

5X)

Assignment

I Verify which of the followings are true or false please, justify your answer

a) If a fn $f = u + iv$ is diffble, Can we say anything about u_n ?

b) If $f(z)$ assumes only real values in a domain, what can you say about the existence of $f'(z)$?

c) where is $|z|$ diffble? Where is $|z|^2$ differentiable?

d) where is $\bar{z}$ diffble? Where is $\bar{z}$ continuously

e) where is $\operatorname{Re}(z)$ diffble? where is $(\operatorname{Re}(z))^2$ diffble?

f) where is $\bar{z} \operatorname{Re}(z)$ diffble? Where is $\bar{z} \operatorname{Re}(z)$ continuous?

g) If a derivative exists, is the derivative a cont fn?

II. If $f(0,0) = 0$, which of the following ~~fun~~ are conts at the origin?

a) $f(x,y) = \frac{x^2 y^2}{x^2 + y^4}$ b) $f(x,y) = \frac{x^2 y^2}{(x^2 + y^2)^2}$

c) $f(x,y) = \frac{x^3 y^2}{(x^2 + y^2)^2}$ d) $f(x,y) = \frac{x + y e^{-x^2}}{1 + y^2}$

e) $f(x,y) = \frac{(x + y^2)^2}{x^2 + y^2}$ f) $f(x,y) = \frac{x \sqrt{xy}}{x^2 + y^2}$

III. Determine where the following ~~fun~~ satisfy the C-R-E, and where the ~~fun~~ are diffble.?

a) $f(z) = \bar{z}^2$ b) $f(z) = x^2 - y^2$ c) $f(z) = 2xyi$

d) $f(z) = x^2 - y^2 + 2xyi$ e) $f(z) = z \operatorname{Re}(z)$

f) $f(z) = z|z|$ g) $|\operatorname{Re}(z) \operatorname{Im}(z)|^{1/2}$ h) $f(z) = z \operatorname{Im}(z)$

IV. ~~Justify~~ Justify your answer?

a) If $f(z)$ is conts at a point z_0 . Then $\overline{f(z)}$ is also conts at z_0 . Is the same true for diffble at z_0 ? What does the ~~for~~ $f(z) = z^2$ ~~show~~ show?

How about $f(z) = \bar{z}$?

b) If a ~~fun~~ f is analytic in a bdd region, is the ~~fun~~ $\overline{f}$ bdd?

c) If $f(z)$ and $\overline{f(z)}$ are both analytic in a domain, what can be said about f throughout D ??

d) Let $f(z)$ be analytic in the unit disk $\Delta = \{z \in \mathbb{C} \mid |z| < 1\}$.

Is $g(z) = \overline{f(\bar{z})}$ analytic in Δ ?

e) Is $f(z) = (\bar{z})^5 - \bar{z}$ diffble at ± 1 ?

Is $f(z)$ analytic?

f) Does there exist a f that is analytic only for $\text{Im}(z) > 2023$ and nowhere else?

g) If a f is analytic in a bdd region, is the f bdd?

h) What important difference are there between diffble f s and analytic f s?

h) what important difference are there between differentiable fns and analytic fns?

i) Does there exist a fn f that is analytic for $\operatorname{Re}(z) \geq 2023$ and is not analytic anywhere else?

j) If $f = u + iv$ is analytic in a domain D such that $au + bv$ is constant, where a, b are real constants, then f is constant.

k) A real-valued fn u in domain D can't be analytic in D unless it is a constant fn.

l) There exists a non-constant real-valued fn u in an open set D which is analytic in D .

m) If $f = u + iv$ and $g = u + i\bar{v}$ are two analytic functions defined in a domain Ω , then $\text{Im}(f - g)$ is necessarily a constant.

n) If $f(z)$ and $\overline{f(z)}$ are analytic functions in a domain D , then f is necessarily a constant.

o) There exist no analytic function f such that $\text{Re}(f(z)) = y^2 - 2x$.

p) There exists no analytic function f such that $\text{Im}(f) = x^3 - y^3$.

q) An analytic function $f = u + iv$ in a domain D such that $v = u^2$ is necessarily a constant.