

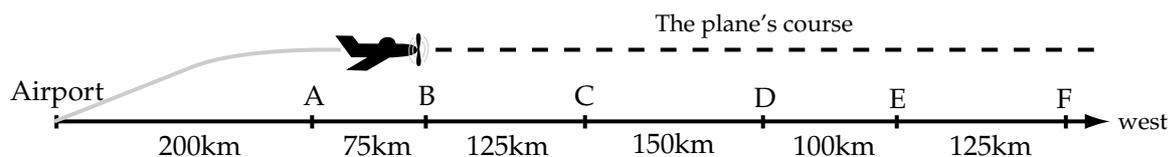
## Project 1 – The Missing Airplane

Last night, an airplane carrying a small number of passengers and some supplies left the Cairo airport flying due west. The engine of the plane had just been replaced. The mechanic asked the pilot to tell him by radio the speed of the airplane every ten minutes of the trip, so that the mechanic could make sure that the engine was functioning normally.

For the first hour, the pilot did as he was told and called back speeds every ten minutes to the mechanic, which the mechanic wrote down. The mechanic reasoned that the engine was working well enough, so he told the pilot to report to him every 15 minutes of the second hour of flight.

Halfway through the second hour of flight, the pilot called in to the mechanic to report his speed, but said suddenly, “Oh no, the engine is giving out! I am going to have to make an emergency landing right away!” Then the radio communication stopped and no one has been able to reach the pilot or any of the other passengers since. They have been declared missing, and have enough supplies to last for a few days. The airport authority needs your help to find the missing plane and passengers before it is too late!

The airport authority has come to you to ask where they should look for the plane. They know that the plane headed due west for its whole trip. Moreover, there are several small towns on the plane’s course. Labeling each town with a letter A through F, their respective distances along the plane’s course are given by the following diagram:



There is a search party ready to look for the plane. Your job is to tell the search party where to go. The search party can cover a total distance of 100km, although they must start from one of the towns. Moreover, half of the search party can go in one direction from the town and the other half in the other direction, although the total distance that they can cover is still 100km (*e.g.* half of the search party can go 75km in one direction from the town, and the other half of the party can go 25km in the other direction from the town). You need to tell them which town to start at and how far in each direction to go.

To help you as best he can, the mechanic has told you all the information he knows. Below is a table of the speeds of the plane for the 90 minutes of its journey.

Time	Speed km/h
:00	0
:10	300
:20	420
:30	390
:40	370
:50	280
:60	380
:75	400
:90	320

The mechanic also says that the plane does not travel at a constant speed due to weather and other factors. He does know, however, that the maximum speed that the plane can reach is 600km/hr, and the minimum speed it can have while still flying is 100km/hr. Moreover, since he just put in the engine, he knows that the maximum acceleration and deceleration of the plane is  $1800 \text{ km/hr}^2$ , the *jerk* of the plane (the rate of change of acceleration) is at most  $6000 \text{ km/hr}^3$ , the “double jerk” (the rate of change of the jerk) is at most  $50,000 \text{ km/hr}^4$ , and the “triple jerk” (the rate of change of the double jerk) is at most  $1.8 \times 10^6 \text{ km/hr}^5$ . (The mechanic keeps very good records of his engine facts.)

**Your job:** Prepare a detailed report telling the search party where to look for the plane, in particular which town to start from and how far to go in each direction from that town. Since no one knows the actual distance that the plane traveled, your report will only include approximations of the total distance. Make sure to include the following components:

- Find initial approximations using the left hand endpoint and right hand endpoint approximations.
- How accurate are these approximations to the plane’s actual distance? In particular, what are the maximum and minimum possible distances the plane could have gone?
- Do the same for the following methods: the midpoint rule, the trapezoid rule, Simpson’s rule (see your Calculus textbook for descriptions of each method, and make sure you read the whole section on them).
- Out of all the methods, which is the most accurate? Which is the least accurate? Why?
- Based on your approximations using all of the methods above, make your final recommendation for the search party. How certain are you that the search party will find the plane using your recommendation?

Note that just addressing the above points is not sufficient for your report. You also need to make sure that your report abides by the requirements given in the “Project Guidelines” handout. Good luck, the pilot and passengers are counting on you to find them!