# Electromagnetics for Working Engineers

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# Messy Math

- To *solve* EM problems, math can be messy
- To *understand* EM math is not too bad!
- We'll minimize the math here
  - Derivatives
  - Integration
  - Maxwell's Equations
  - Faraday's Law → Inductance

# Derivative

- How fast is *something* changing?

$$\frac{d}{dt}\left[something\right]$$

Changing with respect to time

$$\frac{d}{dx}\left[something\right]$$

Changing with respect to position (x)

# Partial Derivative

- How fast is *something* changing for one variable?

$$\frac{\partial}{\partial t}\left[something(t,x)\right]$$

Changing with respect to time (as ‘x’ is constant)

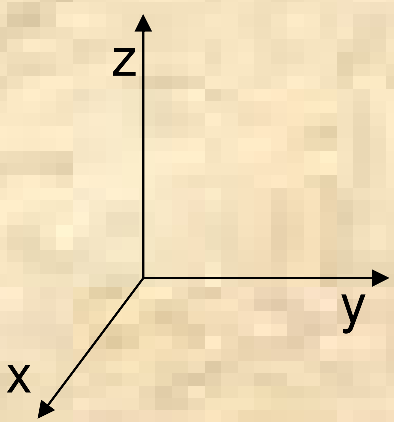
$$\frac{\partial}{\partial x}\left[something(t,x)\right]$$

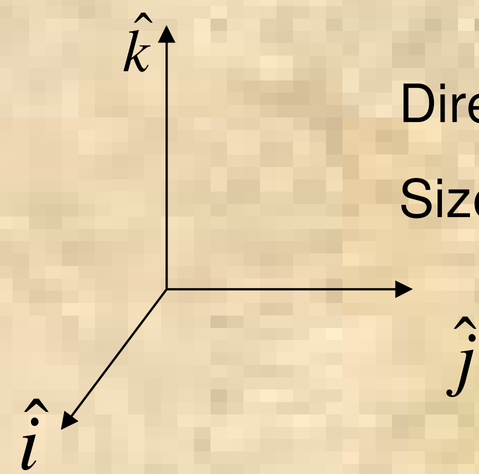
Changing with respect to position (x) (as time is constant)

# Vector Notation

## Dot, Gradient

- Dot Product
  - How much of something is going in a specified direction?




Direction only

Size = 1

# Vector Notation – Dot Product

- Suppose we have an electric field that varies in x, y, z

$$\overrightarrow{E} = E_x + E_y + E_z$$

$$\vec{E} \cdot \hat{i} = E_x$$

$$\vec{E} \cdot \hat{j} = E_y$$

$$\vec{E} \cdot \hat{k} = E_z$$

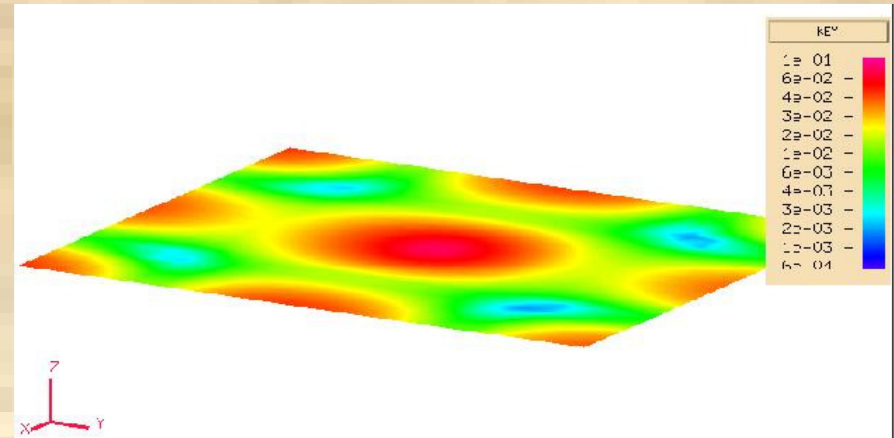
# Vector Notation -- Gradient

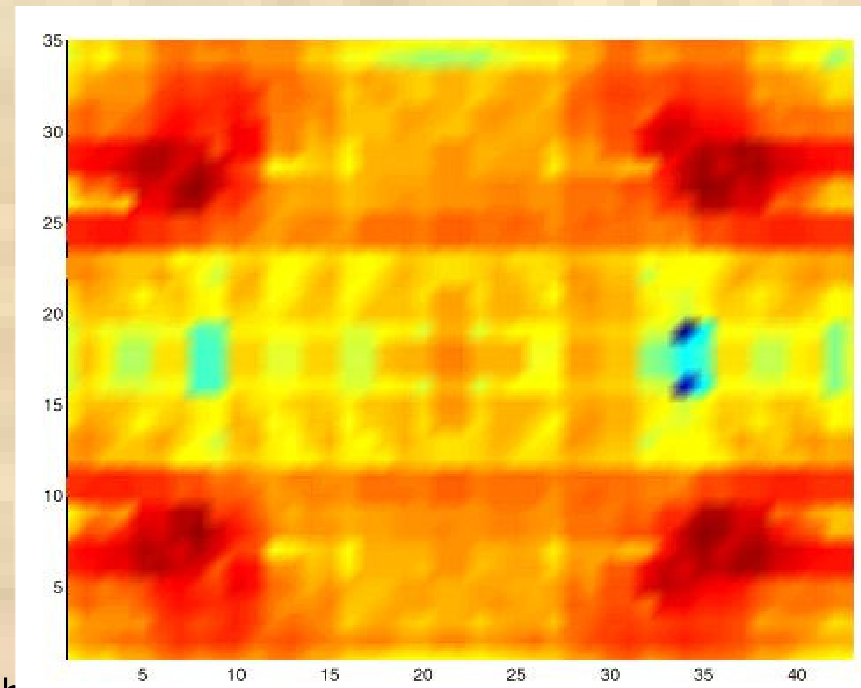
- How fast is something changing, and in what direction is this change?

$$\nabla \vec{E} = \left( \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \right) \vec{E}$$

# Gradient -- Example

- Voltage Distribution between power/gnd planes on printed circuit board
  – Standing wave due to resonance at 800 MHz



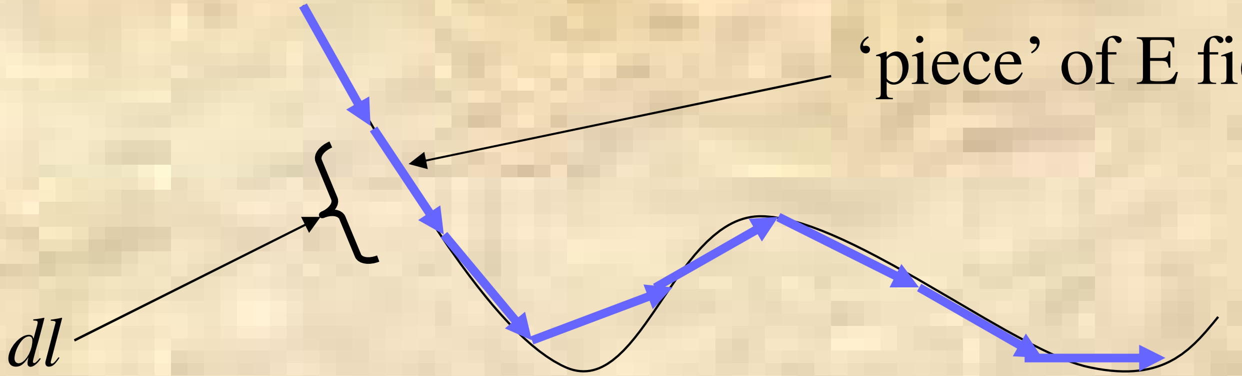



- Voltage Gradient
  – How fast is the voltage between plates changing?

# Integration

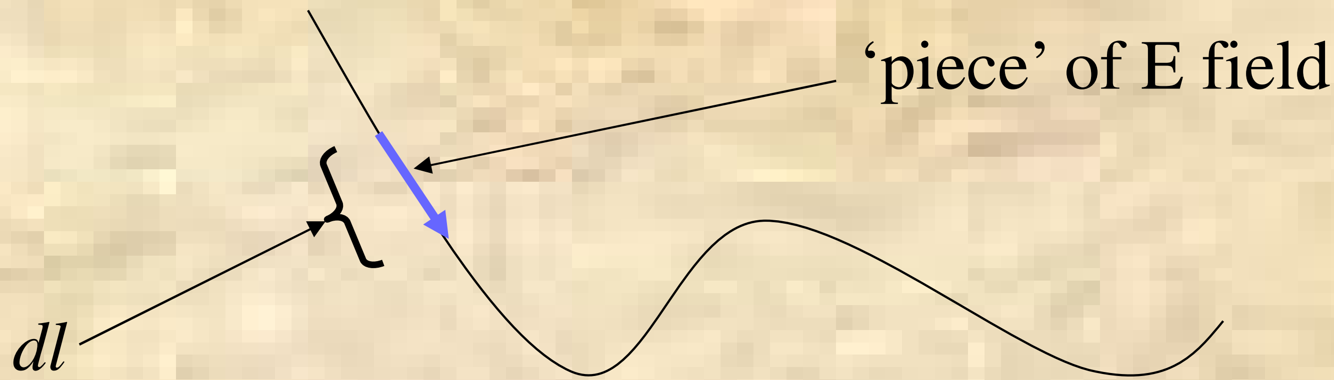
- Simply the sum of parts (when the parts are very small)
  - Line Integral --- sum of small line segments
  - Surface Integral -- sum of small surface patches
  - Volume Integral -- sum of small volume blocks

# Line Integral
## (find the length of the path)




$$V = -\int_{start}^{stop} (\vec{E} \bullet dl)$$

# Line Integral

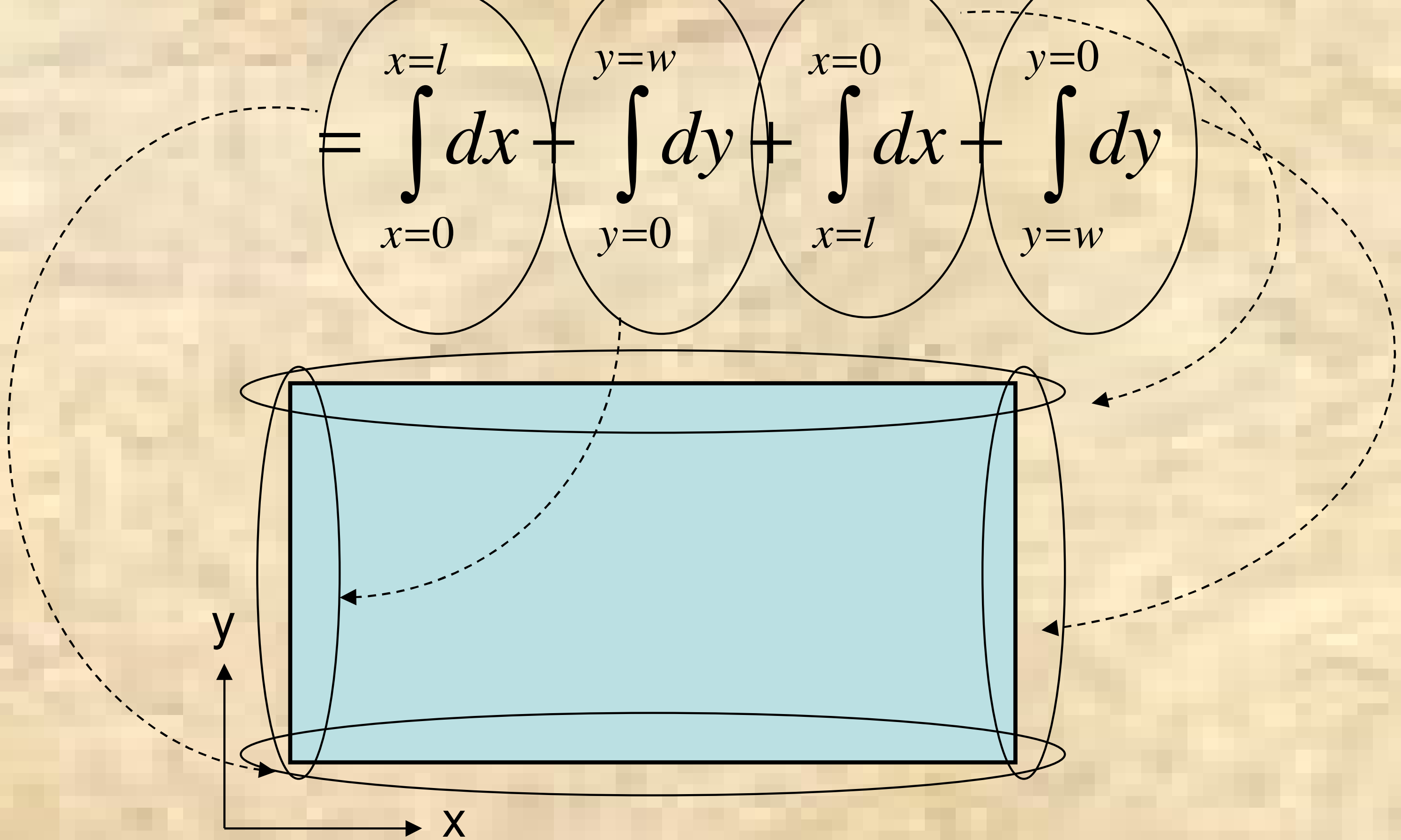



$$V = -\int_{start}^{stop} \left[ (E_x * dx) + (E_y * dy) \right]$$

# Line Integral -- Closed

$$Circumference = \oint path\ around\ box$$

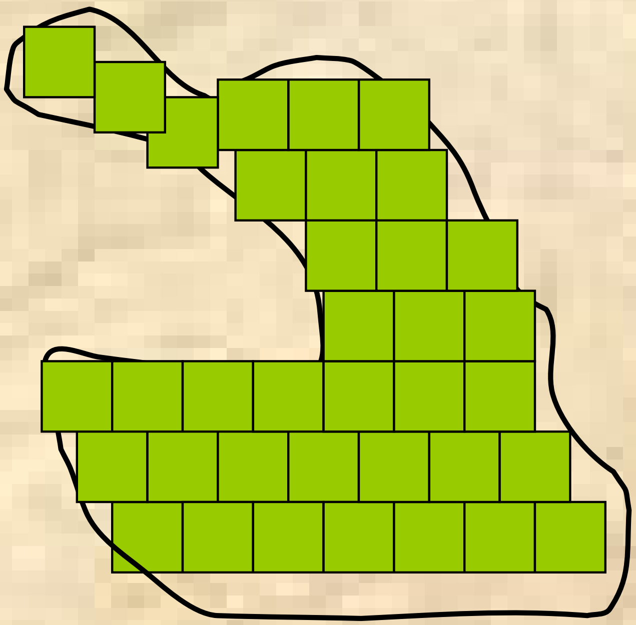
$$= \int_{x=0}^{x=l} dx + \int_{y=0}^{y=w} dy + \int_{x=l}^{x=0} dx + \int_{y=w}^{y=0} dy$$



# Line Integral -- Closed

- Closed line integrals find the path length
- And/or the amount of some quantity along that closed path length

# Surface Integral
## (find the area of the surface)



$$Area = \int da$$

$$da = dx * dy$$

$$Area = \iint dx * dy$$

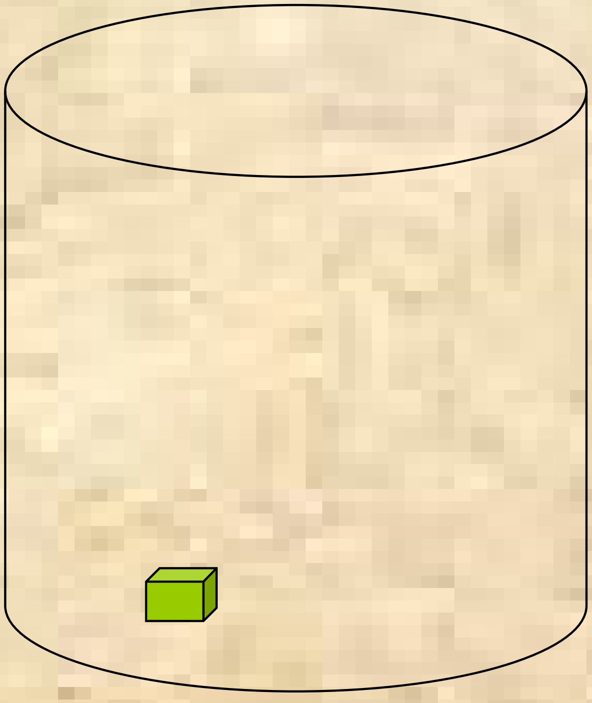
As *dx* and *dy* become smaller and smaller, the area is better calculated

