Topology

1) Let $(X, \tau_1)$ and $(X, \tau_2)$ be two topology

P: $\tau_1 \subset \tau_2$, $\tau_1$ first countable $\Rightarrow$ $\tau_2$ first countable

Q: $\tau_1 \supset \tau_2$, $\tau_1$ first countable $\Rightarrow$ $\tau_2$ first countable

R: $\tau_1 \subset \tau_2$, $\tau_1$ - second countable $\Rightarrow$ $\tau_2$ second countable

S: $\tau_1 \supset \tau_2$, $\tau_1$ - second countable $\Rightarrow$ $\tau_2$ second countable.

W.O.T.F.A. (True)

(a) P (b) Q (c) R (d) S

---

2) $A = \{(x, \sin(1/x)) / x \in \mathbb{R} \setminus 0\} \subset \mathbb{R}^2$ (Euc-met topo)

$B = A \cup \{(0, y) / -1 \leq y \leq 1\} \subset \mathbb{R}^2$ (Eu-met topo)

P :- A is compact if $|x| < 1$

Q :- A is compact if $|x| \leq 1$

R : B is compact if $|x| \leq 1$

S : B is compact if $|x| < 1$

W.O.T.F.A. (True)

(a) P (b) R and Q (c) R (d) S

---

3) Let $A, B \subset (X, \tau_1)$. Then.

P : $A \cup B$ is compact $\Rightarrow$ A (or) B is compact

Q : $A \cap B \neq \phi$ is compact $\Rightarrow$ A (or) B is compact

R :- A compact $\Rightarrow$ $\overline{A}$ compact.

S : $\overline{A}$ compact $\Rightarrow$ A compact.

W.O.T.A. (False)

(a) P (b) Q (c) R (d) S

4) Let $\mathbb{R}(x) \subset (C[0,1],$ with supnorm metric topo)

$A = \{ p(x) \in \mathbb{R}(x) \,/\, |p(x)| \leq k$, where $k>0$ and $k$ is fixed$\}$

$B = \{ p(x) \in \mathbb{R}(x) \,/\, |p(x)| < k$, where $k>0$ and $k$ is fixed$\}$.

P: A is compact.

Q :- B is compact

W.O.T.F.A (True)

(a) P only (b) Q only (c) P and Q (d) Neither.

----

5) Let X be a topo space E be a subset of topological space.

(P) $\overline{E}$ is path connected $\Rightarrow$ E is path connected

(Q) E path connected $\Rightarrow$ $\overline{E}$ is path connected

(R) E path connected $\Rightarrow$ $\partial E$ is path connected

(S) E connected $\Rightarrow$ $\overline{E}$ is path connected.

W.O.T.F.A (False).

(a) P (b) Q (c) R (d) S

-----

6) Let $X = M_n(\mathbb{R})$, $A = \left\{ \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \,/\, \theta \in \mathbb{R} \right\}$. W.O.F.A (True).

(a) A is compact

(b) A need not be bounded

(c) A is not compact

(d) $A = \phi$

-----

7) Let $X$ be an ordered set and $A$ be a subset of $A$. Let $(X, \tau)$ be an ordered topological space and $(A, \tau_1)$ be the topology induced by the subspace topology and $(A, \tau_2)$ be the topology induced by the order in $A$.

$P: (A, \tau_1) = (A, \tau_2)$, $\forall A \subset X$.

$Q: (A, \tau_1) \supset (A, \tau_2)$, for some $A \subset X$.

(a) P only (b) Q only (c) P and Q (d) neither.

- - - -

8) Consider a ~~seq~~ $\{a_n\}$ in $\mathbb{R}$ with usual topology. where $\{a_n\}$ is fixed and $a_n \to a$.

$A = \{a_1, a_2 \cdots a_n, \cdots\}$

$B = A \cup \{a\}$. W.O.F.A. (True)

(a) $A$ is bounded and has both lub and glb inside a

(b) $B$ is not compact

(c) $B$ is closed

(d) There exist a homeomorphism from $B$ to $\mathbb{Z}$.

- - - -

9) Let $(X, \tau)$ be a topology

P: $A$ is connected iff $\overline{A}$ is connected

Q: $A$ and $B$ is not compact then $A \cap B \neq \phi$ is not compact

R: $A_n$ is compact then $\bigcup_{n=1}^{\infty} A_n$ is compact., $\text{car}(A_n) = \infty$ $\forall n$

S: $A_n$ is compat then $\bigcap_{n=1}^{\infty} A_n \neq \phi$ is compact. WOFT (True)

(a) P (b) Q (c) R (d) S

7) Let X be an ordered set and A be a subset of A. Let $(X, \tau)$ be an ordered topological space and $(A, \tau_1)$ be the topology induced by the subspace topology and $(A, \tau_2)$ be the topology induced by the order in A.

