① Let G be the set of all 020346

5×5 symmetric matrices with real entries Then

a) $\forall A \in G, \exists P \in GL_5(\mathbb{R}) \ni$

$A = PDP^T$, for some D-diagonal

b) If $X_0 \neq 0$ in $\mathbb{R}^5$, $\exists A \in G$

$\ni AX_0 = \lambda X_0$ for some $\lambda \in \mathbb{R}$.

c) If $X_1 \neq X_2$ in $\mathbb{R}^5$, $\exists A \in G$

$\ni AX_1 = \lambda_1 X_1$ and $AX_2 = \lambda_2 X_2$.

d) $\forall A \in G, \exists B \in G \ni AB = 0$.

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(2) Which of the followings is true?

a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ is not diagonalizable over $\mathbb{C}$

b) The matrix $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ is diagonalizable over $\mathbb{R}$

c) The matrix $A = \begin{pmatrix} 1 & 0 \\ 9 & 2 \end{pmatrix}$ is diagonalizable over $\mathbb{R}$

d) If A is diagonalizable and A^{-1} exists then $A + A^{-1}$ is diagonalizable over $\mathbb{F}$.

(3) If V is a v.s over $\mathbb{Z}_7$ & $\dim(V(\mathbb{Z}_7)) = 3$. Then the Cardinality of V is

a) 49 b) 672 c) 343 d) infinite

(4) Let $V = M_n(\mathbb{R})$ be v.s over $\mathbb{R}$. Then which of the followings are true?

a) $\forall A \in M_n(\mathbb{R})$, $\exists B \in M_n(\mathbb{R}) \ni$

$AB = BA$

b) If $A \in M_n(\mathbb{R})$ and if A is diagonalizable over $\mathbb{R}$ Then A^T is diagonalizable over $\mathbb{R}$.

c) If $A \in M_n(\mathbb{R})$ and if A^2 is diagonalizable over $\mathbb{R}$ Then A is diagonalizable over $\mathbb{R}$.

d) If $A \in M_n(\mathbb{R})$ then A similar to A^T

5. Let $V = \mathbb{R}^6(\mathbb{R})$ be vector space over $\mathbb{R}$

Then which of the followings

are false?

a) If $S \subseteq \mathbb{R}^6$ then $\text{span}(S) = S$

b) If $S \subseteq \mathbb{R}^6$ and if $\text{span}(S) = \mathbb{R}^6$

Then $\exists\, S_1 \subseteq S \ni S_1$ is basis of $\mathbb{R}^6$

c) If $S \subseteq \mathbb{R}^6$ be L.D then there

exists $S_1 \subseteq S$ such that

S_1 is basis of $\mathbb{R}^6$

d) If $S \neq \{0\}$ be subset of $\mathbb{R}^6$ and if

~~Then~~ $\text{span}(S) = S$ Then

S is L.D

6.) The rank of the 4×6 matrix

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$ is

a) 1 b) 2 c) 3 d) 4.

7) Let $V = P_5(\mathbb{R})$ be vector space. Then which of the followings are not false?

a) $\{f \in V \mid f(\frac{1}{2}) \in \mathbb{Q}\}$ is subspace of V

b) $\{f \in V \mid f(\frac{1}{3}) \notin \mathbb{Q}\}$ is subspace of V

c) $\{f \in V \mid f(\frac{1}{n}) = 0, \forall n = 1,2,3,4,5,6,7\}$ be subspace of V containg only '0' elts

d) $\{f \in V \mid f(1) = f(\frac{1}{2}) = f(-1) = f(2) = f(4) = f(\frac{1}{4})\}$

~~Then~~ be subspace of V containg only '0' elts.

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M	T	W	T	F	S	S	M	T	W	T	F	S	S
		1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24	25	26
27	28	29	30	31									

8) For $A, B \in M_2(\mathbb{R})$

define $(A, B) = AB - BA$

consider the two statements

$P: (A,(B,C)) + (B,(C,A)) + (C,(A,B)) = 0$
$\forall A, B, C \in M_2(\mathbb{R})$

$Q: (A,(B,C)) = ((A,B),C)$
$\forall A, B, C \in M_2(\mathbb{R})$

Then

a) P, Q are true

b) P is true but Q is false

c) P is false, but Q is true

d) Both P and Q are false

(9) For $P \in M_5(\mathbb{R})$ and $i, j \in \{1, 2, \dots, 5\}$

let P_{ij} denote the $(i,j)^{th}$ entry of P

let $S = \{ P \in M_5(\mathbb{R}) \mid P_{ij} = P_{rs}$ for $i, j, r, s \in \{1, 2, \dots, 5\}$

with $i + r = j + s \}$

Then which one of the following is false?

a) S is subspace of $M_5(\mathbb{R})$.

b) $\dim(S) = 5$

c) $\dim(S) = 11$

d) If $P \in S$ and all the entries of P are integers. Then 5 divides the sum of all the diagonal entries of P

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10) Let P be a 3×3 real matrix having eigenvalues $\lambda_1 = 0$, $\lambda_2 = 1$, $\lambda_3 = -1$. Further $v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $v_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ are eigenvectors of the matrix P corresponding to the eigenvalue λ_1, λ_2, and λ_3 respectively. Then the entry in the first row and third column of P is

a) 0 b) 1 c) -1 d) 2.