# Closed Surface Integral

- Find the surface area of a closed shape

$$\oiint shape \; da$$

# Volume Integral
# (find the volume of an object)



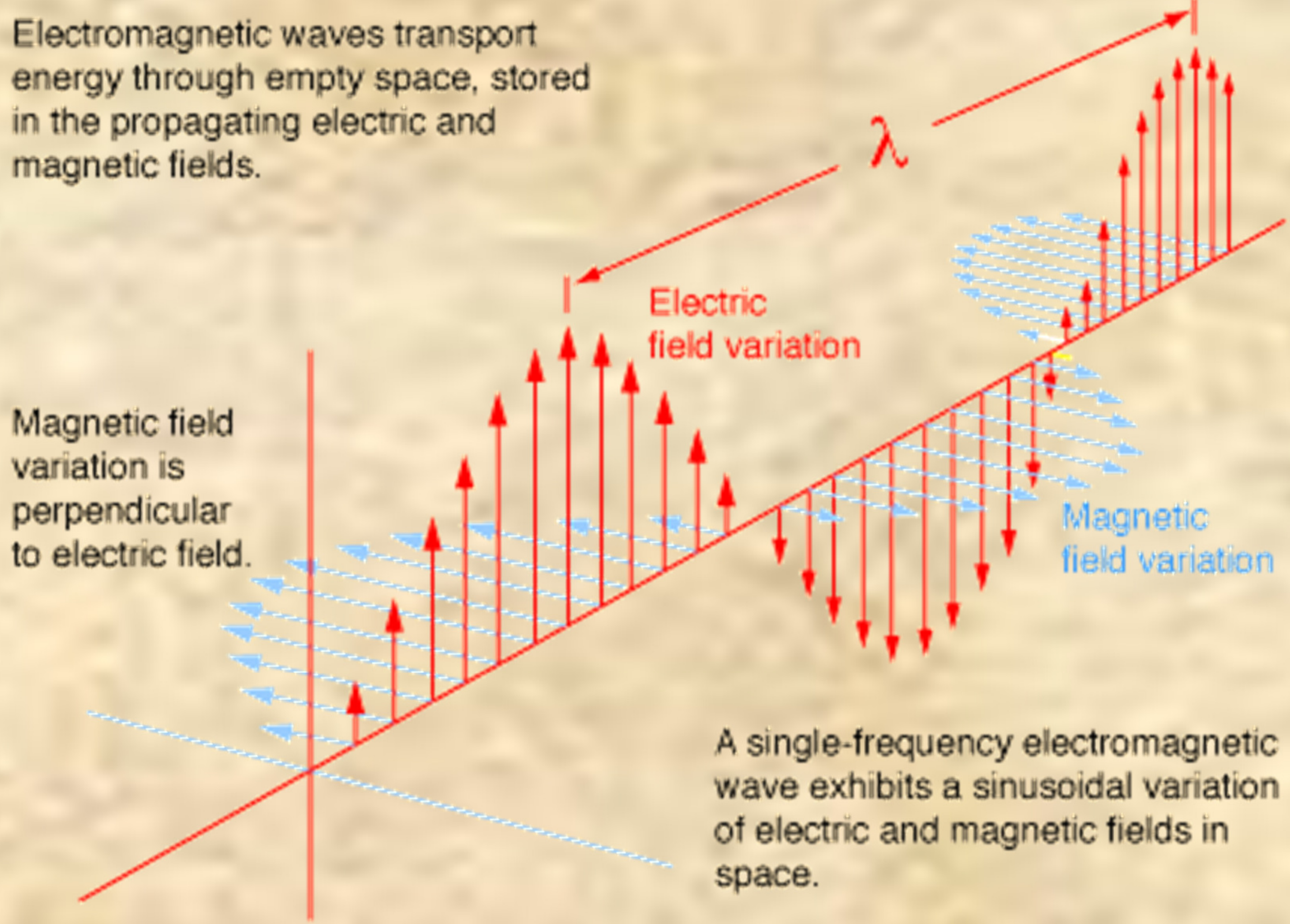
$$Volume = \int dv$$

$$dv = dx * dy * dz$$

$$Volume = \iiint [dx * dy * dz]$$

# From Math to Electromagnetics

# What is Electromagnetic Waves?



# Electromagnetics

## *In the Beginning*

- Electric and Magnetic effects not connected
- Electric and magnetic effects were due to ‘action from a distance’
- Faraday was the 1$^{st}$ to propose a relationship between electric lines of force and time-changing magnetic fields
  - Faraday was very good at experiments and ‘figuring out’ how things work

# Maxwell

- Discovered the link between the “electro” and the “magnetic”
- Scotland’s greatest contribution to the world next to Scotch
- Maxwell, Heaviside and Hertz

# Maxwell's Equations are NOT Hard!

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

# Maxwell's Equations – Differential Form

A difference in Magnetic Field across a small piece of space

A change in Electric Flux Density with respect to time

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

A difference in Electric Field across a small piece of space

A change in Magnetic Flux Density with respect to time

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

# Maxwell's Equations are not Hard!

- Change in H-field across space ~ Change in E-field (at that point) with time
- Change in E-field across space ~ Change in H-field (at that point) with time
- (Roughly speaking, and ignoring constants)

# Other Famous Equations

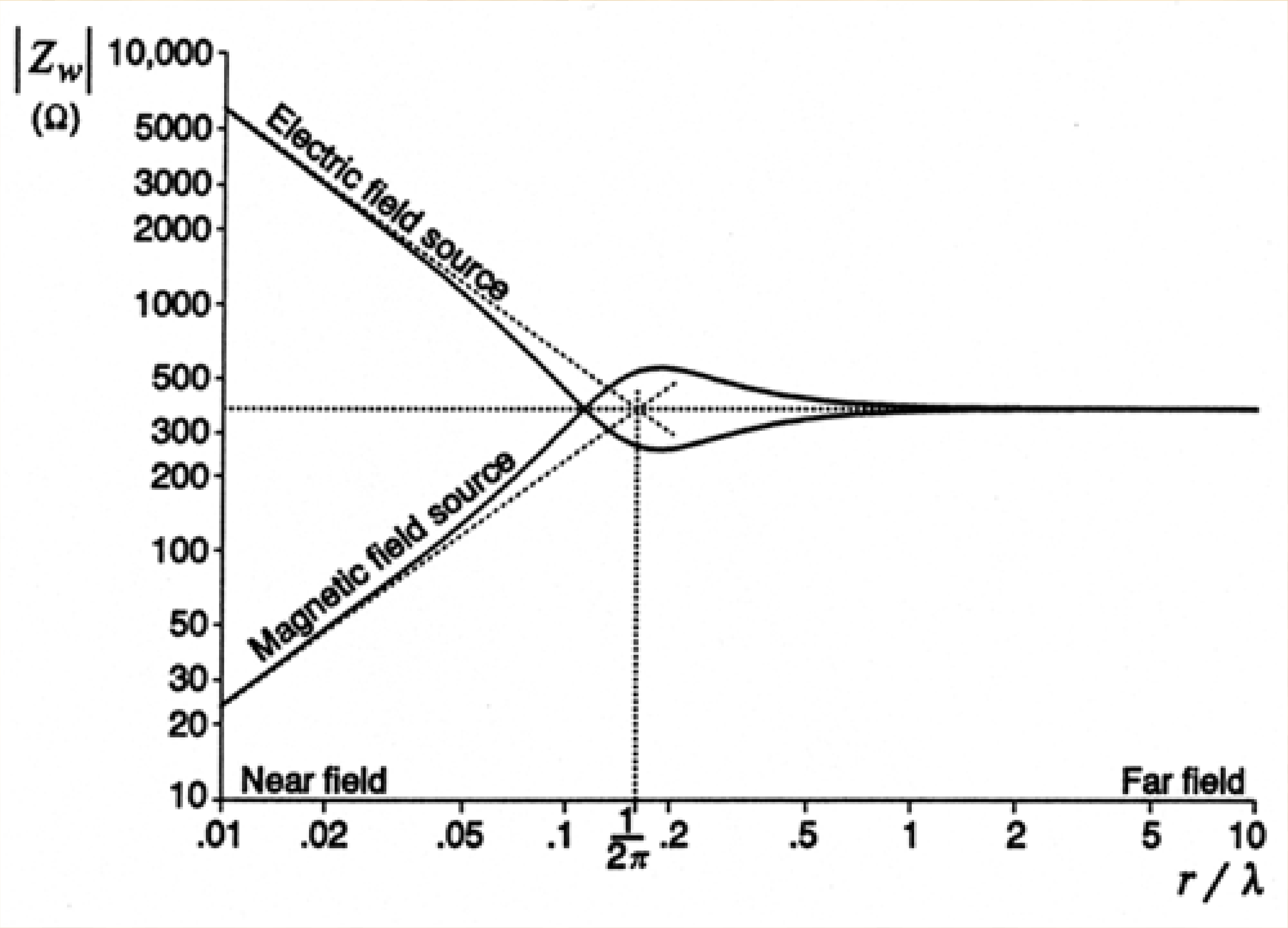
- Faraday's Law
- Gauss' Law
- Ampere's Law
- Stokes Theorem
- Many others

# In the Far Field

$$Z = \frac{\vec{E}}{\vec{H}} = 377\, ohms$$

- Electric field source (dipole, etc) has high impedance near to the source
- Magnetic field source (loop, etc) has low impedance near to the source

# EM Wave Impedance



# Near Field vs. Far Field

- Distance / Frequency
- Source Size
- Transition Distance Depends On Magnitude Of Error Allowed
- If Truly  Far-Field Then Source Can Usually Be Modeled Simply
- Equations and Graphs Assume Far-Field Simplified Case
- Real Life Problems Are Seldom Simple Due to Multiple Effects

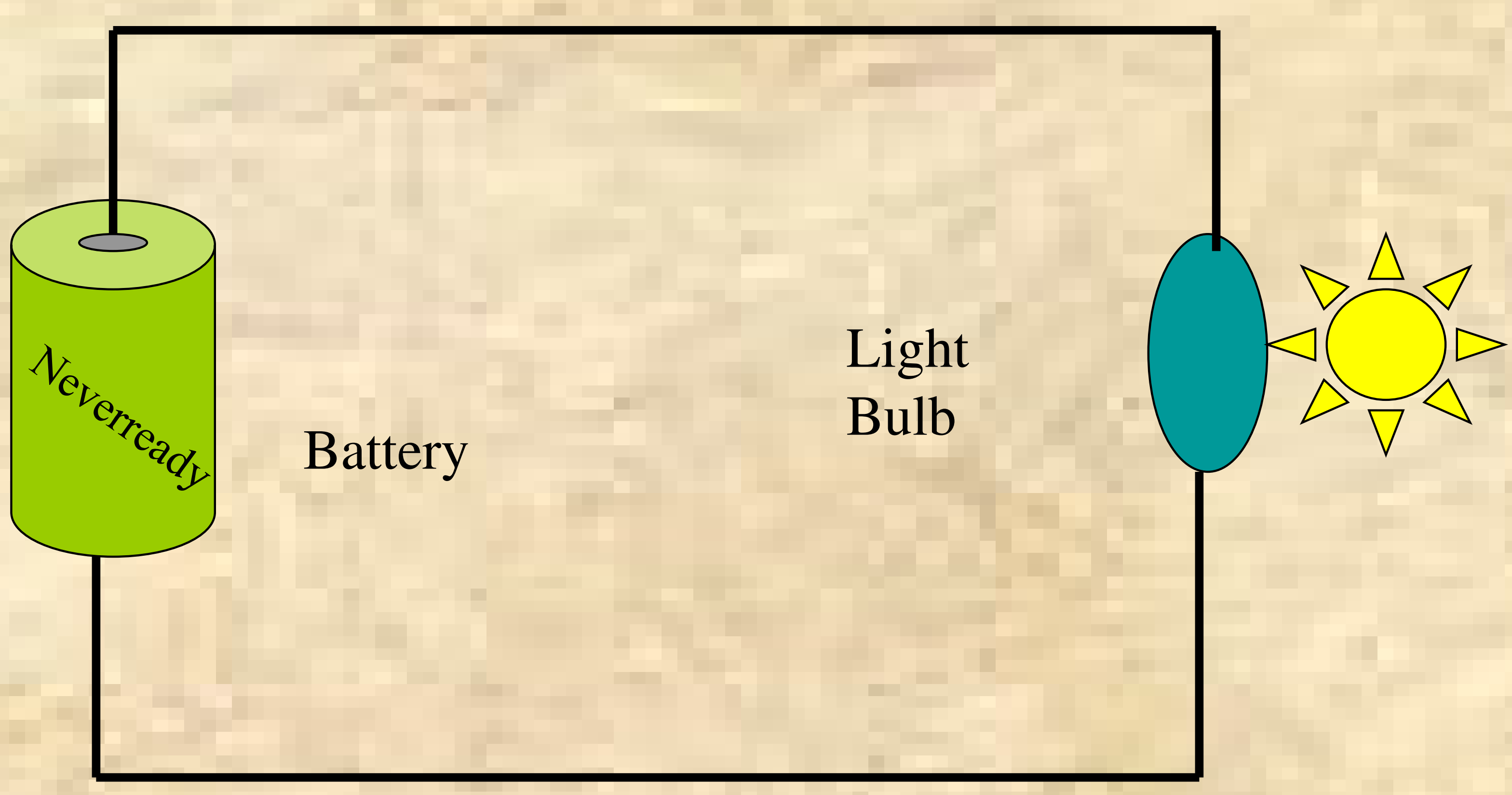
# There is No Such Thing as VOLTAGE!

- Initially shocking!
- Maxwell's equation include Electric Fields, Magnetic Fields and current!
- 'Voltage' defined as

$$V = -\int_{Start}^{Stop} E \cdot dl$$

- ***<u>CURRENT MUST ALWAYS RETURN TO ITS SOURCE</u>***

# Consider a Battery and Light Bulb Direct Current (DC)



# Alternating Current (AC)

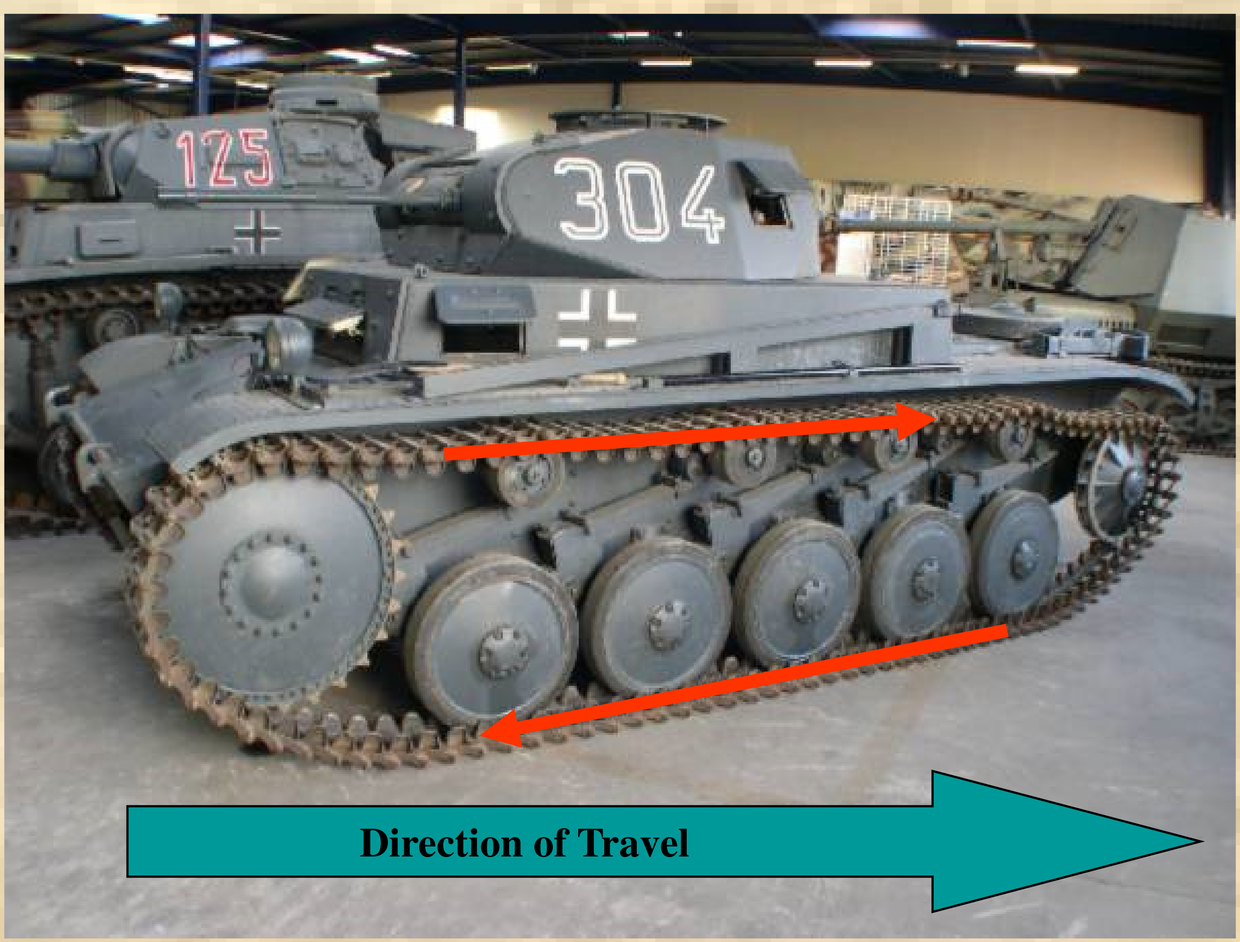
- Sine wave
- Current flows in one direction for first half cycle
- Current reverses and flows in the opposite direction for 2nd half cycle

# Printed Circuit Board Trace

- Assume 500 ps rise time
  - CMOS devices mean current only flows from driver during logic transition (500 ps pulse of current)
- Speed of light = 3 $e^8$ m/s
  - For FR4 eps=~4.2 → propagation velocity slower
    - 1.46 $e^8$ m/s = 14.6 cm/ns
  - 500 ps current pulse is 7.3 cm (2.87 in) long

