KEY

Section 1: Algebra

1.1 a,c
1.2 S^1, the multiplicative group of complex numbers with modulus one
1.3 $N(3) = 1$ or 7; $N(7) = 1$
1.4 a,b
1.5 a,b
1.6 a,b,c
1.7 a,b
1.8
a. rank of $A = 1$
b. $x^T x = 1$
1.9 b,c
1.10
a. $\{\lambda \in \mathbb{C} \mid \lambda \text{ real}, \geq 0\}$
b. $\{\lambda \in \mathbb{C} \mid \lambda \text{ real}\}$
c. $\{\lambda \in \mathbb{C} \mid \mathrm{Re}(\lambda) = 0\}$

Section 2: Analysis

2.1 $\frac{1}{6} \leq x < \frac{1}{2}$
2.2 $\frac{k(k+1)}{2} m$
2.3 a,b
2.4 b,c
2.5 a,b,c
2.6 b,c
2.7 b,c
2.8 $2 \log 2 - 1$
2.9 b,c
2.10 a,b,c

Section 3: Topology

3.1 a,c
3.2 c
3.3 a,b
3.4 a,b
3.5 a,b
3.6 a,b
3.7 a
3.8 a,b,c
3.9 b,c
3.10 b

Section 4: Calculus & Differential Equations

4.1 $\frac{1}{n!}$
4.2

$$\frac{2}{x}\left(e^{x^2} - e^{-x^2}\right)$$

4.3 Minimum at $(3, -1)$.
4.4 $\frac{\pi}{\sqrt{3}}$
4.5

$$x - \frac{1}{2}\frac{x^3}{3} - \frac{1.1}{2.4}\frac{x^5}{5} - \frac{1.1.3}{2.4.6}\frac{x^7}{7} - \cdots$$

4.6 $\frac{1-r^2}{4}$
4.7 a,b
4.8 b,c
4.9 $\frac{n!}{s^{n+1}}$
4.10

$$\int_{x_1}^{x_2} 2\pi y(x)\sqrt{1 + (y'(x))^2}\, dx$$

(The constant 2π can be omitted.)

Section 5: Miscellaneous

5.1

$$\det(A) \;=\; \begin{cases} (-1)^{\frac{n}{2}} n!, & \text{for } n \text{ even} \\ (-1)^{\frac{n-1}{2}} n!, & \text{for } n \text{ odd.} \end{cases}$$

5.2

$$\begin{bmatrix} 2n \\ n \end{bmatrix}$$

5.3 a,b,c
5.4 a,b,c
5.5 $\left(\frac{a}{n}\right)^n$
5.6 a,b,c
5.7 a
5.8 $\frac{5}{36}$
5.9

$$S \;=\; \cup_{\varepsilon>0} \cap_{n=1}^{\infty} \cup_{m=n}^{\infty} E_m(\varepsilon)$$

5.10

$$\frac{1}{1 - z - z^2}$$

Note: Accept any correct equivalent form of the answers.