Part B

1. Which of the followings are true?

a) The number of homomorphism from A_5 to S_4 is 2

b) The number of homo from A_3 to S_4 is 2

c) The number of homo from A_3 to A_5 is 0

d) The number of homo from S_3 to $\mathbb{Z}_6$ is 3

2. let $G = S_{10}$ be gp. Then which of the followings are false.

a) There is an element $\sigma \in S_{10} \ni o(\sigma) = 10$

b) There is an element $\sigma \in S_{10} \ni o(\sigma) = 7$

c) There is an element $\sigma, \tau \in S_{10}$, $\sigma \neq \tau$ such that $o(\sigma) = o(\tau)$

d) There is an element $\sigma' \in S_{10} \ni o(\sigma') = 20$ and $(\sigma')^4 = e$.

3. let $f(x) \in \mathbb{Z}[x]$ be a monic poly. Then the roots of f

a) can belong to $\mathbb{Z}$.

b) always belong to $\mathbb{R} \setminus \{\mathbb{Q}\}$.

c) always belong to $\mathbb{C} \setminus \{\mathbb{R}\}$

d) can belongs to $\mathbb{Q} \setminus \{\mathbb{Z}\}$

④ Let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \frac{\sin(x)}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Then

a) f is not conts at 0

b) f is conts at 0 but not diffble at 0

c) f is not conts at 0 but not bdd

d) f is R.I on $[0, 1]$. but f is conts at '0'

5. Consider the seq $a_n = \left(k + (-1)^n \frac{1}{n}\right)^n$, $\cancel{k \in N}$

Then

a) $\lim_{n\to\infty} \sup\{a_n\} \neq \lim_{n\to\infty} \inf\{a_n\}$, $\forall k \in N$

b) $\lim_{n\to\infty} \sup\{a_n\} = \lim_{n\to\infty} \inf\{a_n\}$, for some $m \in N$

c. $\lim_{n\to\infty} a_n$ exist for some $k \in N$.

d. $\lim_{n\to\infty} a_n = \frac{k}{e}$, if k is even.

6. For $n \geq 1$, let $f_n(x) = x e^{-nx^2}$, $\forall x \in \mathbb{R}$. Then the seq $\{f_n\}$ is

a) Uniformly convergent on $\mathbb{R}$

b) uniformly convergent only on compact subsets of $\mathbb{R}$.

c) bdd and not uniformly convergent on $\mathbb{R}$

d) A seq of ~~b~~ unbdd fns.

7. The rowspace of a 20×50 matrix A has dimension 20 then

a) $\dim(\mathrm{Ker}(A)) = 20$

b) $\dim(\mathrm{Im}(A)) = 30$

c) $\dim(\mathrm{Ker}(A)) = \dim(\mathrm{Im}(A))$

d) $\dim(\mathrm{Im}(A)) \leq \dim(\mathrm{Ker}(A))$.

8. The sum of the series $\frac{1}{1!} + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \cdots$ equals

a) $\frac{e}{4}$ b) $\frac{e}{2}$ c) $\frac{3}{2}e^{-1}$ d) $\frac{3}{2}e$

9. Let $y(x)$ be a conts solution of the initial value problem $y' + 2y = f(x)$, $y(0) = 0$

Where $f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & x > 1 \end{cases}$

Then $y(\frac{3}{2})$ is equal to

a) $\frac{\sinh(1)}{e^3}$ ~~b) cosh(1)~~ b) $\frac{\cosh(1)}{e^3}$ c) $\frac{\sinh(1)}{e^2}$

d) $\frac{\cosh(1)}{e^2}$

10) Let R be the ring of all 2x2 matrices with integer entries. Which of the followings subsets of R is an integral domain?

a) $\left\{\begin{pmatrix} 0 & x \\ y & 0 \end{pmatrix} \mid x, y \in \mathbb{Z}\right\}$

b) $\left\{\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} \mid x, y \in \mathbb{Z}\right\}$

c) $\left\{\begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} \mid x \in \mathbb{Z}\right\}$

d) $\left\{\begin{pmatrix} x & y \\ y & z \end{pmatrix} \mid x, y, z \in \mathbb{Z}\right\}$