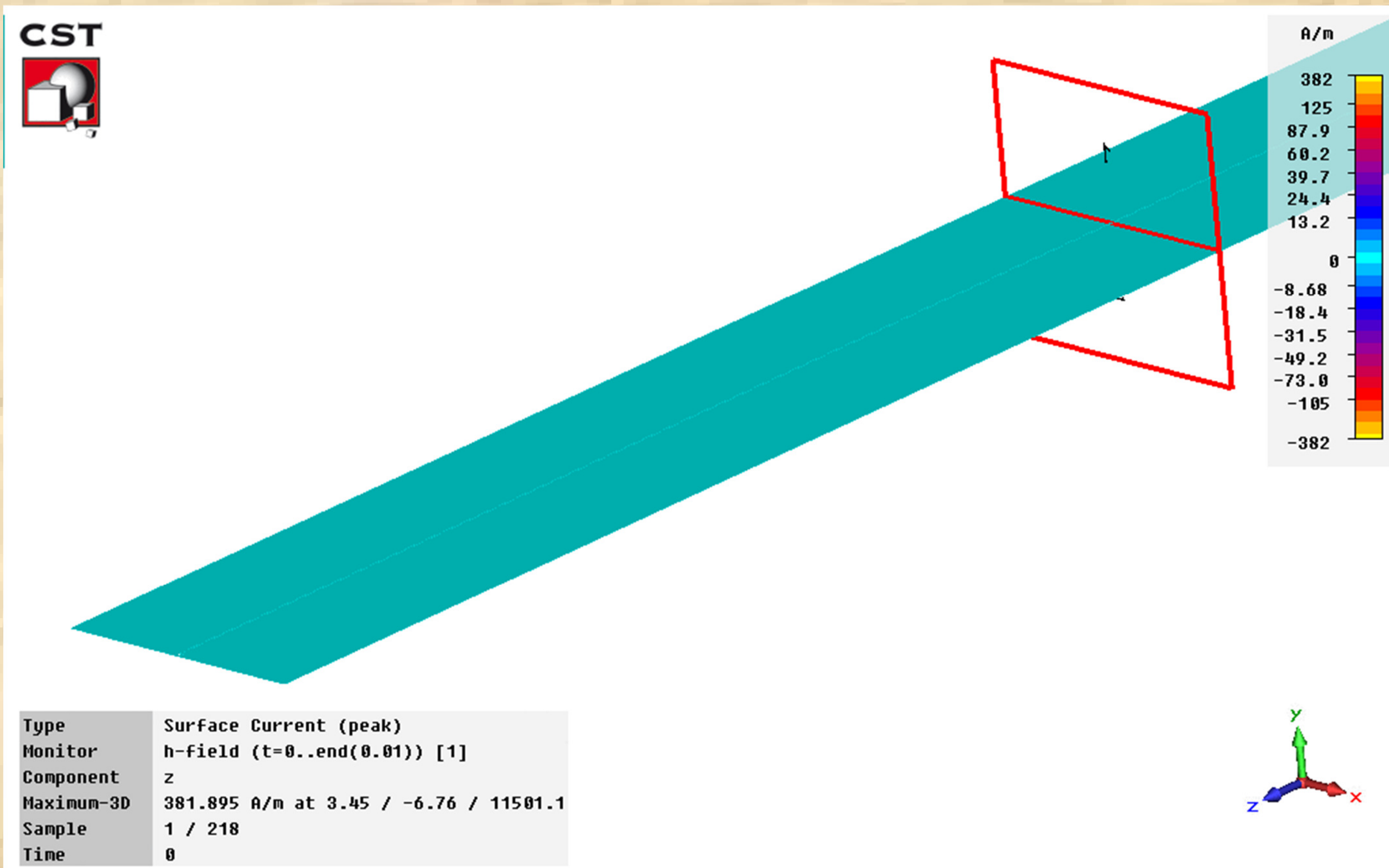
# Pulse of Current

- When Current pulse is shorter than trace



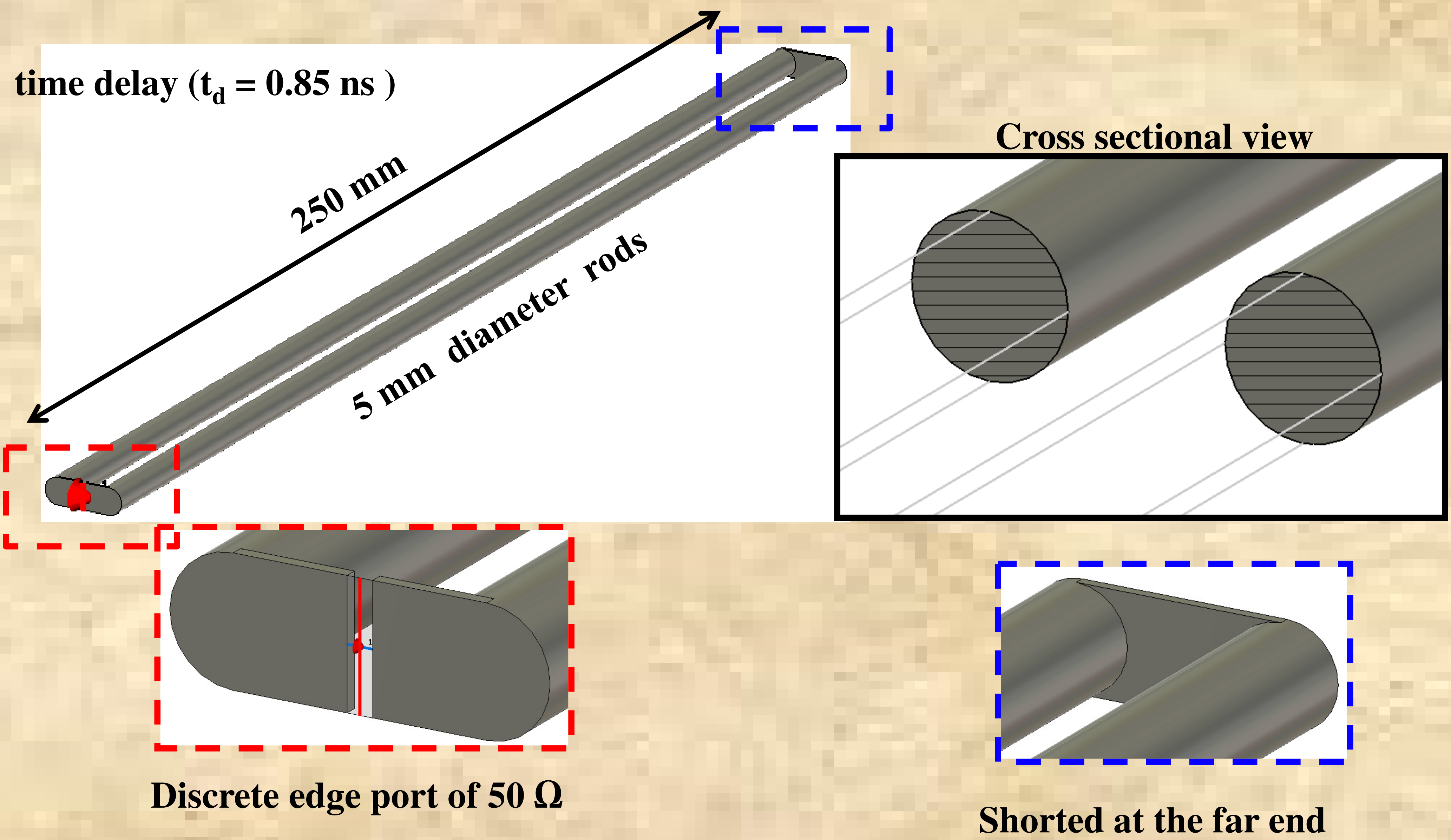

- Current only exists for a limited distance
- But still maintains a loop

# Surface Current



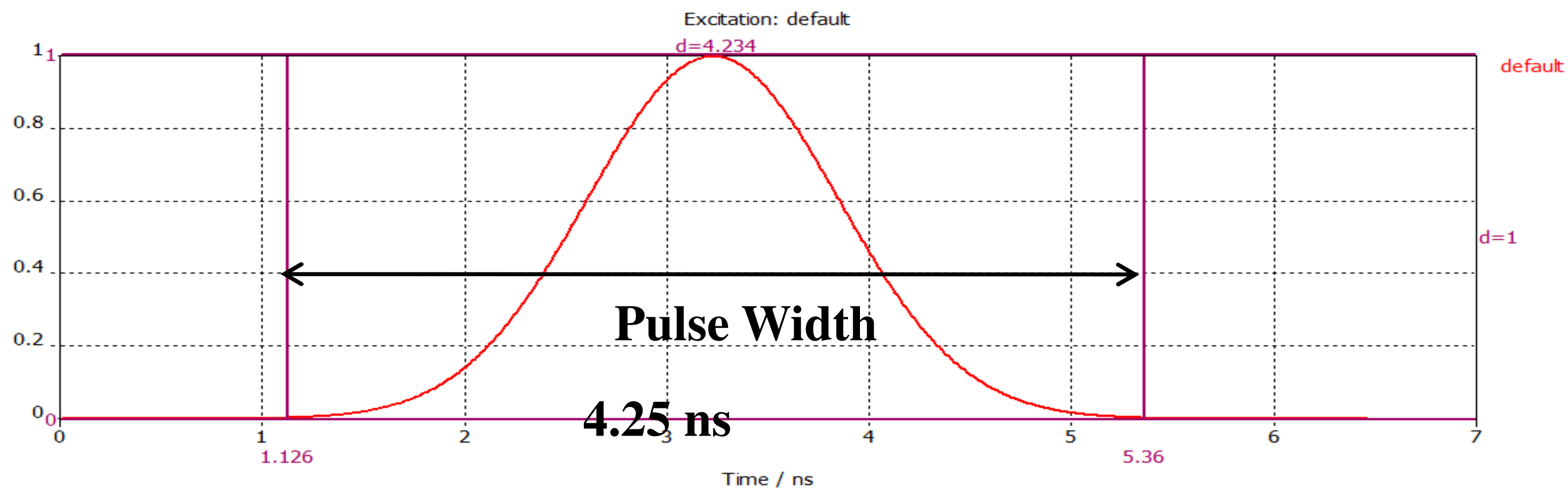
# Side View PCB Trace with Current Pulse

# Transmission-Line Geometry

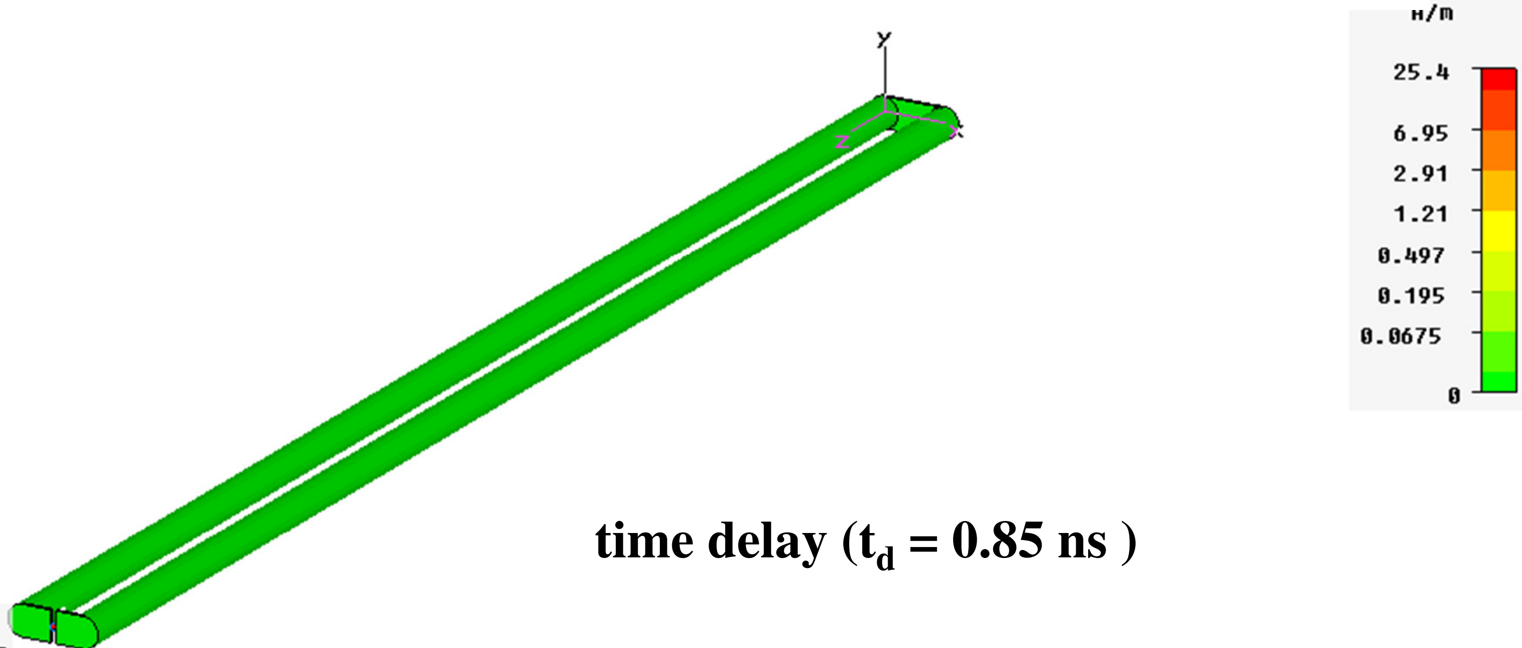


# Electrically Short Twin Lead

$$t_p = 5t_d$$




CST

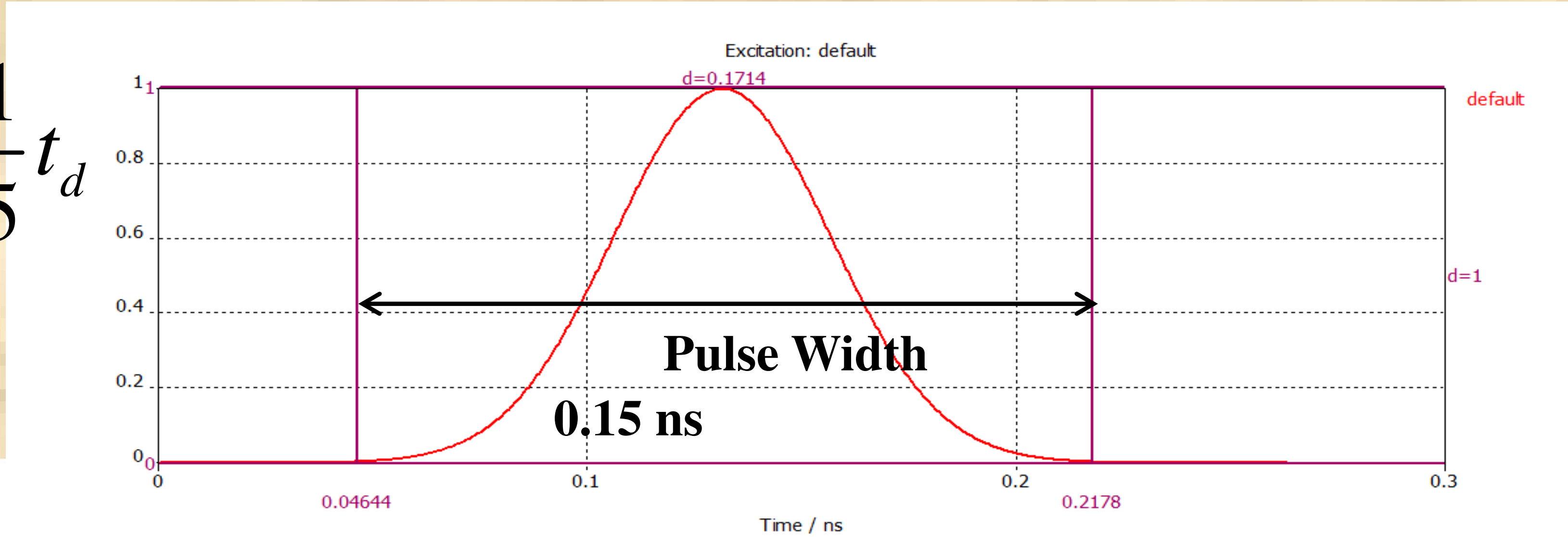



time delay ($t_d$ = 0.85 ns )

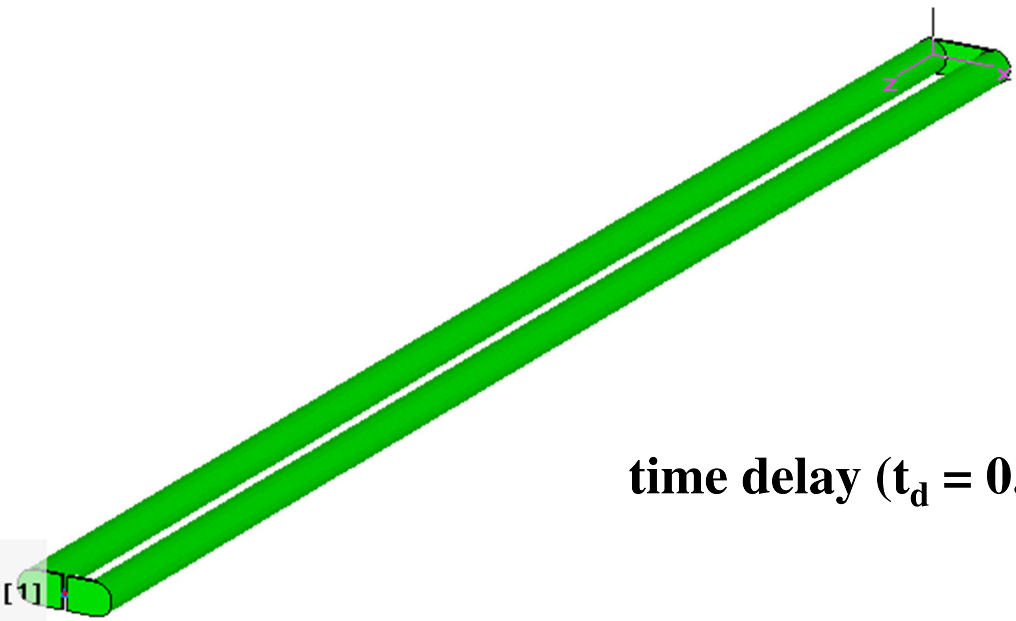
| | |
|---|---|
| Type | Surface Current (peak) |
| Monitor | h-field (t=0..end(1e-9)) [1] |
| Component | Abs |
| Maximum-3d | 25.4365 A/m at 0 / 2.5 / 0.5 |
| Sample | 1 / 141 |
| Time | 0 |

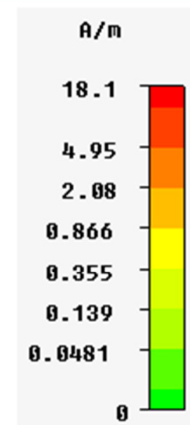
# Transmission Line Behavior

$$t_p = \frac{1}{5} t_d$$




CST



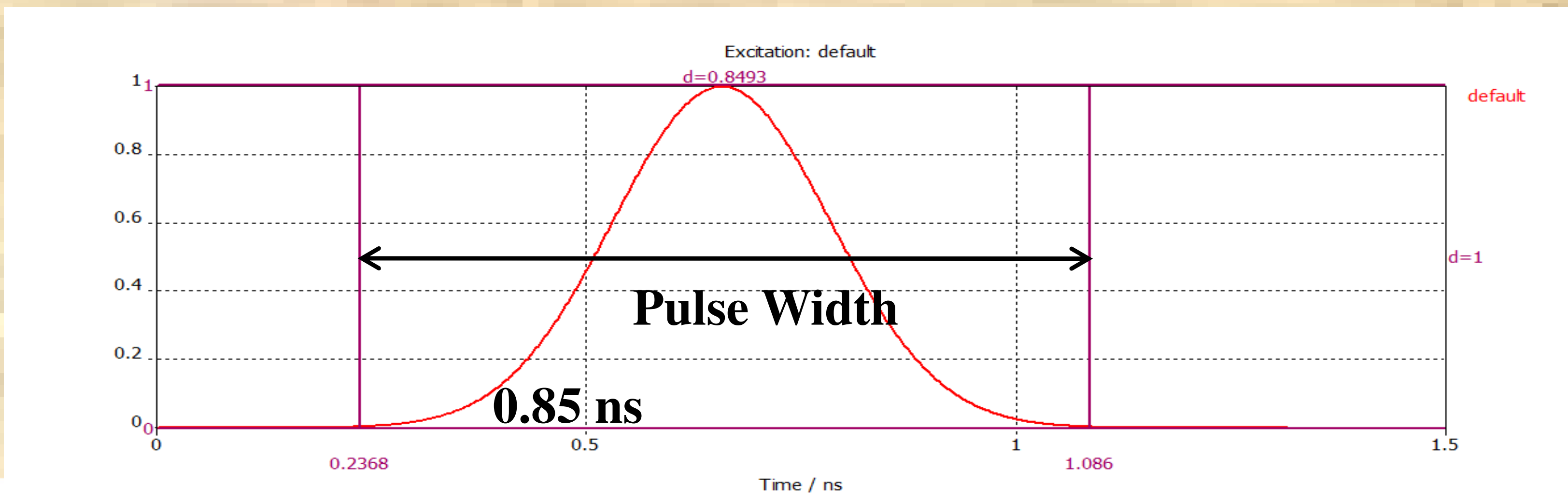



time delay ($t_d$ = 0.85 ns )