$P : (A, \tau_1) = (A, \tau_2)$ , $\forall A \subset X$.

$Q : (A, \tau_1) \supset (A, \tau_2)$ , for some $A \subset X$.

(a) P only (b) Q only (c) P and Q (d) neither.

- - - -

8) Consider a seq $\{a_n\}$ in $\mathbb{R}$ with usual topology. where $\{a_n\}$ is fixed and $a_n \to a$.

$A = \{a_1, a_2 \cdots a_n, \cdots\}$

$B = A \cup \{a\}$. W.O.F.A. (True)

(a) A is bounded and has both lub and glb inside a

(b) B is not compact

(c) B is closed

(d) There exist a homeomorphism from B to $\mathbb{Z}$.

- - - -

9) Let $(X, \tau)$ be a topology

P: A is connected iff $\bar{A}$ is connected

Q: A and B is not compact then $A \cap B \neq \phi$ is not compact

R: $A_n$ is compact then $\bigcup_{n=1}^{\infty} A_n$ is compact., $\text{Car}(A_n) = \infty \ \forall n$

S: $A_n$ is compat then $\bigcap_{n=1}^{\infty} A_n \neq \phi$ is compact. WOFT (True)

(a) P (b) Q (c) R (d) S

10) P: union of connected sets is connected.

Q:- Intersection of connected set is connected set

(a) P (b) Q (c) P and Q (d) Neither.

- - - -

11) P: $f : (X, \tau_1) \rightarrow (X, \tau_2)$ is cts bijective then $f^{-1}$ is cts.

Q :- Are there any non constant cts fn from $f : \mathbb{R}_u \rightarrow (\mathbb{Q}^c, \tau)$, ($\tau$ be some topology)

(a) P (b) Q (c) P and Q (d) Neither.

- - -

Let A

12) P :- $(X, \tau)$ be a topology, Every infinite subset A is open in $(X, \tau)$ the $\tau$ is discrete topology.

Q: similarly if A is closed in $(X, \tau)$ then $\tau$ is discrete topology.

(a) P (b) Q (c) P and Q (d) Neither.

- - -

(13) Let $(X, \tau)$ be topological space, where $A$ is open and $B$ is closed in $(X, \tau)$

P: $A \setminus B$ is open.

Q:- $B \setminus A$ is closed.

(a) P only (b) Q only (c) P and Q (d) neither.

----

14) $U \subseteq \mathbb{R}^n$ be open in Euclidean topology.

P: There exist a function $f: \mathbb{R}^n \to \mathbb{R}$ such that $f$ is cts outside $U$ and dis cts in $U$.

Q:- $U$ can be written as countable union of closed sets.

(a) P only (b) Q only (c) P and Q (d) Neither

----

15) Let $A = \left\{ \begin{bmatrix} P_1(x) & P_2(x) \\ P_3(x) & P_4(x) \end{bmatrix} \Big/ x \in [0,1] \right\}$

$A \subseteq M_n(\mathbb{R})$

P:- A is compact

Q:- A is open.

(a) P only (b) Q only (c) P and Q (d) Neither

---

16) Let $E \subset \mathbb{R}^n$ be compact.

$\{A_n\}$ be the sequence of closed sets contained in $E$

Then P:- $\bigcup_{n=1}^{\infty} A_n$ is compact.

Q:- $\bigcap_{n=1}^{\infty} A_n \neq \phi$ is compact.

(a) P only (b) Q only (c) P and Q (d) Neither.

17) $f :- (X, \tau_1) \to (X, \tau_2)$ be a cts function

$P :-$ f is bijective $\Rightarrow f^{-1}$ is cts

$Q :- \tau_1 = \tau_2 \Rightarrow f^{-1}$ is cts.

(a) P only (b) Q only (c) P and Q (d) Neither.

---

18) $\mathbb{R}_L$ be lower limit topology.

$P :- [a, b]$ is compact

$Q :- [a, b)$ is closed.

(a) P (b) Q (c) P and Q (d) Neither.

---

19) $P : (X, \tau)$ is compact $\Rightarrow (X, \tau)$ is limit pt compact.

$Q :- (X, \tau)$ is limit pt compact $\Rightarrow$ $(X, \tau)$ is compact.

(a) P only (b) Q only (c) P and Q (d) Neither

---

20) $(X, \tau_{d_1})$ be metric space topo with metric $d_1$
$(X, \tau_{d_2})$ be metric space topo with metric $d_2$

P: There exist a bijection between
$f : (\mathcal{B}_{d_1}) \to (\mathcal{B}_{d_2})$, where $\mathcal{B}_{d_1}, \mathcal{B}_{d_2}$ is a fixed Basis for $(X, \tau_{d_1})$ and $(X, \tau_{d_2})$

Q: $(X, d_1)$ and $(X, d_2)$ is first countable

(a) P only (b) Q only (c) P and Q (d) None

---

1) which of the flg are examples of arbitrary union of closed set is closed.