11 Let $R[x]$ be ring with co-effts in $\mathbb{R}$. The number of ideals in the quotient ring $\frac{R[x]}{\langle x^2-3x+2\rangle}$ is

a) 2 b) 3 c) 4 d) 6

12. Let $f(x) = x^3 - 9x^2 + 9x + 3$ Then

a) $f(x)$ is irreducible over $\mathbb{Q}$ but reducible over $\mathbb{Z}_2$

b) $f(x)$ is irreducible over $\mathbb{Q}$ and $\mathbb{Z}_2$

c) $f(x)$ is reducible over $\mathbb{Q}$ but irreducible over $\mathbb{Z}_2$

d) reducible over $\mathbb{Q}$ ~~but~~ and $\mathbb{Z}_2$

13. Which of the followings are false?

a) In the ring $\frac{\mathbb{Z}}{8\mathbb{Z}}$, the eqn $x^2=1$ has ~~No soln~~

b) exactly 2 solns

b) The number 2 is a prime in $\mathbb{Z}[i]$

c) The poly x^8+1 is irreducible in $\mathbb{R}[x]$ and $x^3+3x-2\pi$ is irreducible over $\mathbb{R}$

d) All are false above.

14. ~~Which of the followings are true?~~

14. Let $f:[-1,1]\to\mathbb{R}$ be a conts fn. Then

a) $\sum_{n=1}^{\infty}\left(f(\frac{1}{n})-f(\frac{1}{n+1})\right)$ is convergent

b) $\sum_{n=1}^{\infty}\left(f(\frac{1}{n})-f(\frac{1}{n+1})\right)$ is divergent

c) $\sum_{n=1}^{\infty}\left|f(\frac{1}{n})-f(\frac{1}{n+1})\right|$ is not necessarily convergent on $(0,1)$, where $|f'(x)|<1$, $\forall x\in(0,1)$

d) I can't conclude

15. The set $\left\{\frac{x^2}{1+x^2}\mid x\in\mathbb{R}\right\}$ is

a) Connected but not compact in $\mathbb{R}$.

b) Compact but Not connected in $\mathbb{R}$

c) compact and connected in $\mathbb{R}$

d) Neither compact nor connected in $\mathbb{R}$

16) which of the following seq of fns is uniformly convergent on $(0,1)$

a) x^n b) $\frac{n}{nx+1}$ c) $\frac{x}{nx+1}$ d) $\frac{1}{nx+1}$

17. The set of all x at which the power series $\sum_{n=1}^{\infty} \frac{n}{(2n+1)^2}(x-2)^{3n}$ converges is

a) $[-1,1)$ b) $[-1,1]$

c) $[1,3)$ d) $[1,3]$

18. $X = \{(x,y) \in \mathbb{R}^2 \mid x \in \mathbb{Q}, y \in \mathbb{R} \setminus \mathbb{Q}\}$ Then

a) X is an open and dense subset of $\mathbb{R}^2$

b) X is an open but not dense subset of $\mathbb{R}^2$

c) X is not an open but a dense subset of $\mathbb{R}^2$

d) X is Neither an open nor a dense Subset of $\mathbb{R}^2$

19 Let $T(z)$ be a bilinear map $\ni T(2)=1$, $T(i)=i$, $T(-2)=-1$, Then $T(z)$ is

a) $\frac{3z+2i}{iz+6}$ b) $\frac{3z-2i}{iz+6}$ c) $\frac{3z+2i}{iz-6}$

d) $\frac{3z+2i}{iz-6}$

20. Let $f(z) = \sum_{n=1}^{\infty} \frac{1\cdot3\cdot5\cdots(2n-1)}{2\cdot5\cdot8\cdots(3n-1)} z^n$

be a power series about $z=0$.