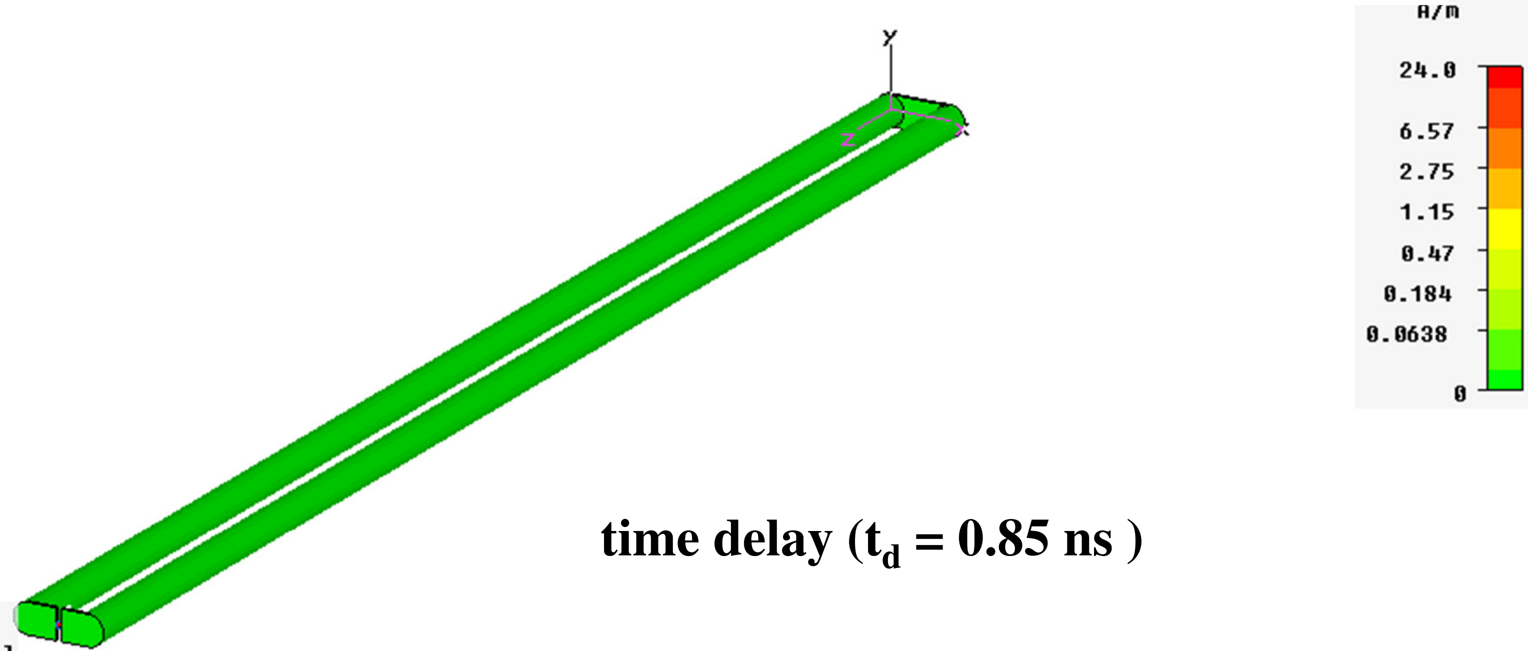
| | |
|---|---|
| Type | Surface Current (peak) |
| Monitor | h-field (t=0..end(1e-9)) [1] |
| Component | Abs |
| Maximum-3d | 18.1429 A/m at 0 / 2.5 / 0.5 |
| Sample | 1 / 31 |
| Time | 0 |

# Pulse Propagation

$$t_p = t_d$$




CST




time delay ($t_d$ = 0.85 ns )

| | |
|---|---|
| Type | Surface Current (peak) |
| Monitor | h-field (t=0..end(1e-9)) [1] |
| Component | Abs |
| Maximum-3d | 24.0457 A/m at 0 / 2.5 / 0.5 |
| Sample | 1 / 31 |
| Time | 0 |

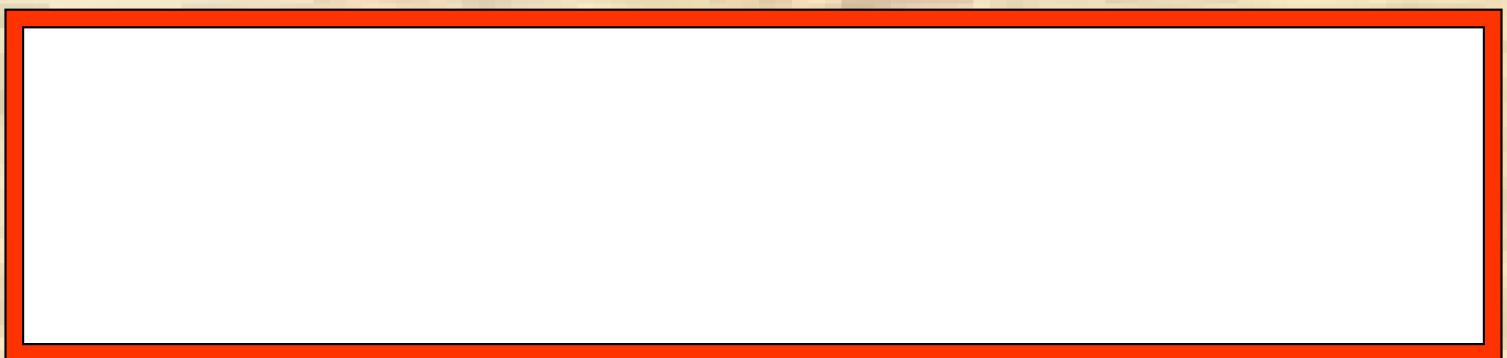
# Skin Depth

- Current flows only near surface at high frequencies

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

| Frequency | Skin Depth | Skin Depth |
|---|---|---|
| 60 Hz | 260 mils | 8.5 mm |
| 1 KHz | 82 mils | 2.09 mm |
| 10 KHz | 26 mils | 0.66 mm |
| 100 KHz | 8.2 mils | 0.21 mm |
| 1 MHz | 2.6 mils | 0.066 mm |
| 10 MHz | 0.82 mils | 0.021 mm |
| 100 MHz | 0.26 mils | 0.0066 mm |
| 1 GHz | 0.0823 mils | 0.0021 mm |

# Current Migrates to Outer Portions of the Conductor at High Frequencies



Resistance is determined by the area of the copper conductor actually used at each frequency!

# At High Frequencies

- Resistive loss and dielectric loss are present
- Inductance will usually dominate

# Inductance

- Current flow through metal = inductance!
- Fundamental element in EVERYTHING
- Loop area first order concern
- Inductive impedance increases with frequency and is MAJOR concern at high frequencies

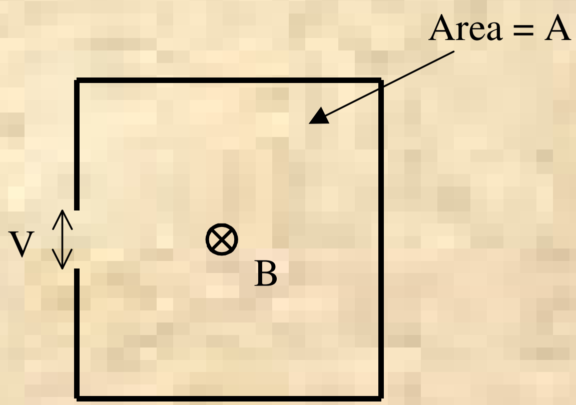
$$X_L = 2\pi f L$$

# Inductance Definition

- Faraday's Law

$$\oint \overline{E} \cdot dl = -\iint \frac{\partial \overline{B}}{\partial t} \cdot d\overline{S}$$

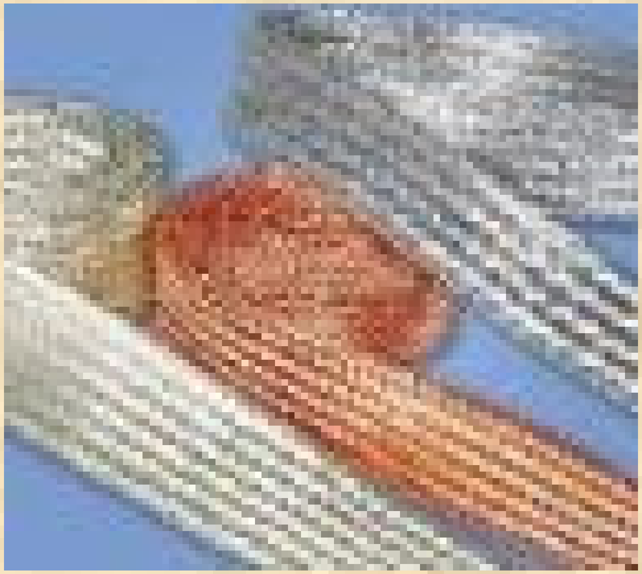
- For a simple rectangular loop




$$V = -A \frac{\partial B}{\partial t}$$

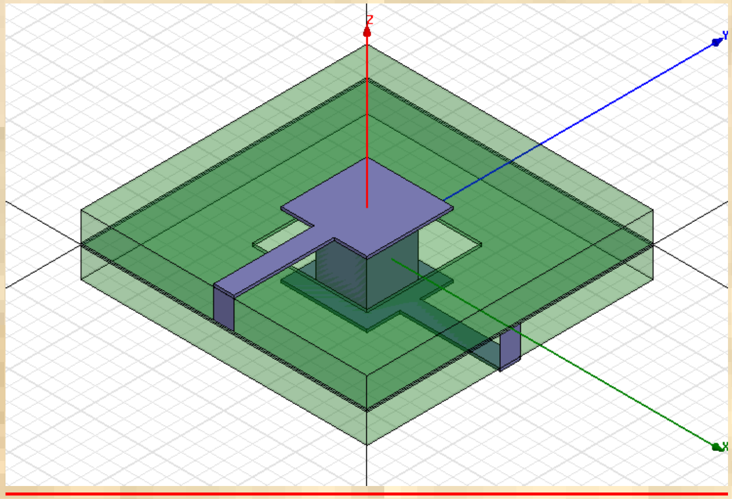
# Given the Definition of Inductance

- Do these have inductance?



"Ground Strap"

SMT Capacitor



PCB Via

**<u>Not until return path for current is identified!</u>**

# Current Loop = Inductance



Courtesy of Elya Joffe

# Self (Loop) Inductance

- Isolated circular loop

$$L \approx \mu_0 a \left( \ln \frac{8a}{r_0} - 2 \right)$$

- Isolated rectangular loop

$$L = \frac{2\mu_0 a}{\pi} \left( \ln \frac{p + \sqrt{1 + p^2}}{1 + \sqrt{2}} + \frac{1}{p} - 1 + \sqrt{2} - \frac{1}{p}\sqrt{1 + p^2} \right)$$

Note that inductance is directly influenced by loop ***AREA*** and less influenced by conductor size!

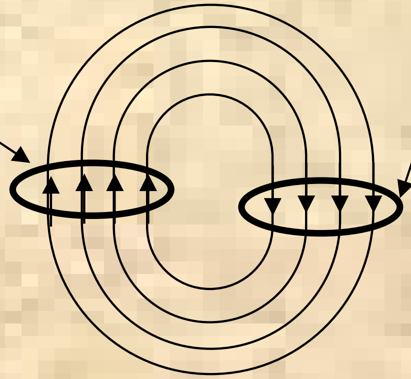
$$p = \frac{\textit{length of side}}{\textit{wire radius}}$$

# Mutual Inductance

$$\Phi_2 = M_{21} I_1$$

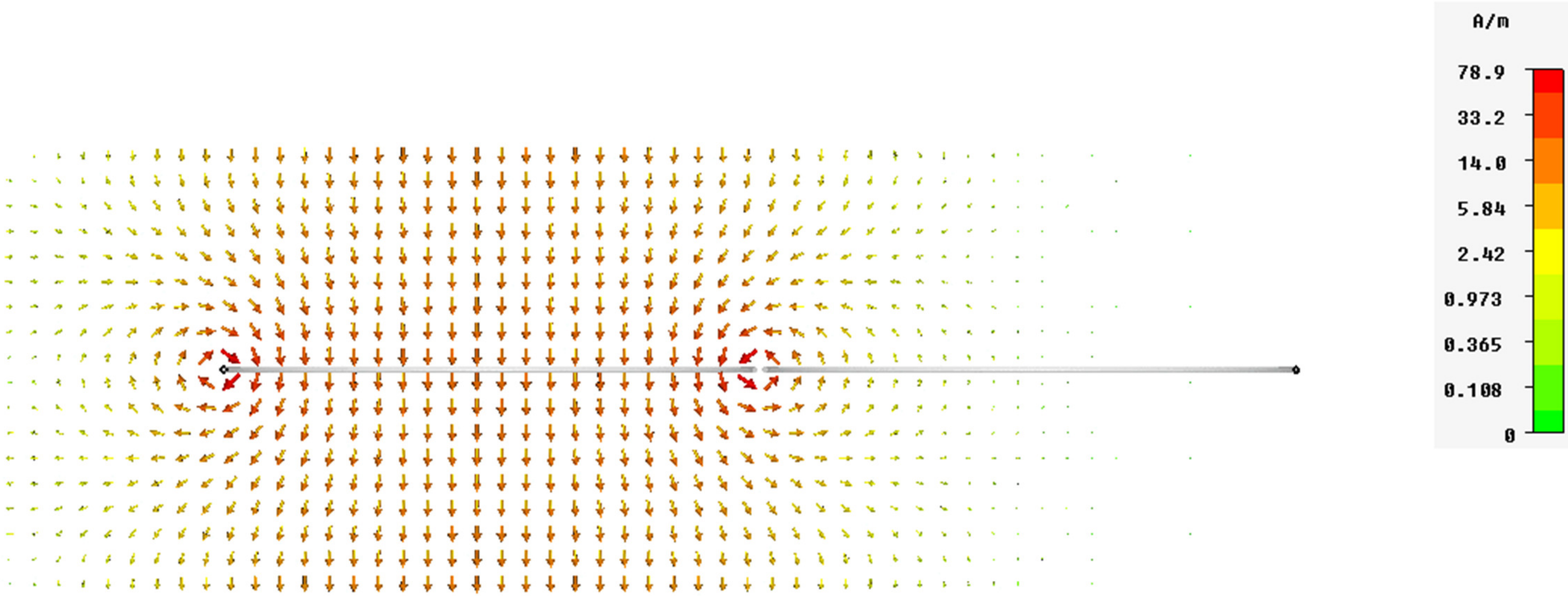
$$M_{21} = \frac{\Phi_2}{I_1}$$

How much magnetic flux is induced in loop #2 from a current in loop #1?

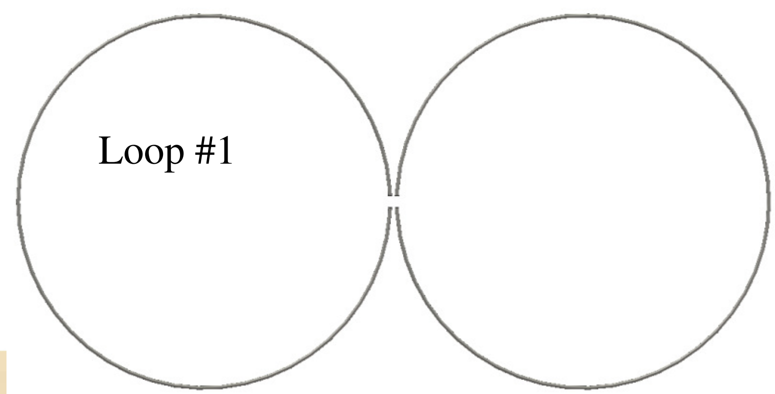



$$\Phi_2 = \int_{S_2} \vec{\mathrm{B}}_1(\mathrm{r}) \cdot \hat{n}\, \mathrm{dS}_2$$

