,* 305 U. S. 47, and that, without them, the invention must be taken to be limited to a structure having an angle double that disclosed by the Abraham formula, which was made the basis of the alleged invention. As already shown, neither the Abraham formula nor the empirical formula describes, or purports to describe, the directional radio activity or defines the angle which affords "the best directional propagation" of the patent for antennae of wire lengths intermediate of multiples of half-wavelengths. The expert testimony shows that, in fact, neither formula serves that purpose. The finding of the trial court that they do not make "a correct showing of what happens when the wires are other than exact multiples of half-wavelengths" is supported by the evidence. The testimony warrants the conclusion that differences in wave effect already noted, [Footnote 10] when wires of other than exact multiples of half-wavelengths are used, produce, through consequent changes in "radiation resistance," differences in directional radio activity not calculable by the formulae of the patent. It follows that Claims 15 and 16, so far as they claim antennae of wire lengths intermediate of multiples of half-wavelengths, are invalid. So far as the patent discloses and claims invention of a structure made in conformity to the Abraham formula, petitioner's structures do not infringe, for none of them conforms to the Abraham formula.

For reasons already indicated it is not material that the variations are small between the angles used by petitioner for wire lengths of multiples of quarter wavelengths

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and those obtained by application of the empirical formula. Further, Carter's advance over prior art was in specifying an exact angle for wires of the prescribed length. Lindenblad had indicated a preferred angle, and Bruce, before Carter, had plotted a rule of thumb graph, which the trial court found to be prior art, showing the directional radio activity of a V antenna and exhibiting relatively small variations from that of Carter. Carter, avoiding prior art by defining his angle for antennae with wires of particular wavelengths with mathematical precision, cannot discard that precision to establish infringement. *Kokomo Fence Machine Co. v. Kitselman, supra; Cimiotti Unhairing Co. v. American Fur Refining Co., supra, cf. General Electric Co. v. Wabash Appliance Corp.,* 304 U. S. 364.

It is unnecessary to discuss the further question whether petitioner's structures avoid infringement because the direction of their principal radio activity is not in the plane of the wires, an operative difference from the antennae described by the patent which the court below found to be due wholly to ground effect, which it thought must be assumed to be envisaged by, though not stated in, the Carter patent.

Reversed.

MR. JUSTICE ROBERTS took no part in the consideration or decision of this case.

[Footnote 1]

Understanding of the disclosure and other features of the patent requires a brief explanation of its terms. The term "long," as applied to an antenna, means a wire which is long in relation to the wavelength used. The term "standing waves," as distinguished from "traveling waves," describes the phenomenon manifested when an oscillating electric current of radio frequency is communicated to one end of a wire which is open at the other (that is, not in a closed circuit) and sufficiently short so that the waves have not completely radiated their energy before reaching the end of the wire. The waves will then be reflected back along the wire, and the energy of the reflected waves tends to unite with that of the oncoming waves of the same periodicity so as to produce standing waves along the wire. As the velocity of the radio wave in space is approximately that of the current waves in the wire, the number of complete standing waves on the wire is always exactly the same as the length of the wire divided by the wavelength. When the length of the wire is a multiple of half-wavelengths, the oncoming and reflected waves, traveling at the same velocity, occur simultaneously, differing in this respect from the waves in a wire of a length intermediate a multiple of half-wavelengths, and with different effects upon the resulting radio energy, presently to be noted.

When oscillating current is so related to the length of wire that the energy of the former is exhausted by radiation before or when the waves reach the end of the wire, there is no reflection of the waves, and they travel in one direction only toward the open end of the wire. They are denominated "traveling waves." In professional parlance, wires producing reflected, and hence "standing," waves are electrically of finite length. Those of sufficient length to avoid reflection, and thus carry waves flowing in only one direction, are said to be electrically of infinite length.

[Footnote 2]

The specifications state:

"By considering a long wire the equivalent of a very large number of very short (Hertz) oscillators and by adding up the field components at any point P having a direction angle relative to the axis of the wire, where the point P is a great distance from the wire as compared to the length of the wire such that all lines from point P to any point on the wire are essentially parallel, it can be shown that the field strength H is given by the following proportionality for a conductor an odd number of half-wavelengths long:"

cos(*n* �/2 cos �)

Ha----------------------

sin �

"The letter '*n*' indicates the number of half-wavelengths contained in the wire."

"For a wire an even number of half-wavelengths long, in similar fashion, the field strength 'H' is given by the following proportionality:"

sin(*n* �/2 cos �)

Ha----------------------

sin �

"Where *n,* as above, indicates the number of half-wavelengths on the wire."

[Footnote 3]

"For practical purposes the empirical formula"

a = 50.9(*l*/�)-0.513 degrees

"is sufficiently accurate where *l* equals the length of the wire and � the wavelength, both in the same units of measurement."

[Footnote 4]

"15. An antenna comprising a pair of relatively long conductors disposed with respect to each other at an angle substantially equal to twice"

50.9(*l*/�)-0.513

degrees, *l* being the length of the wire and � the operating wavelength in like units, and means in circuit with said antenna for exciting the conductors in phase opposition whereby standing waves of opposite instantaneous polarity are formed on the conductors throughout their length.

Claim 16 claims an antenna arranged in conformity to the empirical formula, as in Claim 15, with "a similar parallel pair of conductors spaced an odd number of quarter wavelengths away from said first mentioned pair. . . ." These parallel wires constitute the "reflector," which, as already noted, was old in the art.

[Footnote 5]

"Still a further object of the present invention is to disclose the proper angle for conductors or radiators either an even number of half-wavelengths long or an odd number of half-wavelengths long, and, in general, to disclose the angle for best directional propagation for wires of any finite length."

After the present suit was brought, this paragraph was amended to read:

"Another object of the invention is to disclose the angle for the best directional propagation for open-ended wires of any finite length, preferably longer than the operating wavelength, having standing waves thereon and arranged in the manner proposed."

[Footnote 6]

"Moreover, it should be clearly understood that the wires of each unit can be of any length whatsoever, provided they are placed at the correct angle for their length. For best tuning, the total overall length of both of the wires and the 'U' loop terminating them should be effectively an integral number of half-wavelengths, but the portion forming the radiation element can be of any length. The law giving the correct angle for lengths between odd and even number of half-wavelengths is not given, due to its complexity, but the empirical formula and the curve of figure 12 will be found accurate for all practical purposes where the length of wire dealt with does not correspond to a whole number of half-wavelengths."

[Footnote 7]

See note 1 *supra.*

[Footnote 8]

See note 5 *supra.*

[Footnote 9]

See note 5 *supra*.

[Footnote 10]

See note 1 *supra*.