Then Radius of convergence is

a) $R = \frac{1}{2}$ b) $R = 2/3$ c) $\frac{3}{2}$ d) $\frac{1}{3}$

21. Evaluate $\int_C \frac{e^{3z}}{z+i} dz$, $C := |z+1+i| = 2$

a) $2\pi i$ b) 0 c) $\frac{e^{-3i}}{2\pi i}$ d) $\frac{2\pi i}{e^{3i}}$

22. Consider the differential equ

$(x-1)y'' + xy' + \frac{1}{x}y = 0$. Then

a) $x = 1$ is the only singular point

b) $x = 0$ is the only singular point

c) Both $x=0$ and $x=1$ are singular points

d) Neither $x=0$ nor $x=1$ are singular points.

23. The matrix $\begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$ is

a) +ve definite b) Non-negative definite but not positive definite

c) Negative definite d) Neither +ve nor -ve

24) Let a, b, c be a +ve real numbers such that $b^2 + c^2 < a < 1$, Consider the 4x4 matrix $A = \begin{pmatrix} 1 & b & c \\ b & a & 0 \\ c & 0 & 1 \end{pmatrix}$.

a) All the eigenvalues of A are negative real numbers

b) All the eigenvalues of A are +ve real numbers

c) A can have a +ve as well as a negative eigenvalues

d) Eigenvalues of A can be non-real complex numbers.

25. The soln of the initial value problem

$y'' + k^2 y = g(t), \; y(t_0) = 0, \; y'(t_0) = 0$

where k is a real +ve constant is

a) $y = \int_{t_0}^{t} \sin k(t-s)\, g(s)\, ds$

b) $y = \int_{t_0}^{t} \sin k(s-t)\, g(s)\, ds$

c) $y = \int_{t_0}^{t} \sin(t-s)\, g(s)\, ds$

d) $y = \int_{t_0}^{t} \sin(s-t)\, g(s)\, ds$

part c

26 Which of the following statements is/are true?

a) Let $f: \mathbb{Q} \to \mathbb{Q}$ be a non-trivial homomorphism then f is an isomorphism

b) If gp G is finite then $Aut(G)$ is finite

c) If gp G is cyclic then $Aut(G)$ is cyclic

d) If $Aut(G)$ is cyclic then G is cyclic

27. Which of the followings is/are not necessarily true?

a) If G be gp $\ni G = Z(G)$ Then G is cyclic

b) $\frac{\mathbb{R}}{2\pi\mathbb{Z}} \cong S^1 := \{z \in \mathbb{C}^* \mid |z| = 1\}$.

c) Let G be a gp, ~~with~~ then there is a subgp H of G such that $\frac{N(H)}{C(H)} \cong Aut(H)$

d) The number of elements of order 8 in S_7 are 630

28. Let G be a gp with generators 'a' and 'b' which satisfies $a^4 = b^2 = e$ and $aba = b$.

Then $\frac{G}{Z(G)}$ is isomorphic to

a) the trivial gp b) $\mathbb{Z}_2$ c) $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ d) $\mathbb{Z}_4$.

29. Let f be a twice differentiable fn on $\mathbb{R}$

Given that $f''(x) > 0$, $\forall x \in \mathbb{R}$

a) $f(x)=0$ has exactly two soln on $\mathbb{R}$

b) $f(x)=0$ has a +ve soln if $f(0)=0$ and $f'(0)=0$

c) $f(x)=0$ has no +ve soln if $f(0)=0$ and $f'(0)>0$

d) $f(x)=0$ has no +ve soln if $f(0)$ and $f'(0)<0$

30. Let $f: \mathbb{R} \to \mathbb{R}$ be a real-valued fn

such that $|f(x)-f(y)| \le \alpha |x-y|$, $\forall x, y \in \mathbb{R}$,

$0 < \alpha < 1$. Let $A = \{x \in \mathbb{R} \mid f(x) = x\}$

Then the number of elements of A is

a) 0 b) 1 c) finite but more than one

d) Infinite

31. Let $f: \mathbb{R} \to \mathbb{R}$ be a diffble fn having infinite number of zero in $[0,1]$

and $A = \{x \in [0,1] \mid f'(x) = f(x) = 0\}$ Then

a) A is countably infinite set

b) $A \neq \phi$ finite set

c) $A \neq \phi$ but need not be finite

d) $A = \phi$

32. Let A be a complex 5×5 matrix with $A^5 = I$ such that $A \neq I$,

which of the followings ~~are~~ not correct?