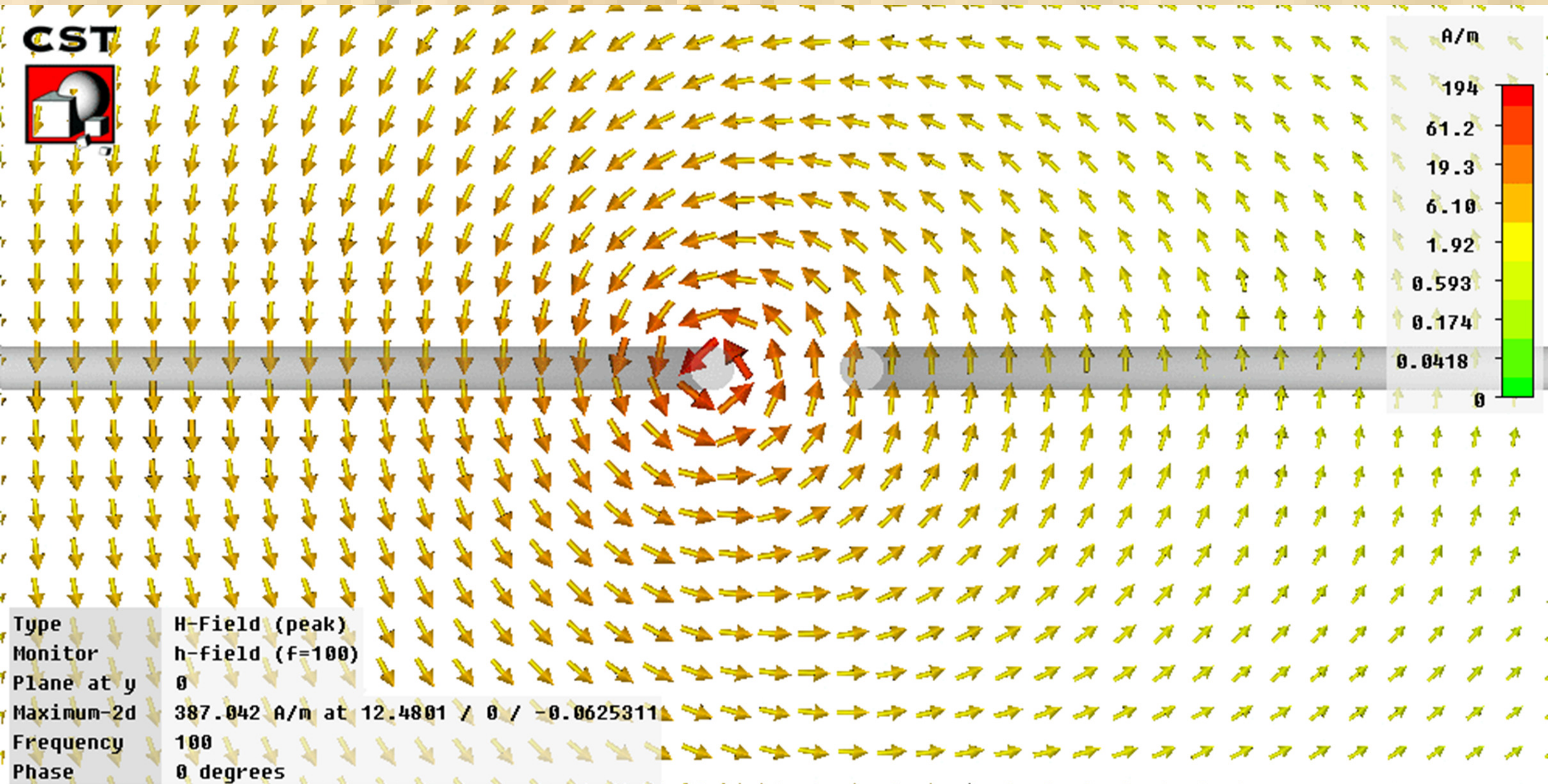
# Flux from Current in Loop #1



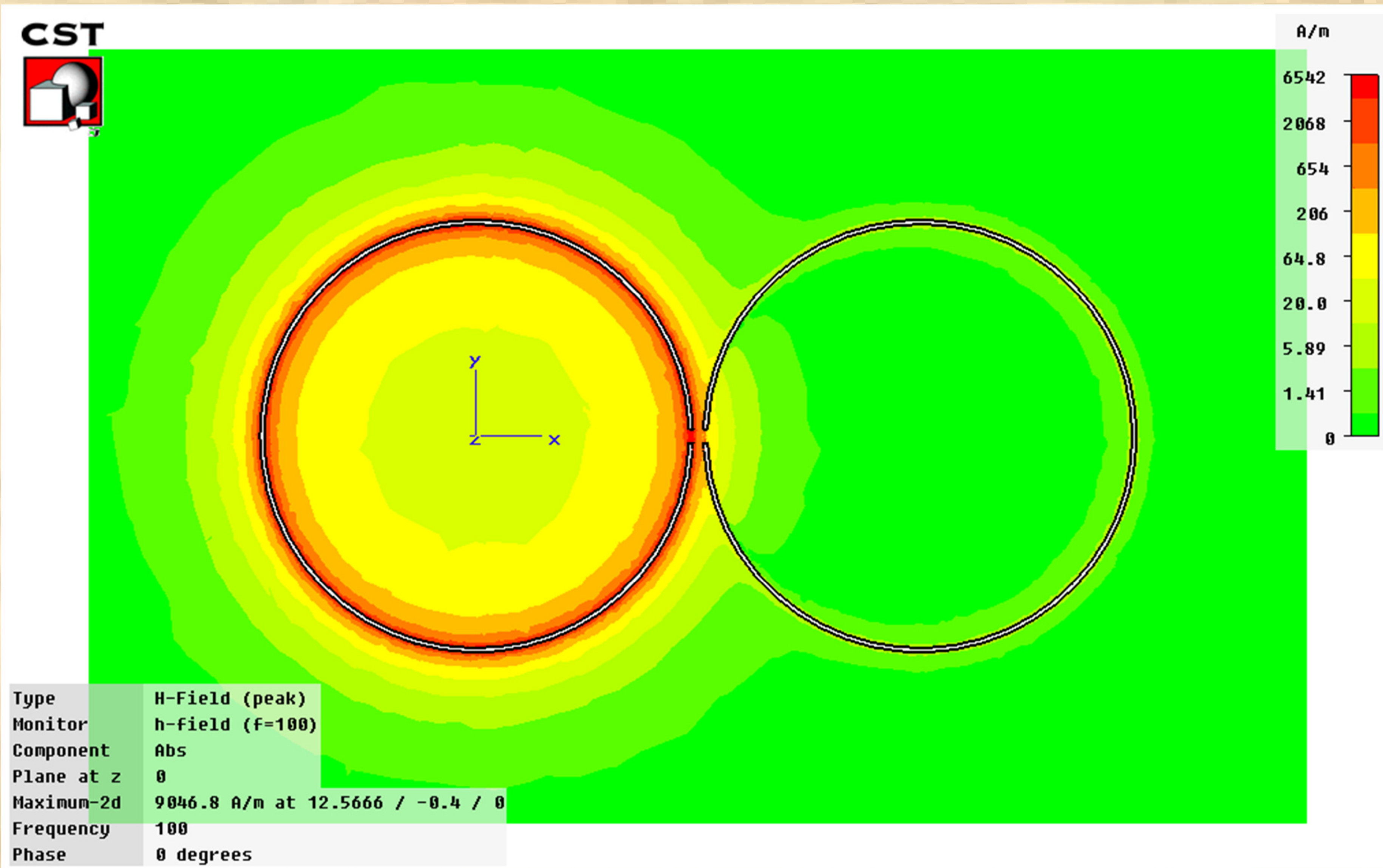

| | |
|---|---|
| Type | H-Field (peak) |
| Monitor | h-field (f=100) |
| Plane at y | 0 |
| Maximum-2d | 78.866 A/m at -12.2246 / 2.11637e-015 / 0.595706 |
| Frequency | 100 |
| Phase | 0 degrees |

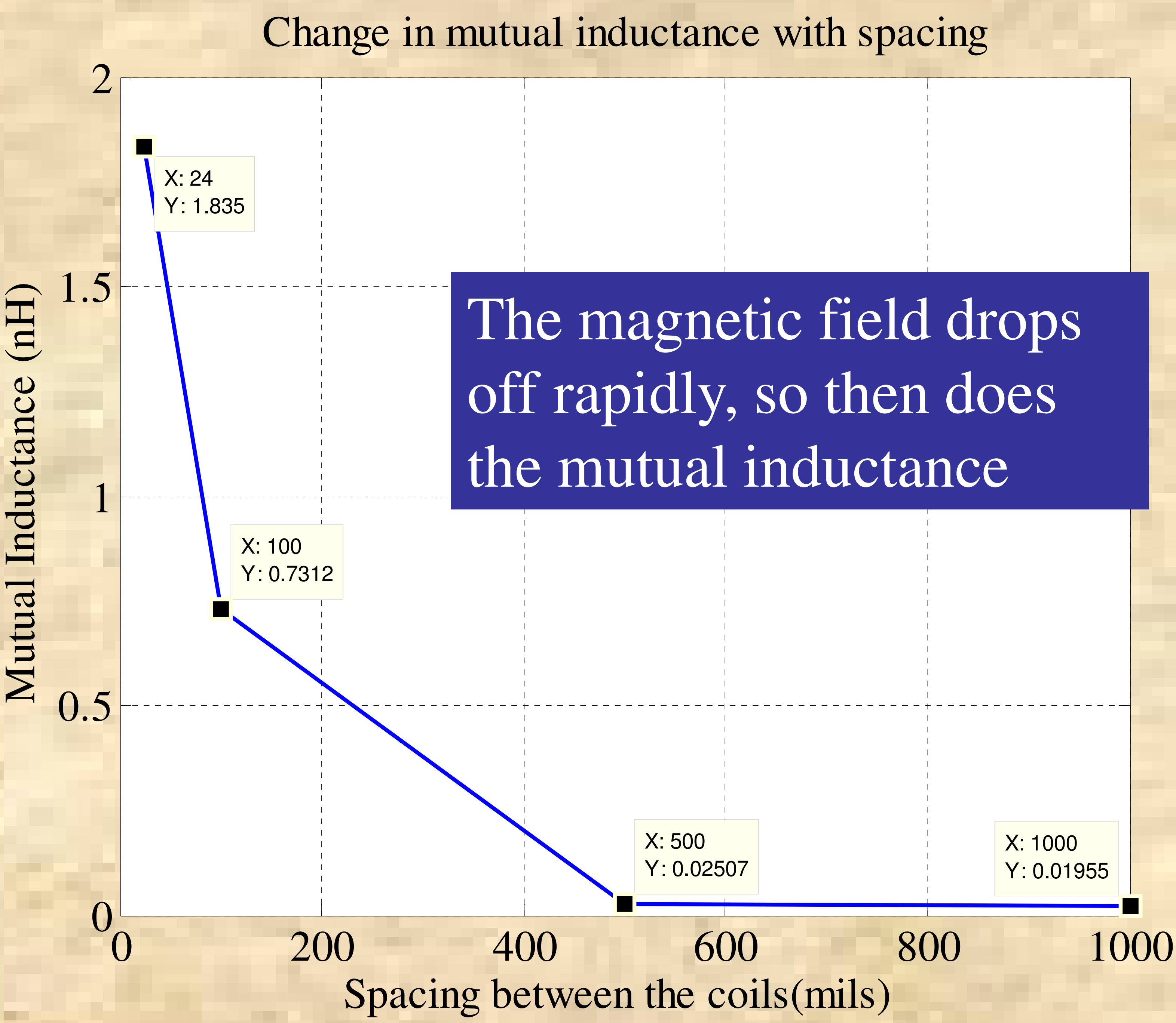


# Flux from Current in Loop #1



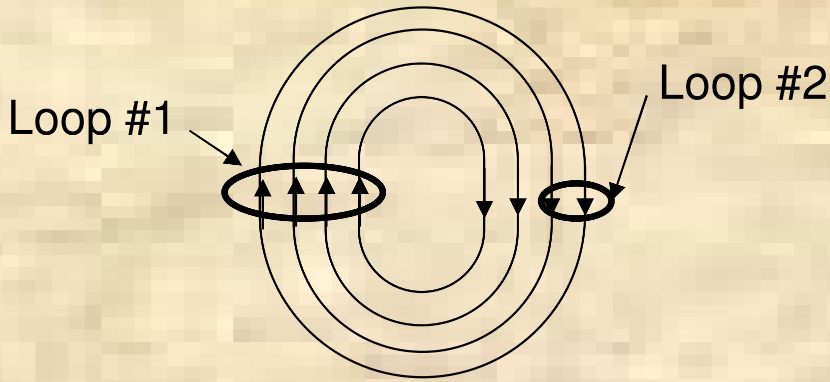
# Flux from Current in Loop #1



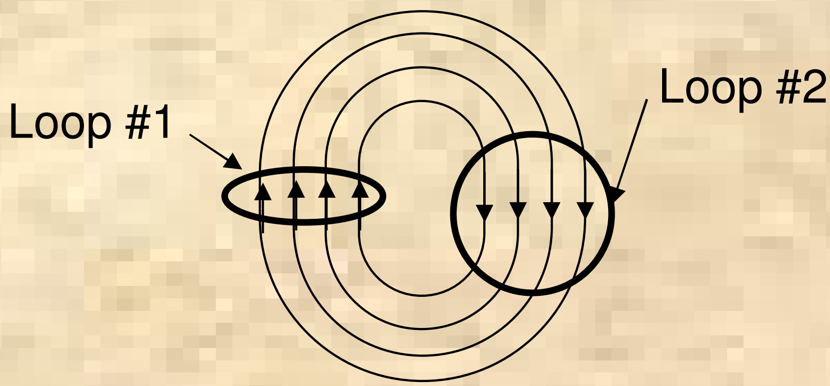



The magnetic field drops off rapidly, so then does the mutual inductance

# Mutual Inductance




Less loop area in loop #2 means less magnetic flux in loop #2 and less mutual inductance

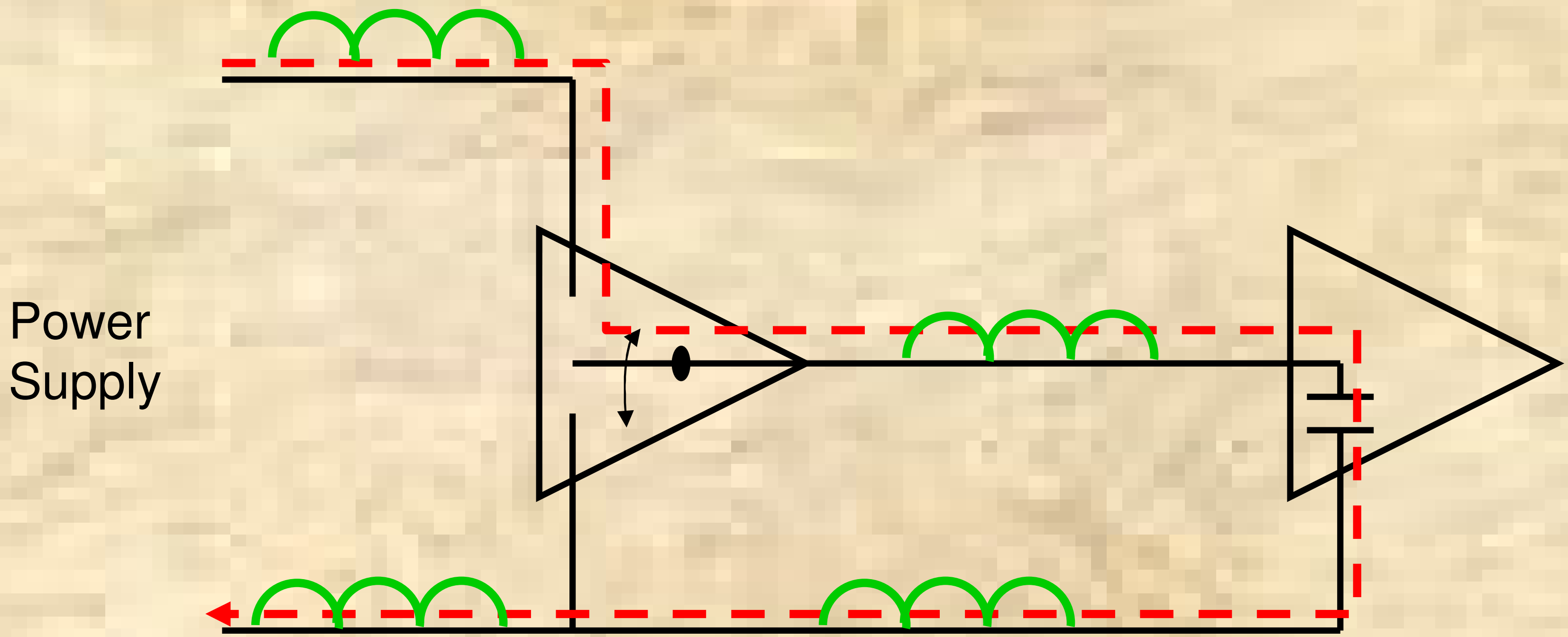



Less loop area perpendicular to the magnetic field in loop #2 means less magnetic flux in loop #2 and less mutual inductance

# Partial Inductance

- We now know that a loop of current has inductance
- We now know that there must be a complete loop to have inductance
- But where do we place this inductance in a circuit?

# Zero-to-One Transition
# Where's the Inductance Go??

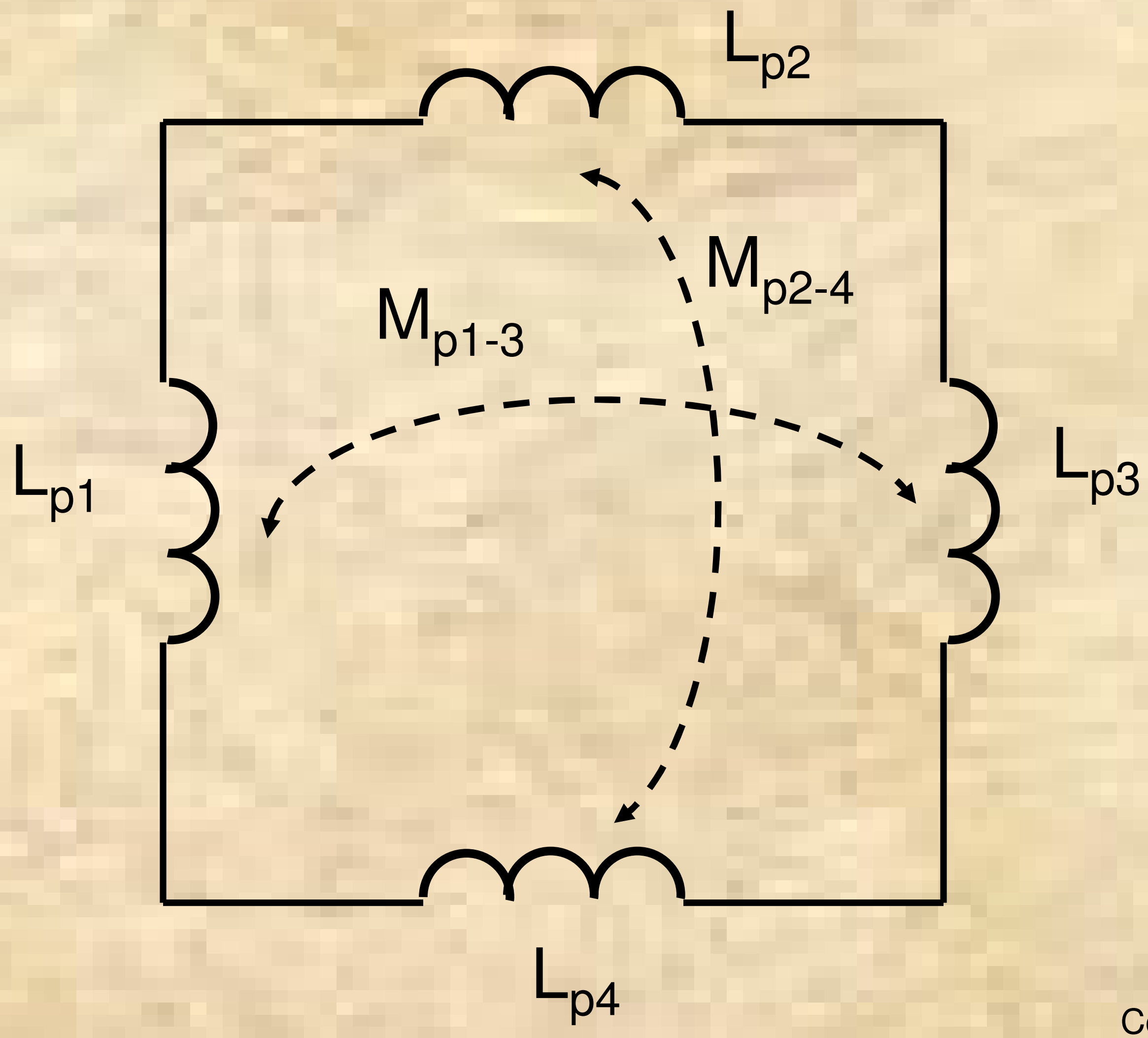



And how could you possibly calculate it?

