

1) If $S_n = \frac{(-1)^n}{2^n + 3}$ and $t_n = \frac{(-1)^n}{4n-1}$, $n=0, 1, 2, \dots$ find (2)

Then.

a) $\sum_{n=0}^{\infty} S_n$ is absolutely convergent

b) $\sum_{n=0}^{\infty} t_n$ is absolutely convergent

c) $\sum_{n=0}^{\infty} S_n$ is conditionally convergent

d) $\sum_{n=0}^{\infty} t_n$ is conditionally convergent.

2) which one of the following series is divergent?

a) $\sum_{n=1}^{\infty} \frac{1}{n} \sin^2 \frac{1}{n}$

b) $\sum_{n=1}^{\infty} \frac{1}{n} \log n$

c) $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{1}{n}$

d) $\sum_{n=1}^{\infty} \frac{1}{n} \tan \frac{1}{n}$

3) Suppose that S is the sum of a convergent series

$\sum_{n=1}^{\infty} a_n$. Define $t_n = a_n + a_{n+1} + a_{n+2}$. Then the series $\sum_{n=1}^{\infty} t_n$?

a) diverges

b) converges to $3S - a_1 - a_2$

c) converges to $3S - a_1 - 2a_2$

d) converges to $3S - 2a_1 - a_2$

4) The sum of the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 + n - 2}$ is

a) $\frac{1}{3} \ln(2) - \frac{5}{18}$

b) $\frac{1}{3} \ln(3) - \frac{5}{6}$

c) $\frac{2}{3} \ln(2) - \frac{5}{18}$

d) $\frac{2}{3} \ln(2) - \frac{5}{6}$

5) let $a_n = e^{-2n} \sin n$ and $b_n = e^{-n} n^2 (\sin n)^2$ for $n \geq 1$.

Then

a) $\sum_{n=1}^{\infty} a_n$ Converges but $\sum_{n=1}^{\infty} b_n$ does not Converges.

b) $\sum_{n=1}^{\infty} b_n$ Converges but $\sum_{n=1}^{\infty} a_n$ does not Converges.

c) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ Converges.

d) neither $\sum_{n=1}^{\infty} a_n$ nor $\sum_{n=1}^{\infty} b_n$ Converges.

b) For, $n \geq 1$ let $a_n = \begin{cases} n 2^{-n} & \text{if } n \text{ is odd} \\ -3^{-n} & \text{if } n \text{ is even.} \end{cases}$ which of the

following statements is (are) True?

a) The Seq $\{a_n\}$ Converges b) The Seq $\{|a_n|^{1/n}\}$ Converges

c) The Series $\sum_{n=1}^{\infty} a_n$ Converges d) The Series $\sum_{n=1}^{\infty} |a_n|$ Converges.

7) Suppose $\{a_n\}, \{b_n\}$ are sequence such that $a_n > 0, b_n > 0$

for all $n \geq 1$. Given that $\sum a_n$ Converges and $\sum b_n$ diverges.

Which of the following statements is (are) necessarily False?

a) $\sum (a_n + b_n)$ Converges b) $\sum \frac{a_n}{b_n}$ Converges

c) $\sum \frac{b_n}{a_n}$ Converges d) $\sum a_n b_n$ Converges.

8) Consider the series S_1 and S_2 given by.

$$S_1: \sum_{n=1}^{\infty} \frac{n^2+n+1}{n(n+1)} \quad \text{and} \quad S_2: \sum_{n=1}^{\infty} \frac{n^2+1}{n^2(n+1)} \quad \text{Then:}$$

- a) Both S_1 and S_2 converge b) S_1 converges and S_2 diverges
c) S_2 converges and S_1 diverges d) Both S_1 & S_2 diverges

9) Let $u_n = \left(4 - \frac{1}{n}\right)^{\frac{(-1)^n}{n}}$, $n \in \mathbb{N}$. and let $l = \lim_{n \rightarrow \infty} u_n$. Which of the following statements is True?