a) A has atleast two complex eigenvalue such that $\lambda_1 = a+ib,\ b\neq 0$,

$\lambda_2 = c+id,\ d\neq 0$.

b) A is diagonalizable over $\mathbb{C}$ but not $\mathbb{R}$

c) $\text{Rank}(A) \leq n$

d) $\text{Rank}(A) = \text{Rank}(A^2) = \text{Rank}(A^3)$

33.) A linear operator T on a complex vector-space V has char poly $x^3(x-5)^2$ and minimal poly is $x^2(x-5)$. Then

a) T has J.C.F over $\mathbb{R}$.

b) $\text{Rank}(T) = 3$.

c) $\dim\left(\frac{V}{\ker(T)}\right) = 2$

d) T is diagonalizable over $\mathbb{C}$ but not $\mathbb{R}$.

34. Let $\mathbb{F} = \frac{\mathbb{F}_3[x]}{\langle x^3+2x-1\rangle}$, where $\mathbb{F}_3$ is the field with 3-elts Then

a) $\mathbb{F}$ is field with 29 elements

b) $\mathbb{F}$ is a seperable but not a normal extension of $\mathbb{F}_3$

c) The autonomorphism gp of $\mathbb{F}$ is cyclic

d) The automorphism gp of $\mathbb{F}$ is abelian but not cyclic.

35. which of the polys are irreducible over the given rings?

a) $x^5+3x^4+9x+15$ over $\mathbb{Z}$.

b) x^3+2x^2+x+1 over $\mathbb{Z}_7$

c) x^3+x^2+x+1 over $\mathbb{Q}$.

d) $x^4+x^3+x^2+x+1$ over $\mathbb{C}$

36. Consider the bddry value problem

$$-u''(x) = \pi^2 u(x), \quad x \in (0,1)$$

$$u(0) = u(1) = 0.$$

If u and u' are conts on $[0,1]$, then

a) $\int_0^1 u^3(x)\,dx = 0$

b) $u'^2(x) + \pi^2 u^2(x) = u'^2(0)$

c) $u'^2(x) + \pi^2 u^2(x) = u'^2(1)$

d) $\int_0^1 u^2(x)\,dx = \frac{1}{\pi^2}\int_0^1 u'^2(x)\,dx.$

37. Consider the following primal linear programming problem.

$$\text{Max } z = -3x_1 + 2x_2$$

Subject to $x_1 \le 3$

$$x_1 - x_2 \le 0$$

$$x_1, x_2 \ge 0$$

Then

a) The primal problem has an optimal soln

b) The primal problem has an unbdd soln

c) The dual problem has an unbdd soln.

d) The dual problem has no feasible soln

~~39~~. 38 Let A be a subset of $\mathbb{R}$. ~~which~~
Then which of the followings are true?

a) $\exists$ a differentiable $f: \mathbb{R} \to \mathbb{R}$ such that $|f'(x)| < 1$ but not uniformly conts on $\mathbb{R}$.

b) Every seq of points in A is convergent in A, then A is compact

c) $\exists$ a subset $A \subseteq \mathbb{R}$ such that A is compact but not limit point compact.

d) $\nexists$ a 1-1 conts fn $f: \mathbb{R} \to [0,1]$

~~40~~. 39 Let f be a Monotonically increasing fn from $[0,1]$ to $[0,1]$ then

a) f has fixed point in $[0,1]$

b) f is R.I on $[0,1]$

c) f is bdd variation on $[0,1]$

d) $A = \{x \in [0,1] \mid \cancel{\lim_{x_n \to x} f(x)}\ \lim_{n\to\infty} f(x_n)$ does not exist whenever $x_n \to x\}$ is Countable set

40. $\left(C([0,1]), \|\cdot\|_1\right), \left(C([0,1]), \|\cdot\|_\infty\right)$

$$\|f\|_1 = \int_0^1 |f(t)|\,dt.$$

$$\|f\|_\infty := \sup\{|f(t)| \mid t \in [0,1]\}.$$

Let U_1 be unit ball in $\|\cdot\|_1$.