Courtesy of Dr. Clayton Paul

# Total Loop Inductance from Partial Inductance

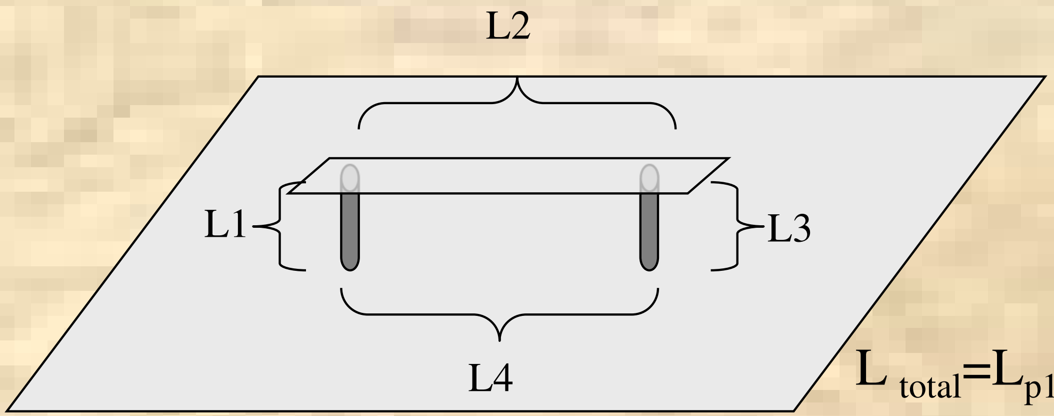
$$L_{total} = L_{p1} + L_{p2} + L_{p3} + L_{p4} - 2M_{p1\text{-}3} - 2M_{p2\text{-}4}$$




Courtesy of Dr. Clayton Paul

# Partial Inductance

- Simply a way to break the overall loop into pieces in order to find total inductance



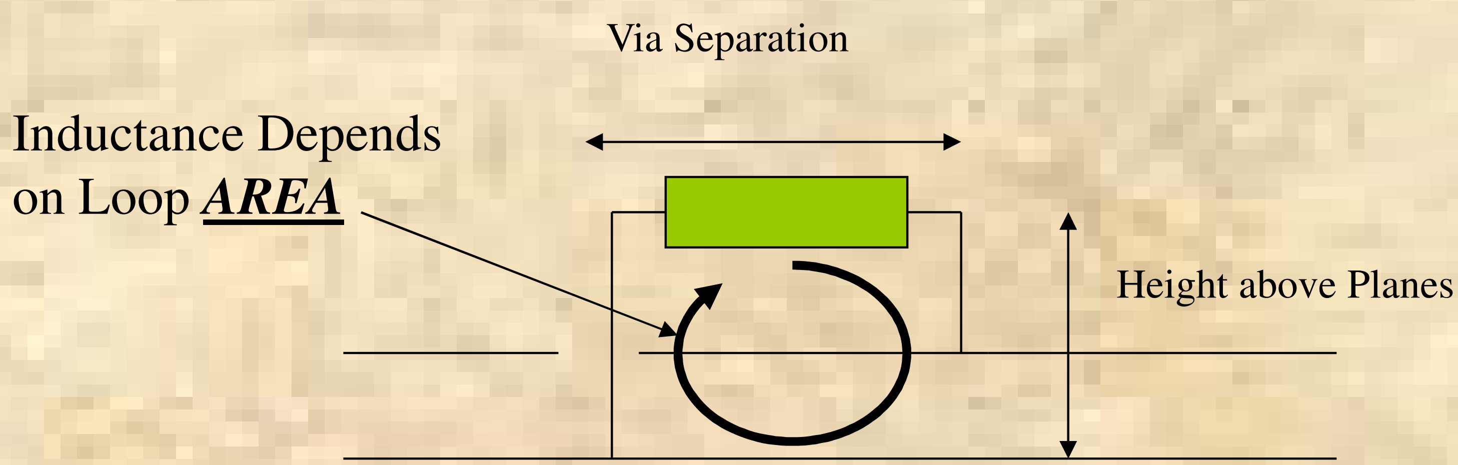

$$L_{total} = L_{p11} + L_{p22} + L_{p33} + L_{p44} - 2L_{p13} - 2L_{p24}$$

# Important Points About Inductance

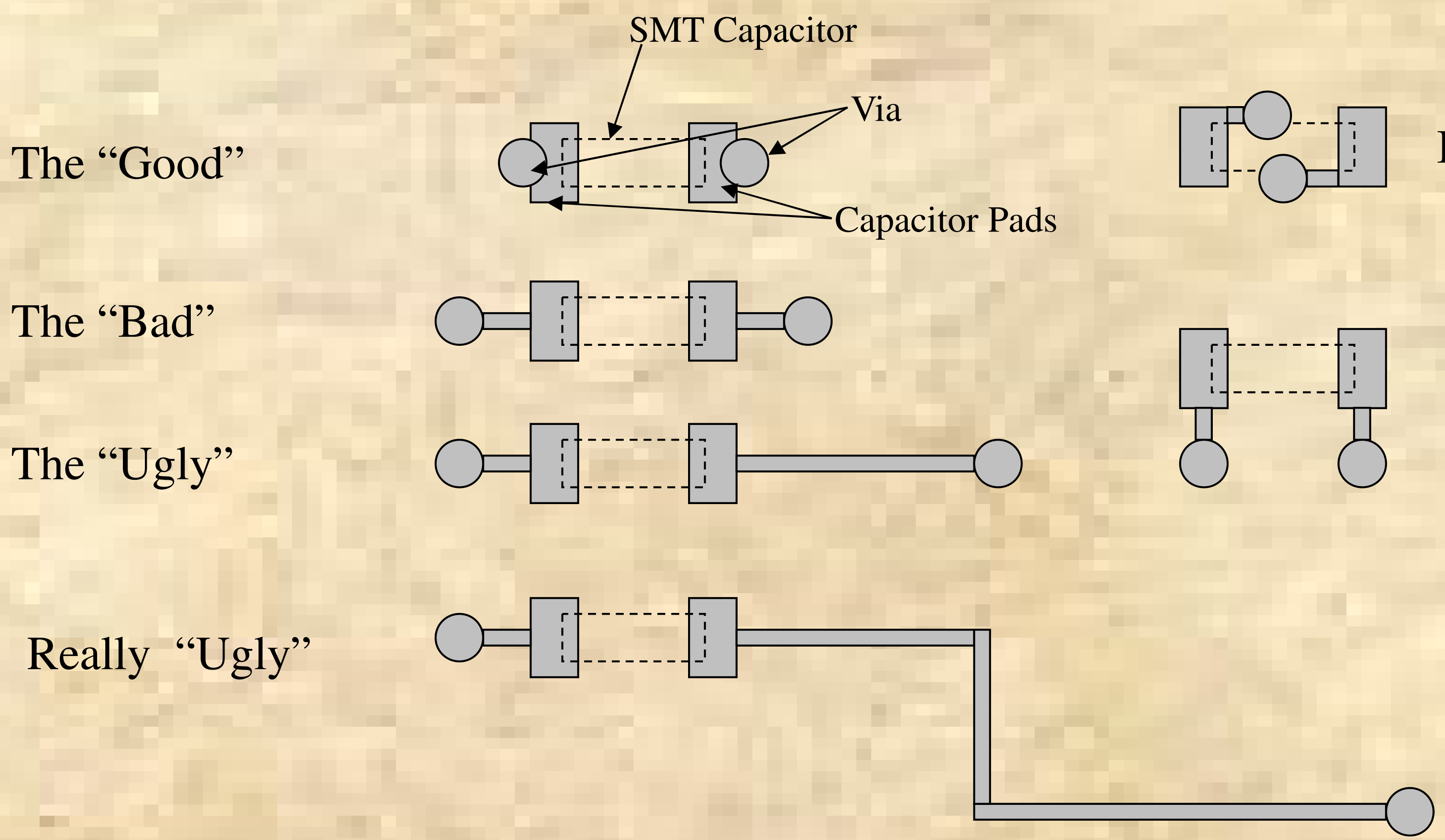
- Inductance is everywhere
- Loop area most important
- Inductance is everywhere

# Example
## Decoupling Capacitor Mounting

- **Keep vias as close to capacitor pads as possible!**



# Via Configuration Can Change Inductance



# What is Capacitance?

$$C = \frac{Q}{V}$$

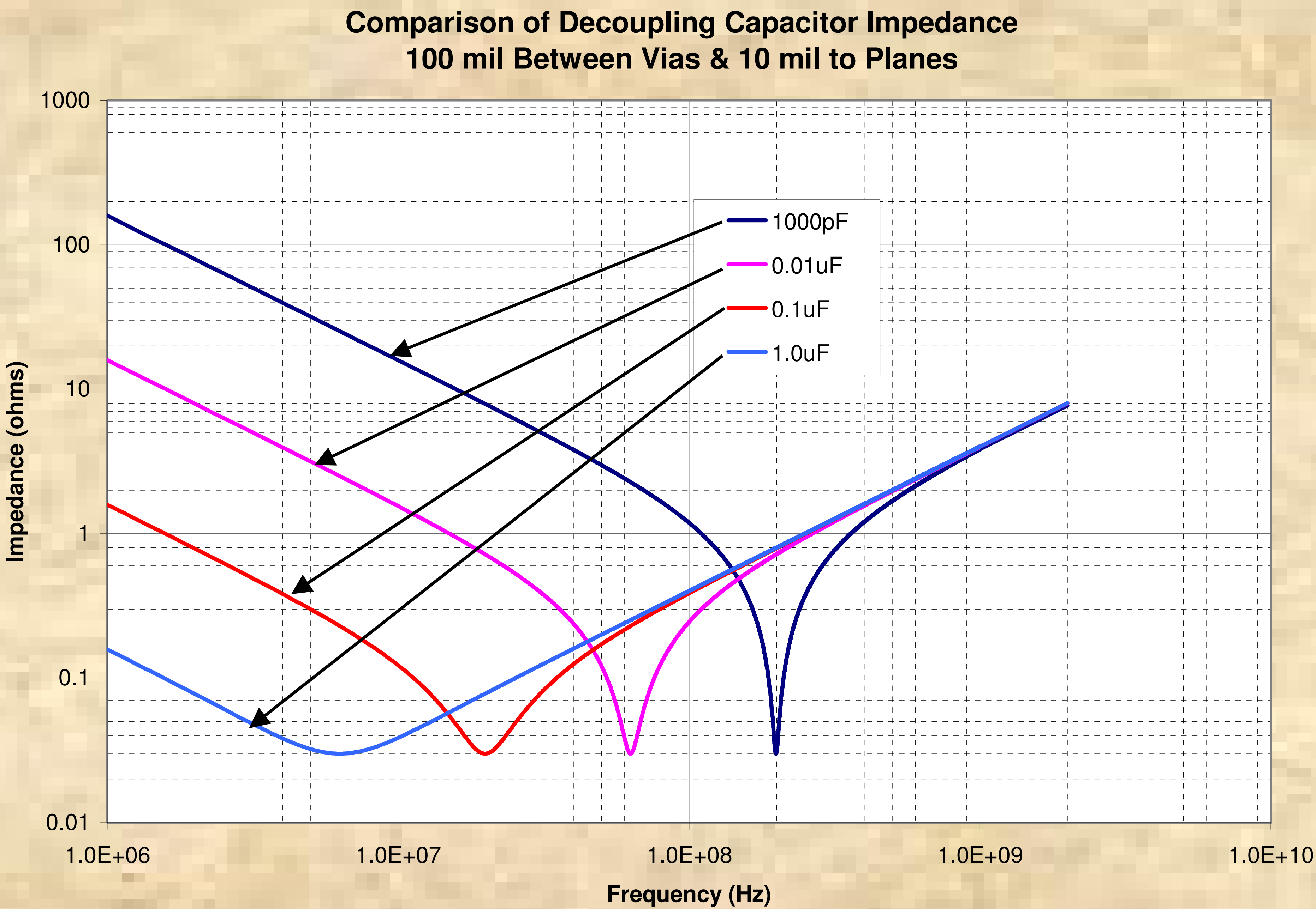
- Capacitance is the ability of a structure to hold charge (electrons) for a given voltage

$$Q = CV$$

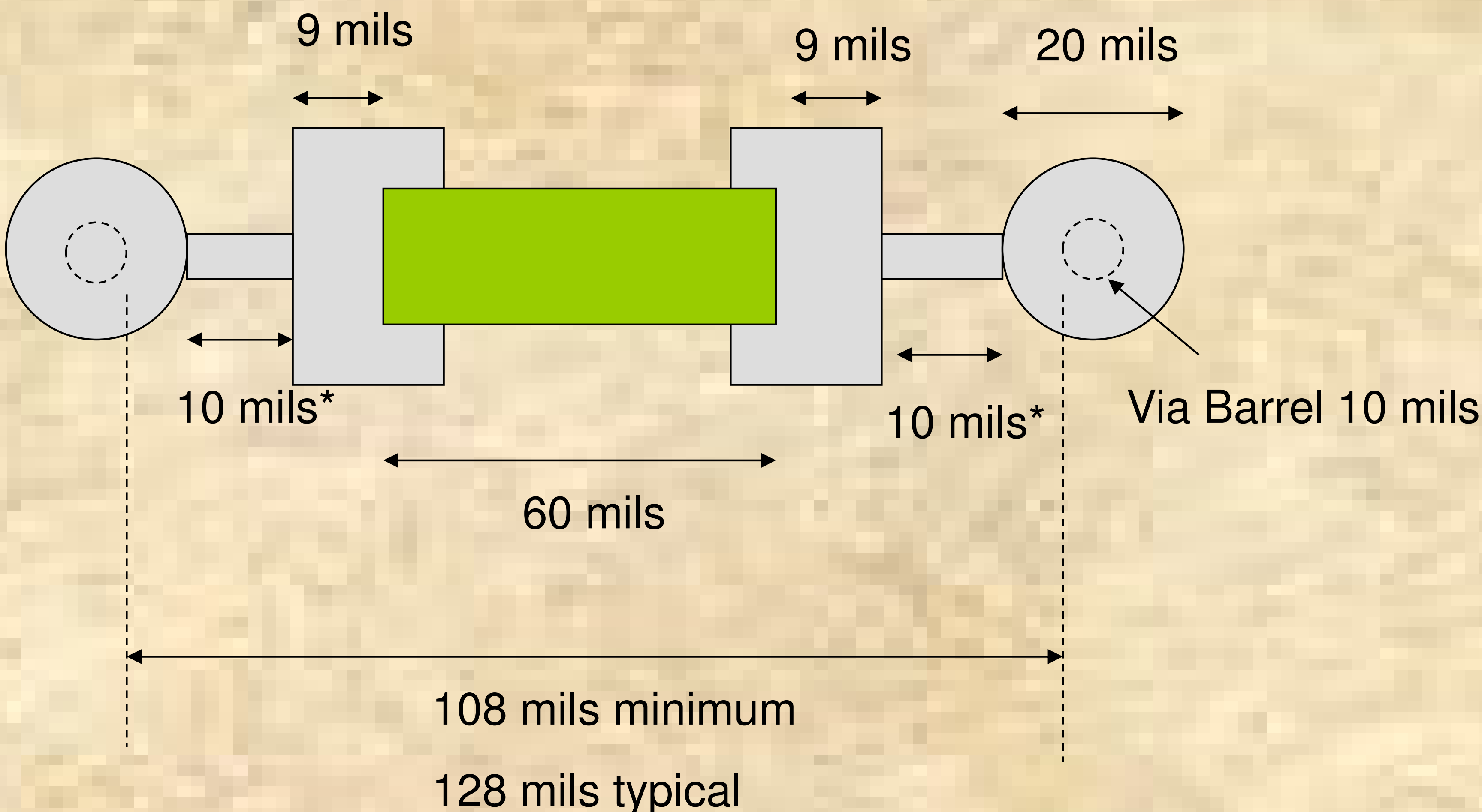
- Amount of charge stored is dependant on the size of the capacitance (and voltage)

# High Frequency Capacitors

- Myth or Fact?