(a) $\left(\bigcup_{n=1}^{\infty}\left[5-\frac{1}{n}, 7+\frac{1}{n}\right]\right) \cup \left(\bigcup_{n=1}^{\infty}\left[5+\frac{1}{n}, 7-\frac{1}{n}\right]\right)$

in $(\mathbb{R}, \mathcal{T}_u)$. $\mathcal{T}_u$ - usual topology.

(b) $\left(\bigcup_{n=1}^{\infty}\left[3+\frac{1}{n}, 7-\frac{1}{n}\right]\right) \cup \left(\bigcup_{n=1}^{\infty}\left[5-\frac{1}{n}, 5+\frac{1}{n}\right]\right)$

in $(\mathbb{R}, \mathcal{T}_u)$.

(c) $\left(\bigcup_{n=1}^{\infty}\left[5-\frac{1}{n}, 7+\frac{1}{n}\right]\right) \cup \left(\bigcup_{n=1}^{\infty}\left[5+\frac{1}{n}, 7-\frac{1}{n}\right]\right)$

in $(\mathbb{R}, \mathcal{T}_d)$ $\mathcal{T}_d$ - discrete topology.

(d) $\left(\bigcup_{n=1}^{\infty}\left[3+\frac{1}{n}, 7-\frac{1}{n}\right]\right) \cup \left(\bigcup_{n=1}^{\infty}\left[5-\frac{1}{n}, 5+\frac{1}{n}\right]\right)$

in $(\mathbb{R}, \mathcal{T}_{Id})$ $\mathcal{T}_{Id}$ - Indiscrete topology.

2) Consider The following Statements :-

P - $X$ and $Y$ be a topological Space induced by Same topology. let $f : X \to Y$ is a continuous map then Inverse image of compact set is compact.

Q - There is a continuous function $f : (\mathbb{R}, T_d) \longrightarrow (\mathbb{R}, T_u)$ which has the property that is inverse image of compact set is compact.

which of the Statements are true.

(a) P only (b) Q only (c) both P and Q.
(d) Both are false.

3) let $\mathbb{R}^2$ be a topological space induced by product topology of $\mathbb{R}$ with usual topology.

$A = \{(x, \sin(1/x)) \in \mathbb{R}^2 / x \in (0,1]\}$

$\overline{A} = A \cup \{(0, y) \in \mathbb{R}^2 / y \in [-1, 1]\}$

let $f : (\overline{A}, \tau) \longrightarrow (\mathbb{R}^2, \tau_d)$ defined by $f(x) = x$.

then: (a) $f$ is continuous. (b) $f(\overline{A})$ is connected but not continuous

(c) $f(\overline{A})$ is not compact (d) NOTA.

Date :

4) Let $(X=\mathbb{R}, \tau_f)$ be a topological space.

Consider, $A=\{1-\frac{1}{n} / n\in\mathbb{N}\}$, $B = A\cup\{1\}$.

Then

(a) $B$ is compact in $\mathbb{R}_L$.

(b) $A\cup\{0\}$ is compact on $(A\cup\{0\}, \tau)$
where $\tau=\{U\cup\{0\} / U\in\tau_f\}\cup\{\phi\}$

(c) $A\cup\{x\}$ is compact on $(A\cup\{x\}, \tau)$
where $\tau=\{U\cup\{x\} / U\in\tau_d\}\cup\{\phi\}$.

(d) NOTA

5) $X = \mathbb{R}^3$, $A = \{(x, 1/x, 0) / x \in \mathbb{R}^+\}$ on product topology induced by usual topology. then.

(a) $A \cup \{X\text{-axis}\}$ is connected.

(b) $A \cup \{Y\text{-axis}\}$ is connected.

(c) $A \cup \{X \text{ axis}\} \cup \{Y \text{ axis}\} \cup \{Z \text{ axis}\}$ is connected

(d) $A \cup \{Z \text{ axis}\} \cup \{(x, 1/x, 0) / x \in \mathbb{R}^-\}$ is connected

6) $X = \mathbb{R}^3$, $A = \{(x, 1/x, 0) / x \in \mathbb{R}^+\}$ on product topology induced by usual topology then.

(a) $A \cup \{X\text{-axis}\}$ path connected.

(b) $A \cup \{(x, x, 0) / x \in \mathbb{R}\}$ is path connected.

(c) $\{(x, 1/x, 0) / x \in \mathbb{R}^-\} \cup \{(x, -x, 0) / x \in \mathbb{R}\}$ is path connectd.