Let U_∞ be unit ball in $\|\cdot\|_\infty$.

Then

a) $U_\infty \subseteq U_1$ c) $U_1 = U_\infty$

b) $U_1 \subseteq U_\infty$ d) $U_\infty \nsubseteq U_1$; $U_1 \nsubseteq U_\infty$

41. Let $A \in M_n(\mathbb{R}) \ni A^T A = A A^T = I$

Then

a) $\langle Ax, Ay \rangle = \langle x, y \rangle$, $\forall x, y \in \mathbb{R}^n$

b) All the eigenvalues lies on unit disk

c) columns of A forms an orthonormal basis of $\mathbb{R}^n$

d) $\langle x, y \rangle = 0$, x, y are different columns of A. w.r.t usual I.p.s on $\mathbb{R}^n$

42. Let f be a non-zero symmetric bilinear form on $\mathbb{R}^3$. Suppose that there exist linear transformation $T_i : \mathbb{R}^3 \to \mathbb{R}$, $i = 1, 2$ such that $\alpha, \beta \in \mathbb{R}^3$, $f(\alpha, \beta) = T_1(\alpha)\, T_2(\beta)$

Then

a) rank$(f) = 1$

b) rank$(f) \geq 1$

d. c) $\{x \mid f(x, x) = 0\}$ is subspace.

c d) f is +ve semi-definite or negative semi-definite

43. Let $f : \mathbb{C} \to \mathbb{C}$ be a meromorphic fn analytic at '0' satisfying $f(\frac{1}{n}) = \frac{n}{2n+1}$, $n \geq 1$

Then

a) $f(0) = f(1)$

b) f has pole at $z = -2$

c) $f(0) = \frac{1}{2}$

d) No such fn exist

44. Which of the followings are not true?

a) $\exists$ a non-constant entire f such that $(\text{Range}(f))^{\circ}$ is uncountable.

b) $\exists$ a non-constant entire fn such that $\text{Range}(f) = (\text{Range}(f))^{\circ}$

c) $\exists$ a non-Constant entire such that $f(A) \subseteq \mathbb{R}$, where A is open subset of $\mathbb{C}$

d) $\exists$ a two entire fn $f, g \ni$ $\text{Im}(f) \neq \text{Im}(g)$.

45. Which of the followings are true?

a) $\exists\, A \in M_3(\mathbb{R}) \ni A^2 = I,\ A \neq I$.

b) $\exists$ a conts fn $f: \mathbb{R} \to \mathbb{R} \ni |f'(x)| < 1$ but f has only unique ~~fixed~~ fixed point in $\mathbb{R}$.

c) $\exists$ a non-abelian gp G of order is 77

d) $\exists$ a non-constant entire such that $\text{Im}(f(x+iy))$ is countably infinite where $y > 0$.

Part A

46 1) Find the number of divisors of 720 (including 1 and 720).
(A)16 (B)22 (C)25 (D)30

2)The highest power of 3 that completely divides 40! Is
47 (A)18 (B)16 (C)24 (D)30

3)The difference of $10^{25} - 7$ and $10^{24} + x$ is divisible by 3 for x = ? 48
(A)1 (B)2 (C)4 (D)5

4)The mean of 1, 2, 2^2 …. 2^{31} lies in between
(A) 2^{24} to 2^{25} (B) 2^{25} to 2^{26} (C) 2^{26} to 2^{27} (D) 2^{29} to 2^{30} 49

5) After striking a floor a rubber ball rebounds (5/6)th of the height from which it has fallen. Find
50 the total distance (in metres) that it travels before coming to rest, if it is gently dropped from a height of 210 metres.
(A)1540 (B)2310 (C)4315 (D)5024

6)Find the sum of all three-digit natural numbers, which on being divided by 7, leave a
51 remainder equal to 6.
(A) 70,208 (B) 70,780 (C) 70,680 (D) 71,270

7)In a family of 5 males and a few ladies, the average monthly consumption of grain per head is
52 9 kg. If the average monthly consumption per head be 12 kg in the case of males and 8 kg in the case of females, find the number of females in the family.