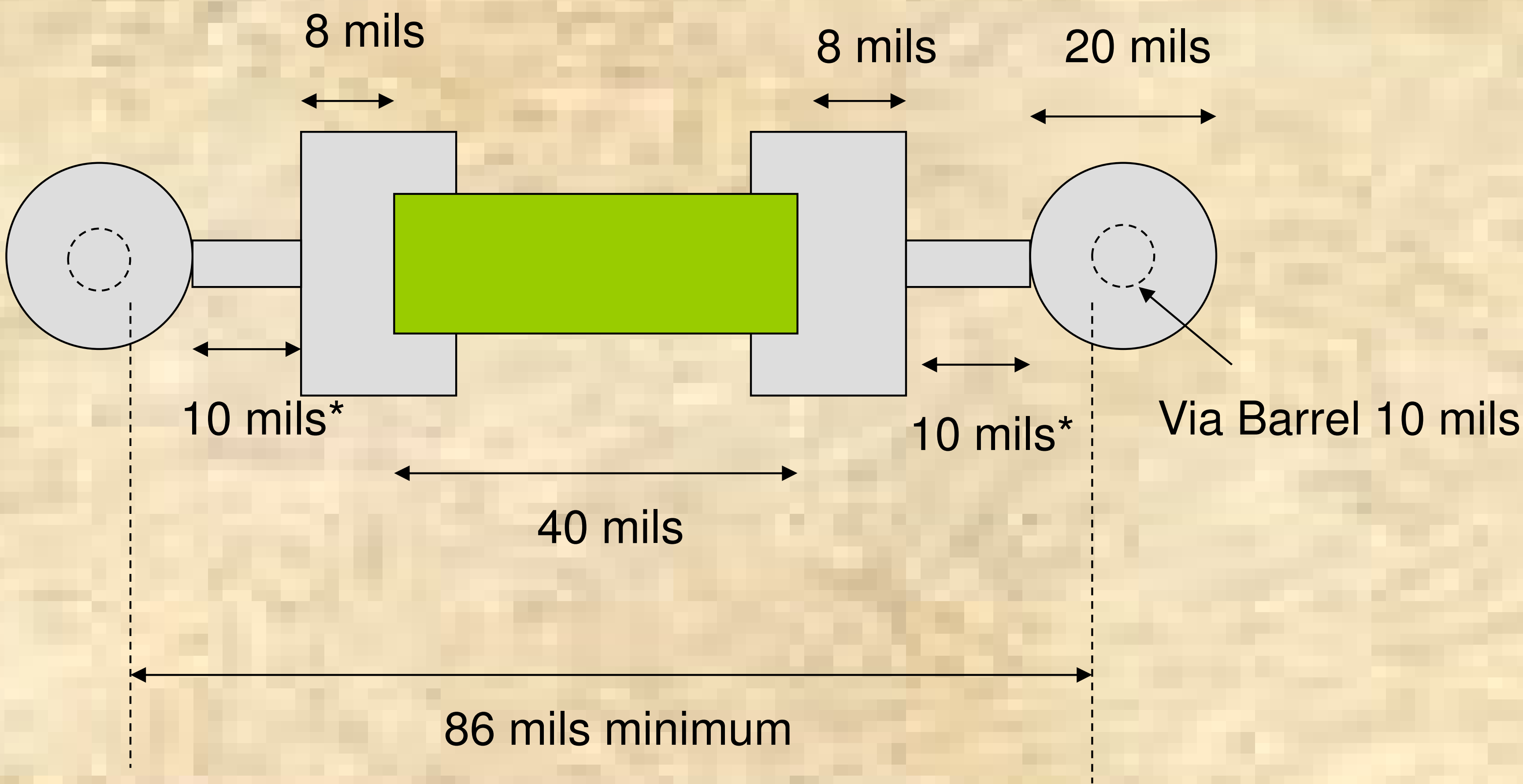


Comparison of Decoupling Capacitor Impedance
100 mil Between Vias & 10 mil to Planes
1000
100
10
1
0.1
0.01
Impedance (ohms)
1000pF
0.01uF
0.1uF
1.0uF
1.0E+06
1.0E+07
1.0E+08
1.0E+09
1.0E+10
Frequency (Hz)

# 0603 Size Cap Typical Mounting




*Note: Minimum distance is 10 mils but more typical distance is 20 mils

# 0402 Size Cap Typical Mounting




*Note: Minimum distance is 10 mils but more typical distance is 20 mils

## Connection Inductance for Typical Capacitor Configurations

| Distance into board to planes (mils) | 0805 typical/minimum (148 mils between via barrels) | 0603 typical/minimum (128 mils between via barrels) | 0402 typical/minimum (106 mils between via barrels) |
|---|---|---|---|
| 10 | 1.2 nH | 1.1 nH | 0.9 nH |
| 20 | 1.8 nH | 1.6 nH | 1.3 nH |
| 30 | 2.2 nH | 1.9 nH | 1.6 nH |
| 40 | 2.5 nH | 2.2 nH | 1.9 nH |
| 50 | 2.8 nH | 2.5 nH | 2.1 nH |
| 60 | 3.1 nH | 2.7 nH | 2.3 nH |
| 70 | 3.4 nH | 3.0 nH | 2.6 nH |
| 80 | 3.6 nH | 3.2 nH | 2.8 nH |
| 90 | 3.9 nH | 3.5 nH | 3.0 nH |
| 100 | 4.2 nH | 3.7 nH | 3.2 nH |

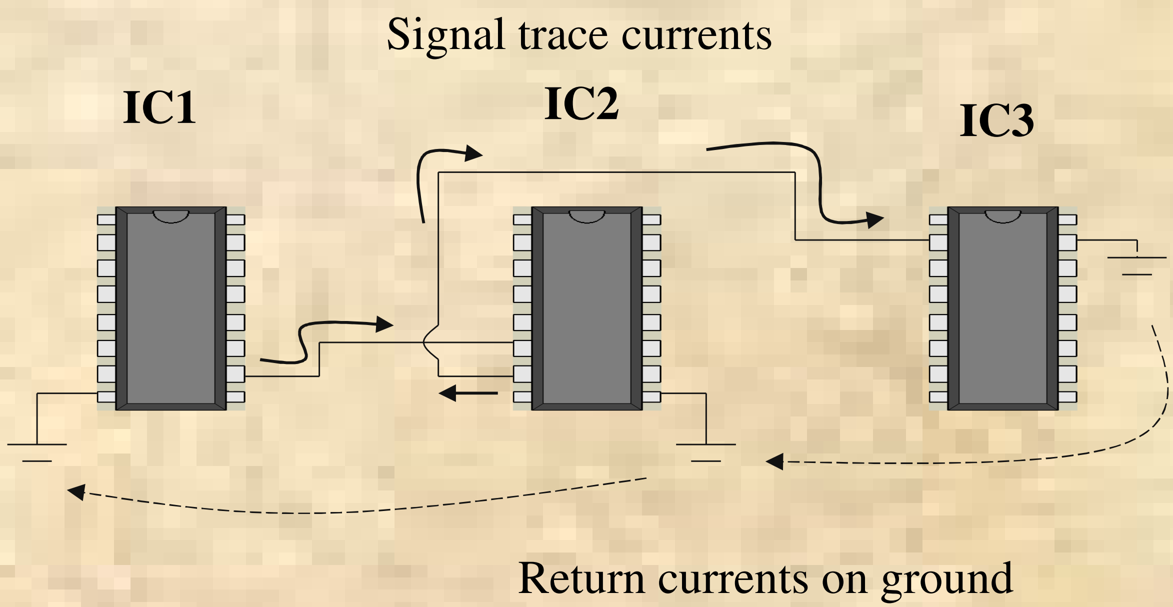
## Connection Inductance for Typical Capacitor Configurations with 50 mils from Capacitor Pad to Via Pad

| **Distance into board to planes (mils)** | **0805 (208 mils between via barrels)** | **0603 (188 mils between via barrels)** | **0402 (166 mils between via barrels)** |
|---|---|---|---|
| 10 | 1.7 nH | 1.6 nH | 1.4 nH |
| 20 | 2.5 nH | 2.3 nH | 2.0 nH |
| 30 | 3.0 nH | 2.8 nH | 2.5 nH |
| 40 | 3.5 nH | 3.2 nH | 2.8 nH |
| 50 | 3.9 nH | 3.5 nH | 3.1 nH |
| 60 | 4.2 nH | 3.9 nH | 3.5 nH |
| 70 | 4.5 nH | 4.2 nH | 3.7 nH |
| 80 | 4.9 nH | 4.5 nH | 4.0 nH |
| 90 | 5.2 nH | 4.7 nH | 4.3 nH |
| 100 | 5.5 nH | 5.0 nH | 4.6 nH |

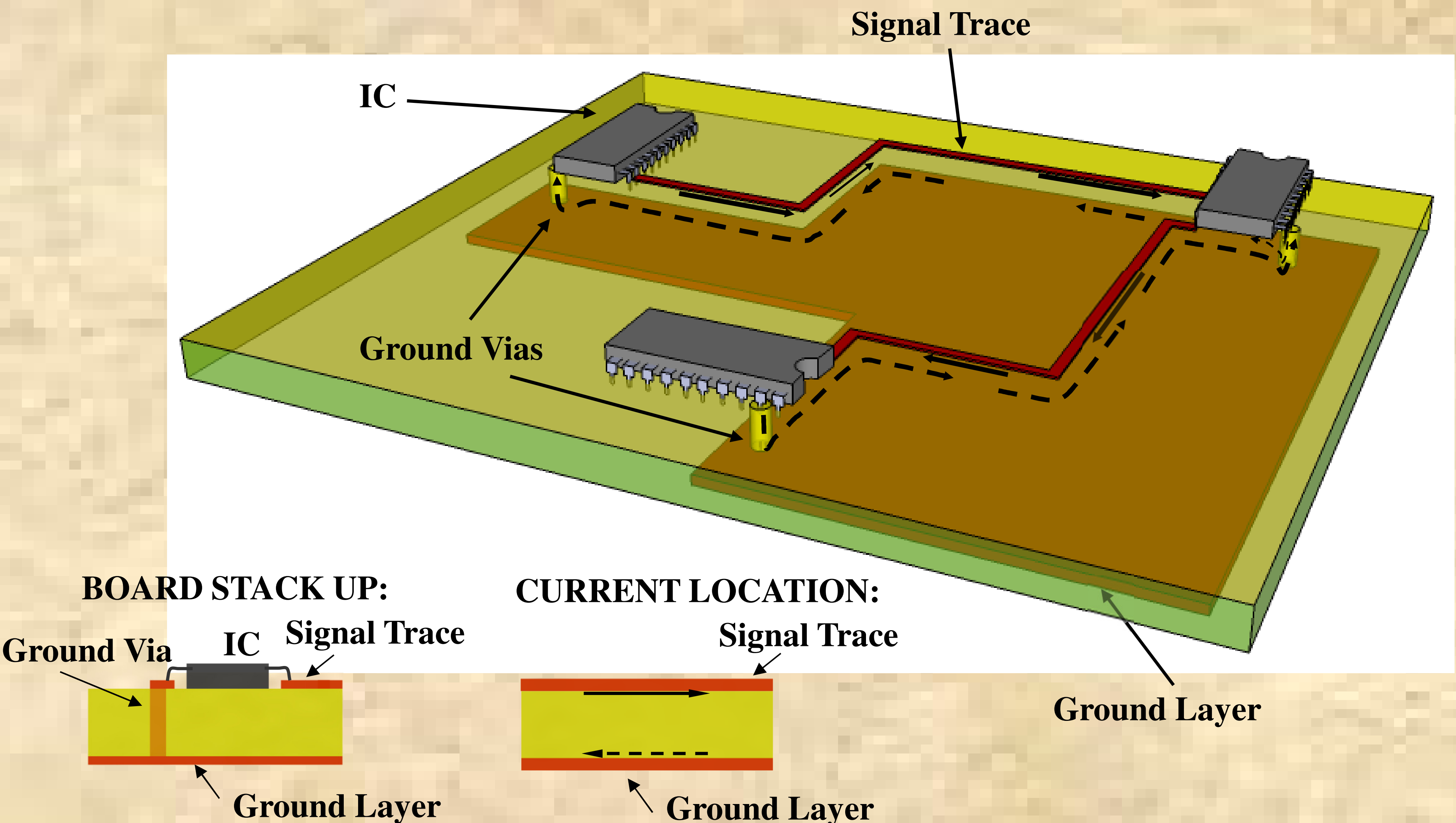
# Current Path

- Current will ALWAYS follow the path of least *impedance*
  - Low frequencies → lowest *resistance*
  - High frequencies → lowest *inductance*
  - Change over ~ 100 KHz

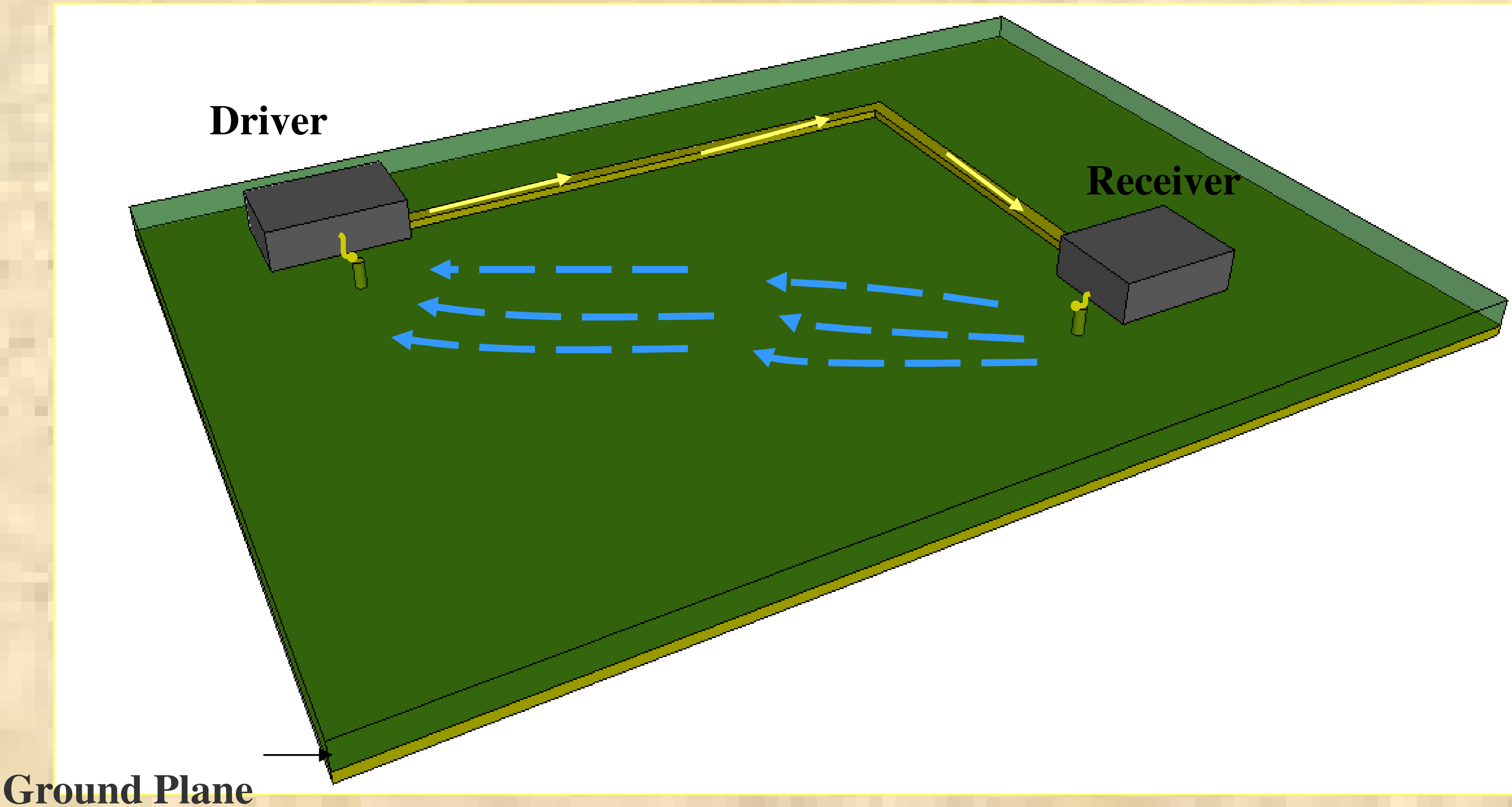
# Schematic with return current shown



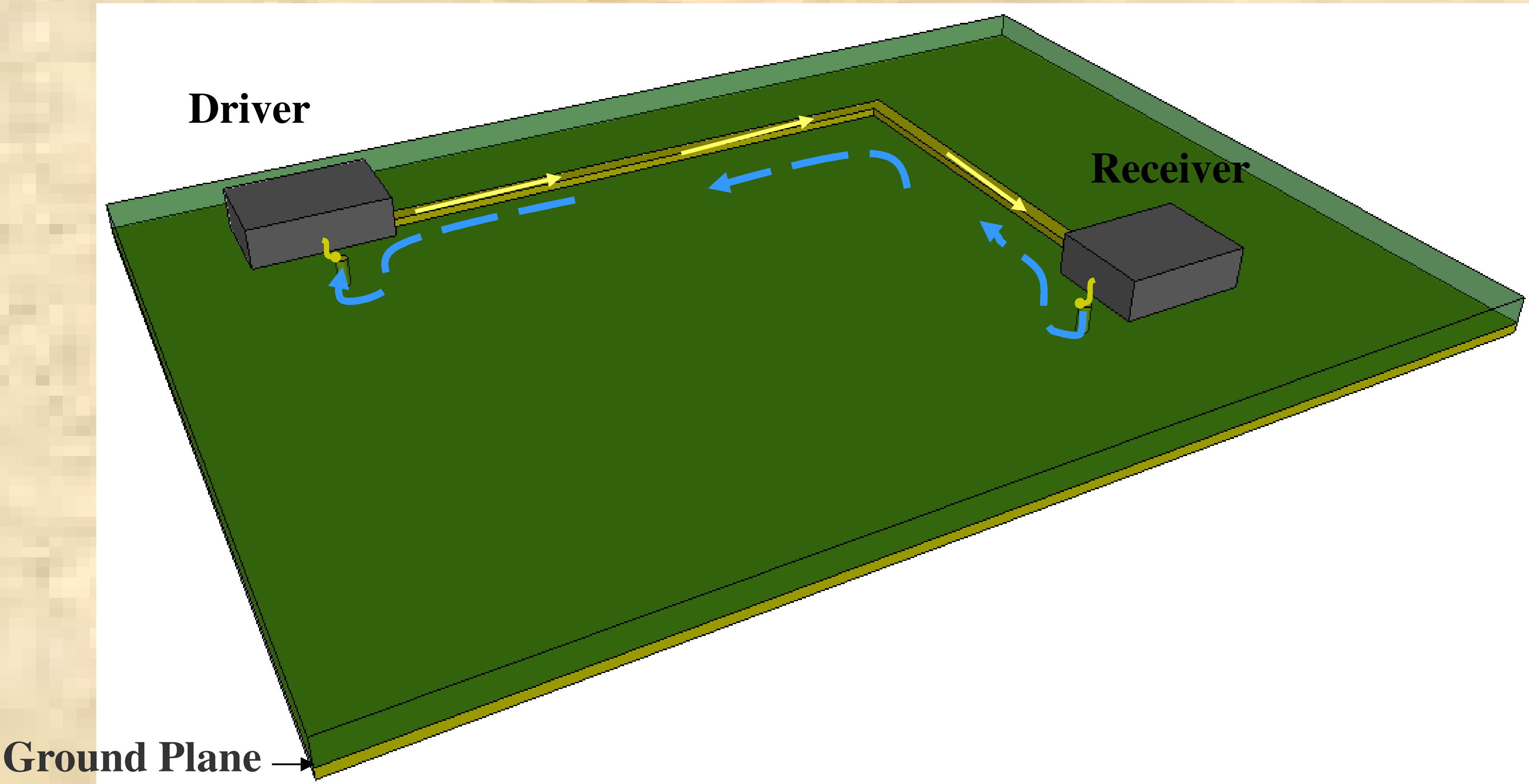
# Actual Current Return is 3-Dimensional