(d) $A \cup \{X\text{-axis}\} \cup \{Y\text{-axis}\}$ has 2 path component and connected.

7) let $f : \mathbb{R} \cup \{\infty\} \longrightarrow \{(x,y) \in \mathbb{R}^2 / x^2+y^2=1\}$ is a onto continuous function defined by,

(a) $f(x) = (e^x \cos x, e^x \sin x)$.

(b) $f(x) = (e^x \cos x, i \sin x)$

(c) $f(x) = (\cos x, \sin x)$.

(d) NOTA.

Date :

8) Which of the following ~~for~~ are basis for usual topology on $\mathbb{R}$ ?

(a) $\mathcal{B} = \{(a,b) \in \mathbb{R} \,/\, a<b \,,\, a,b \in \mathbb{Z}\}$.

(b) $\mathcal{B} = \{(a,b) \in \mathbb{R} \,/\, a<b \,,\, a,b \in \mathbb{R}\}$.

(c) $\mathcal{B} = \{(a,b) \in \mathbb{R} \,/\, a<b \,,\, a,b \in \mathbb{Q}\}$

(d) $\mathcal{B} = \{(a,b) \in \mathbb{R} \,/\, a<b \,,\, a,b \in \mathbb{R}\} \cup \{\mathbb{R}, \phi\}$.

9) Which of the following are basis for a lower limit Topology $\mathbb{R}_l$.

(a) $\mathcal{B} = \{ [a,b) \in \mathbb{R} \,/\, a<b \,,\, a,b \in \mathbb{Q} \}$.

(b) $\mathcal{B} = \{ [a,b) \in \mathbb{R} \,/\, a<b \,,\, a \in \mathbb{R} \,,\, b \in \mathbb{Q} \}$.

(c) $\mathcal{B} = \{ [a,b) \in \mathbb{R} \,/\, a<b \,,\, a \in \mathbb{Q} \,,\, b \in \mathbb{R} \}$.

(d) $\mathcal{B} = \{ [a,b) \in \mathbb{R} \,/\, a<b \,,\, a \in \mathbb{Q} \,,\, b \in \mathbb{Q}^c \}$.

16) Which of the following are connected? on Standard topology.

(a) $\{(x,y)\in\mathbb{R}^2 / (x-a)^2+(x-b)^2=r^2\} \cup$
$\{(x,y)\in\mathbb{R}^2 / (x-a-2r)^2+(x-b)^2=r^2\}$.

(b) $(a,b) \cup \left(\frac{b-a}{2}, \frac{b-a}{2}+1\right)$.

(c) $\{(x,y)\in\mathbb{R}^2 / x\in\mathbb{R}, y>0\} \cup$
$\{(x,y)\in\mathbb{R}^2 / x\in\mathbb{R}, y\leq 0\}$.

(d) $\{(x,y)\in\mathbb{R}^2 / x<0, y<0\} \cup$
$\{(x,y)\in\mathbb{R}^2 / x\geq 0, y\geq 0\}$.

11) Which of the following are path connected?

(a) $A = \{ (x, x\sin(\frac{1}{x})) / x \in (0, \infty) \}$

(b) $A \cup \{ (0, y) / y \in [-1, 1] \}$.

(c) $A \cup \{ (0, 0) \}$.

(d) NOTA.

(2) Which of the following open covers of the open interval $(0,1)$ admit a finite subcover?

(a) $\{ (0, \frac{1}{2} - \frac{1}{n+1}) \cup (\frac{1}{n}, 1) / n \in \mathbb{N} \}$.

(b) $\{ (\frac{1}{n}, 1 - \frac{1}{n+1}) / n \in \mathbb{N} \}$

(c) $\{ (\sin^2(\frac{n\pi}{100}), \cos^2(\frac{n\pi}{100})) / n \in \mathbb{N} \}$

(d) $\{ (\frac{1}{2} e^{-n}, 1 - \frac{1}{(n+1)^2}) / n \in \mathbb{N} \}$.

13) Let $X$ be a connected subset of $\mathbb{R}$. If every elmnt of $X$ is rational, then $|X|$ is

(a) Infinite (b) Uncountable (c) $\phi$ (d) NOTA.

14) Let A be a closed subset of $\mathbb{R}$, $A \neq \phi$, $A \neq \mathbb{R}$
Then A is

(a) Not open in $(\mathbb{R}, T_u)$.

(b) open in $(\mathbb{R}, T_d)$ (c) compact on $(\mathbb{R}, T_u)$

(d) Uncountable in $(\mathbb{R}, T_d)$.

15) $A = \{(x, y) \in \mathbb{R}^2 / (x-1)(x-2)(y-3)(y+4) = 0\}$.

WOFS are True?

(a) A is compact (b) A is dense

(c) A is closed. (d) A is connected.