(A) 18 (B) 12 (C) 9 (D) 15

53 8)One-fifth of a certain journey is covered at the rate of 20 km/h, one-fourth at the rate of 50 km/h and the rest at 55 km/h. Find the average speed for the whole journey.
(A) 53 km/h (B) 40 km/h (C) 35 km/h (D) 38 km/h

9)There are two mixtures of milk and water, the quantity of milk in them being 20% and 80% of
54 the mixture. If 2 liters of the first are mixed with three liters of the second, what will be the ratio of milk to water in the new mixture? (A) 11 : 12 (B) 11 : 9 (C) 19 : 11 (D) 14 : 11

55 10)A mixture of 75 liters of alcohol and water contains 20% of water. How much water must be added to the above mixture to make the water 25% of the resulting mixture?
(A) 5 liters (B) 1.5 litre (C) 2 liters (D) 2.5 liters

56 11) In an examination, 80% students passed in Physics, 70% in Chemistry while 15% failed in both the subjects. If 3250 students passed in both the subjects. Find the total number of students who appeared in the examination. (A) 7500 (B) 8,000 (C) 3000 (D) 5,000

57 12)The price of a certain product was raised by 20% in India. The consumption of the same article was increased from 400 tons to 440 tons. By how much percent will the expenditure on the article rise in the Indian economy? (A) 32% (B) 25% (C) 27% (D) 26%

58 13) If the length, breadth and height of a cube are decreased, decreased and increased by 5%, 5% and 20%, respectively, then what will be the impact on the surface area of the cube (in percentage terms)? (A) 7.25% (B) 5% (C) 8.33% (D) 6.0833%

59 14)A man sells a plot of land at 8% profit. If he had sold it at 15% profit, he would have received ` 630 more. What is the selling price of the land? (A) ` 9320 (B) ` 9600 (C) ` 9820 (D) ` 9720

60 15)David sells his Laptop to Goliath at a loss of 20% who subsequently sells it to Hercules at a profit of 25%. Hercules, after finding some defect in the laptop, returns it to Goliath but could recover only ` 4.50 for every ` 5 he had paid. Find the amount of Hercules' loss if David had paid ` 1.75 lakh for the laptop. (A) ` 3500 (B) ` 2500 (C) ` 17,500 (D) None of these

61 16) An orange vendor makes a profit of 20% by selling oranges at a certain price. If he charges ` 1.2 higher per orange he would gain 40%. Find the original price at which he sold an orange. (A) ` 5 (B) ` 4.8 (C) ` 6 (D) None of these

62 17)A precious stone weighing 35 grams worth ` 12,250 is accidentally dropped and gets broken into two pieces having weights in the ratio of 2 : 5. If the price varies as the square of the weight then find the loss incurred. (A) ` 5750 (B) ` 6000 (C) ` 5500 (D) ` 5000

63 18)14. A tank of capacity 25 litres has an inlet and an outlet tap. If both are opened simultaneously, the tank is filled in 5 minutes. But if the outlet flow rate is doubled and taps opened the tank never gets filled up. Which of the following can be outlet flow rate in litres/min? (A) 2 (B) 6 (C) 4 (D) 3

64 19)Two poles of height 6 m and 11 m stand vertically upright on a plane ground. If the distance between their foot is 12 m, find the distance between their tops.
(A) 12 m (B) 14 m (C) 13 m (D) 11 m

Courses	*STUDENTS*			
	English		*Maths*	
	MALES	*FEMALES*	*MALES*	*FEMALES*
Part-time MBA	30	10	50	10
Full-time MBA only	150	8	16	6
CA only	90	10	37	3
Full time MBA & CA	70	2	7	1

65

20) The percentage increase in students of full-time MBA only over CA only is (A) less than 20 (B) less than 25 (C) less than 30 (D) more than 30