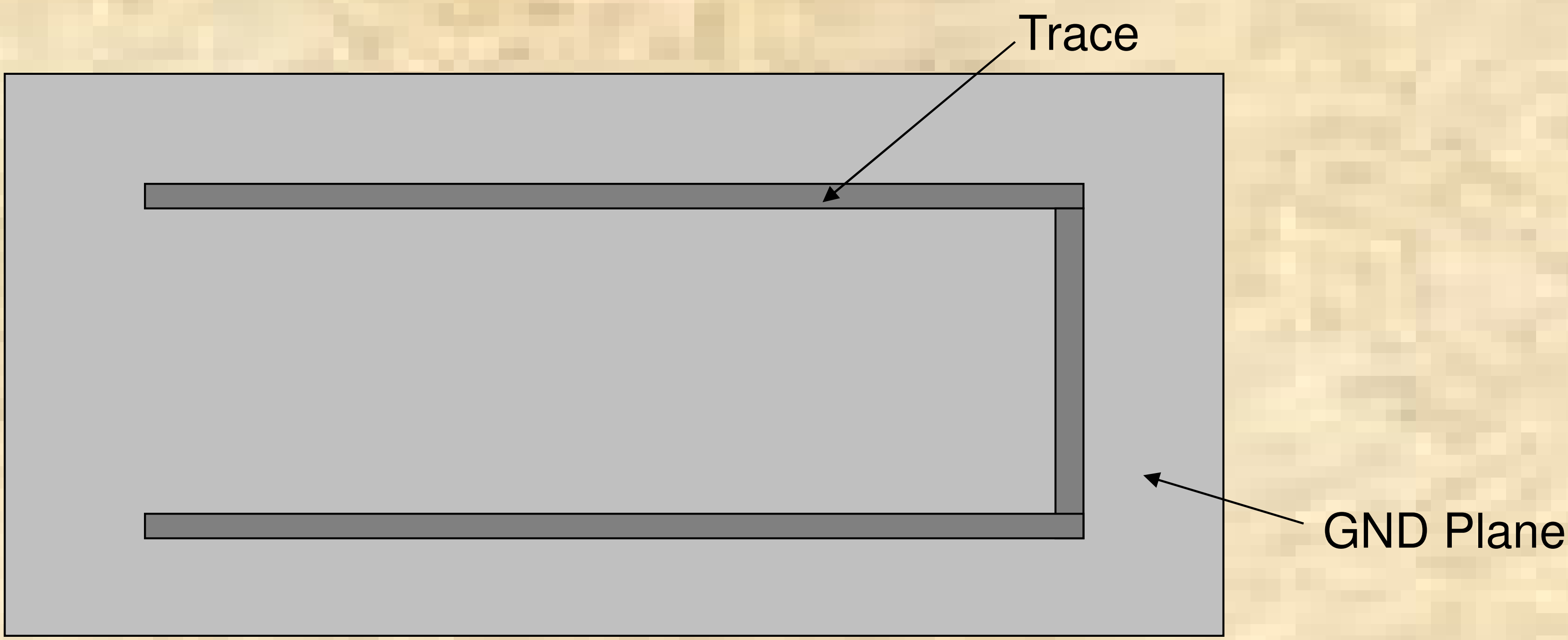
# Low Frequency Return Currents Take Path of Least ***Resistance***



# High Frequency Return Currents Take Path of Least ***Inductance***



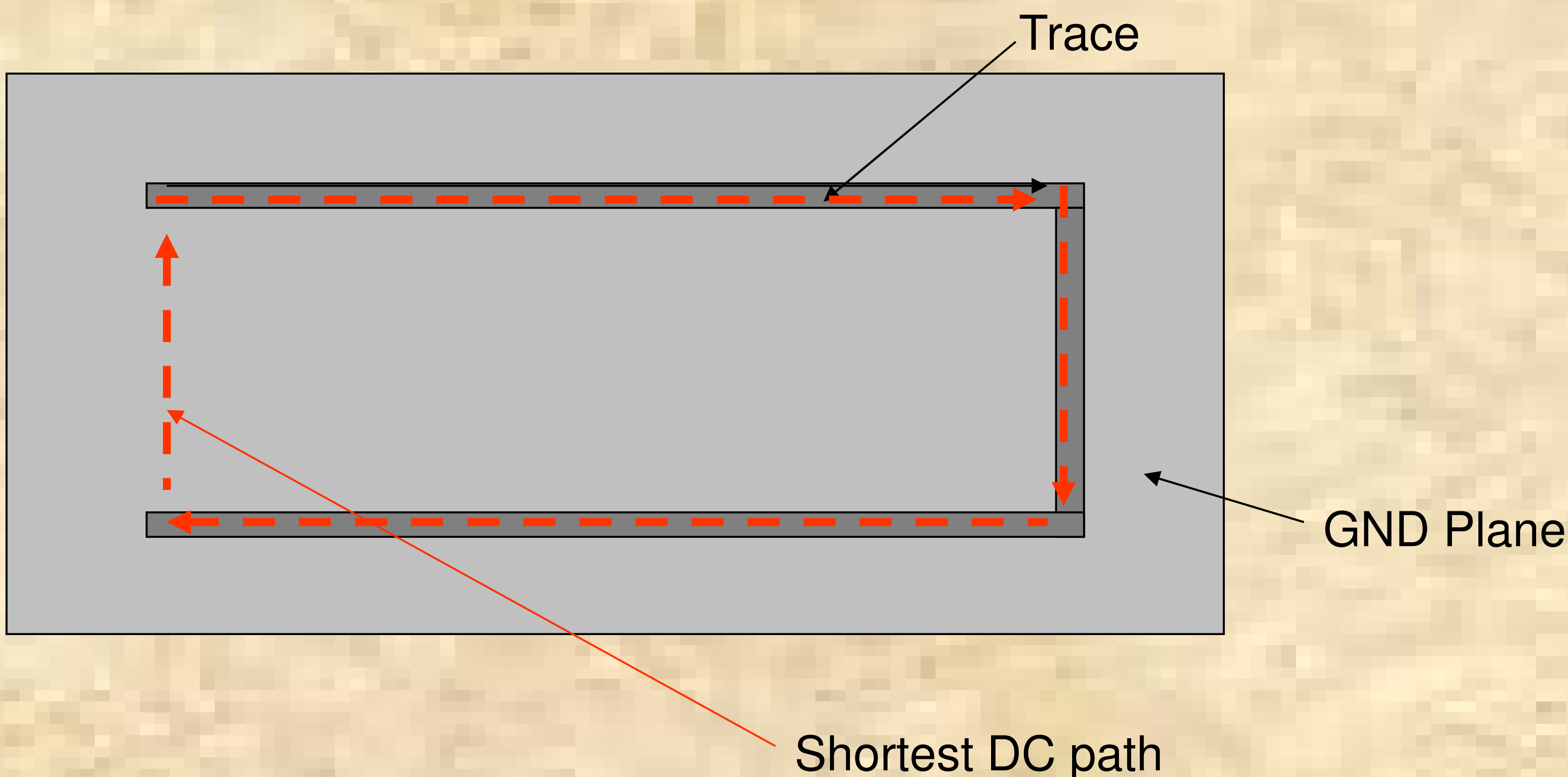
# PCB Example for Return Current Impedance




22" trace

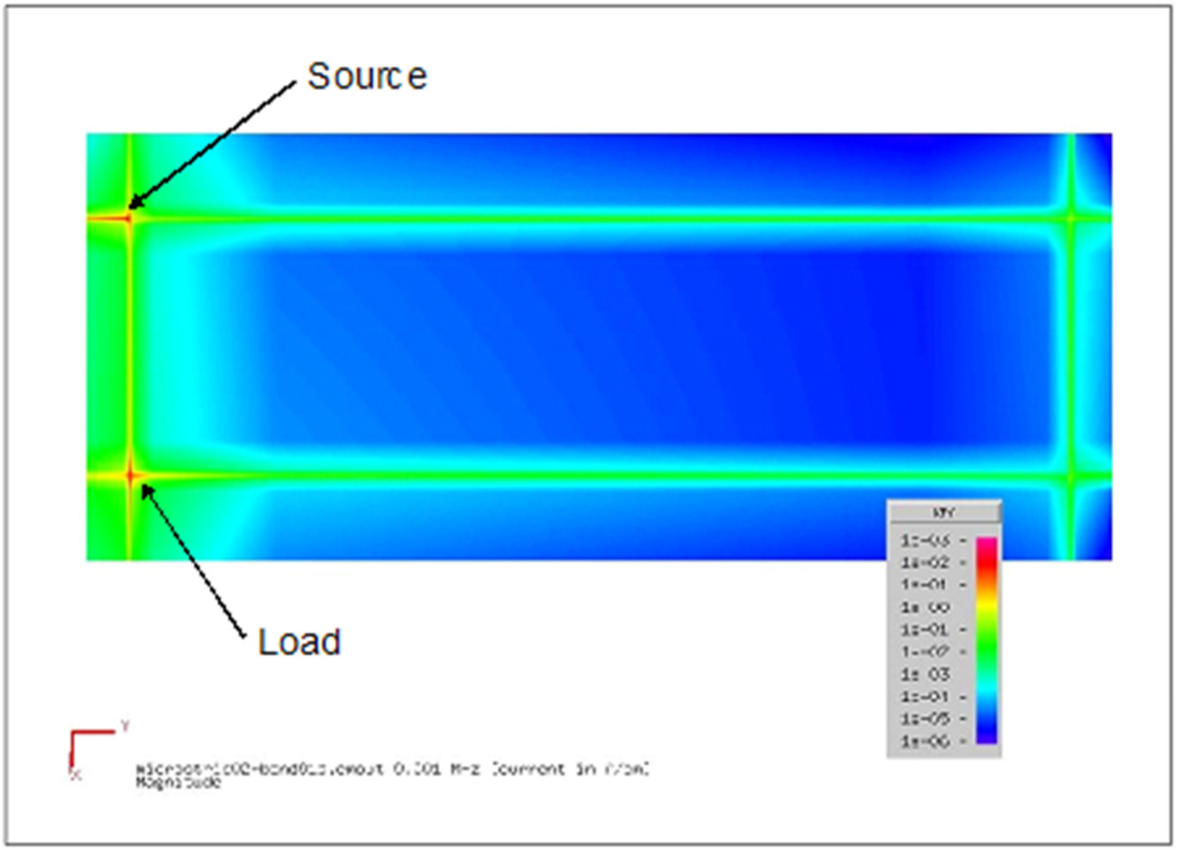
10 mils wide, 1 mil thick, 10 mils above GND plane

# PCB Example for Return Current Impedance

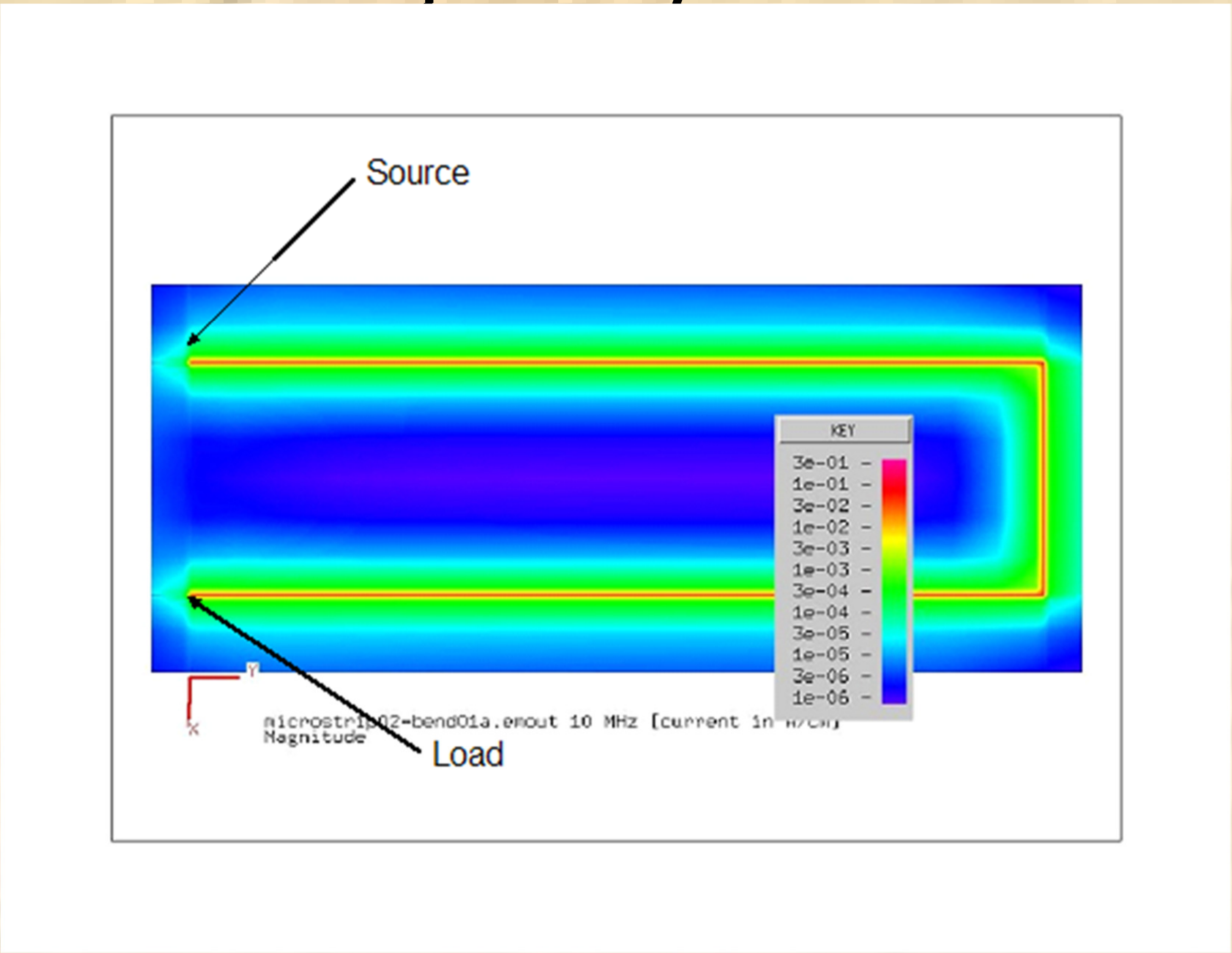


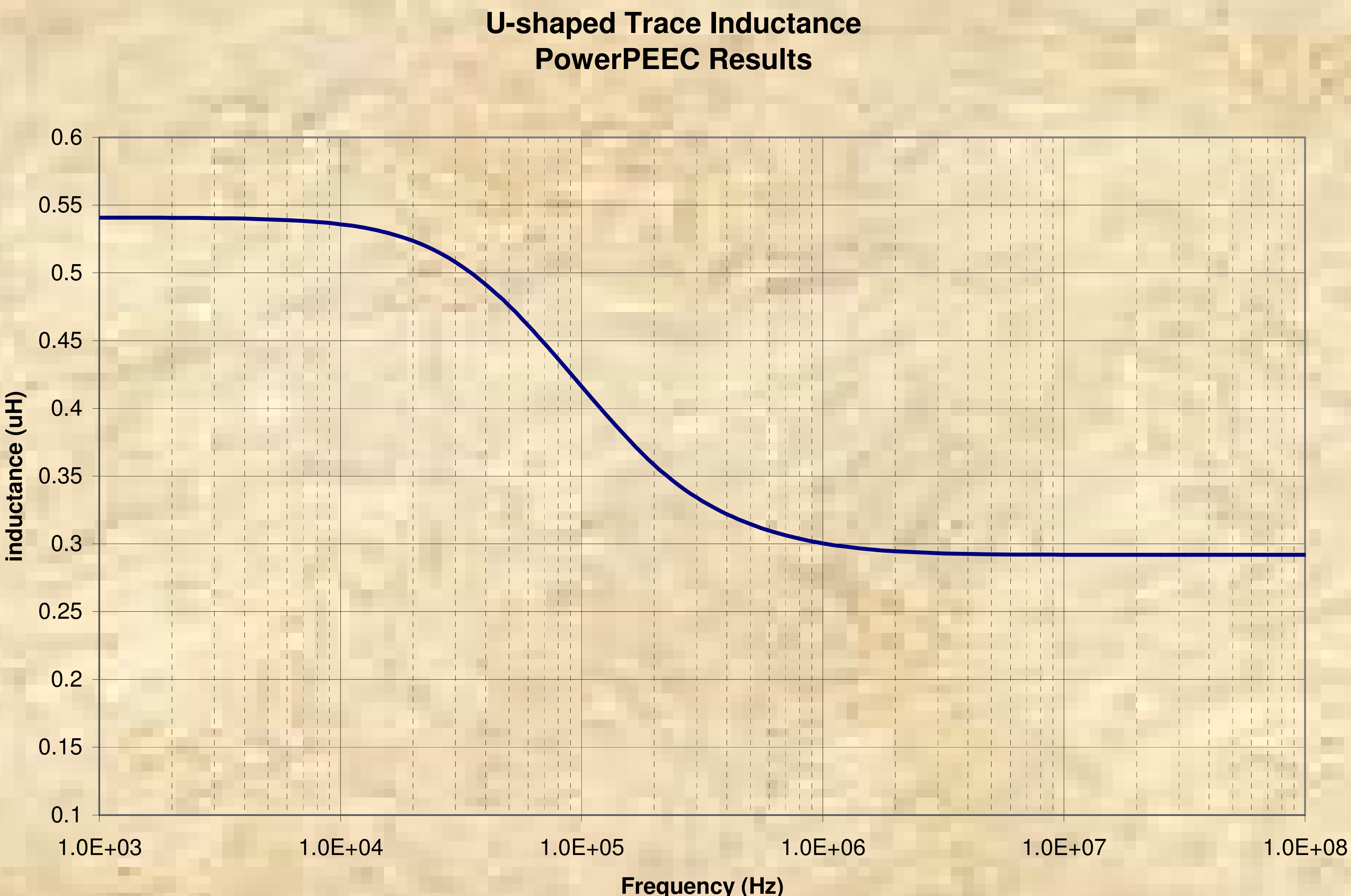

For longest DC path, current returns under trace

# MoM Results for Current Density Frequency = 1 KHz



# MoM Results for Current Density Frequency = 1 MHz




U-shaped Trace Inductance
PowerPEEC Results
inductance (uH)
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
1.0E+03
1.0E+04
1.0E+05
1.0E+06
1.0E+07
1.0E+08
Frequency (Hz)

# EM Summary

- Electromagnetics is not hard
  - Must get past the messy math
- Understanding what the basic equations mean is important
- ***<u>CURRENT</u>*** is important
- “Ground” is a place for potatoes and carrots!
- Where does the return current flow?
  - #1 cause of EMC related problems
  - Use “ground-return” when current flows