- a) $l = 0$ and $\sum_{n=1}^{\infty} u_n$ is convergent
b) $l = \frac{1}{4}$ and $\sum_{n=1}^{\infty} u_n$ is divergent
c) $l = \frac{1}{4}$ and $\{u_n\}_{n \geq 1}$ is oscillatory
d) $l = 1$ and $\sum_{n=1}^{\infty} u_n$ is divergent.

10) The series $\sum_{n=1}^{\infty} \frac{\log(1+\frac{1}{n})}{n^r}$, Then

- a) converges if $r > 0$ b) converges $\forall r \in \mathbb{R}$
c) converges if $r = 0$ d) converges if $r < 0$.

11) Let n be a fixed natural number. Then the series $\sum_{m \geq n} \frac{(-1)^m}{m}$ is.

- a) Absolutely convergent b) divergent
c) Absolutely convergent if $n > 100$ d) convergent.

12) Given $\{a_n\}, \{b_n\}$ two monotone sequences of real numbers and that $\sum a_n b_n$ is convergent, which of the following is true?

- a) $\sum a_n$ is convergent and $\sum b_n$ is convergent.
- b) At least one of $\sum a_n, \sum b_n$ is convergent.
- c) $\{a_n\}$ is bdd and $\{b_n\}$ is bounded (bdd-bounded)
- d) At least one of $\{a_n\}, \{b_n\}$ is bounded.

13) Which of the following are convergent?

- a) $\sum_{n=1}^{\infty} n^2 2^{-n}$
- b) $\sum_{n=1}^{\infty} n^{-2} 2^n$
- c) $\sum_{n=2}^{\infty} \frac{1}{n \log n}$
- d) $\sum_{n=1}^{\infty} \frac{1}{n \log(1 + \frac{1}{n})}$

14) Let $\{a_n\}$ be sequence of real numbers. Which of the following is true?

- a) If $\sum a_n$ converges, then so does $\sum a_n^2$
- b) If $\sum |a_n|$ converges, then so does $\sum a_n^2$
- c) If $\sum |a_n|$ diverges then so does $\sum a_n^3$
- d) If $\sum |a_n|$ diverges then so does $\sum a_n^2$

15) The value of the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$ is

- a) 1 b) 2 c) 3 d) 4.

16) The series $\sum_{n=1}^{\infty} \frac{\cos(3^n x)}{2^n}$, then

- a) diverges; for all rational $x \in \mathbb{R}$
b) Diverges, for some irrational $x \in \mathbb{R}$
c) Converges; for some but not all $x \in \mathbb{R}$
d) converges for all $x \in \mathbb{R}$.

17) Let $\{x_n\}$ be a sequence of positive real numbers.

Then which of the following is false?

- a) If $\sum_{n=1}^{\infty} x_n$ is convergent then $\sum_{n=1}^{\infty} \sqrt{x_n}$ is convergent
b) If $\sum_{n=1}^{\infty} x_n$ is convergent then $\sum_{n=1}^{\infty} x_n^2$ is convergent.
c) If $\sum_{n=1}^{\infty} x_n^2$ is convergent then $\lim_{n \rightarrow \infty} x_n = 0$
d) If $\sum_{n=1}^{\infty} \sqrt{x_n}$ is convergent then $\lim_{n \rightarrow \infty} x_n = 0$.

18) The limit $\sum_{k=2}^n \frac{1}{k^3 - k}$ is equal to _____.

19) for $a > 0$ the series $\sum_{n=1}^{\infty} a^{\ln(n)}$ is convergent if and only if

a) $0 < a < e$

b) $0 < a \leq e$

c) $0 < a < 1/e$

d) $0 < a \leq 1/e$

20) For $n \geq 2$, let $a_n = \frac{1}{n \log n}$. Then.

a) The sequence $\{a_n\}_{n=2}^{\infty}$ is convergent

b) The series $\sum_{n=2}^{\infty} a_n$ is convergent

c) The series $\sum_{n=2}^{\infty} a_n^2$ is convergent

d) The series $\sum_{n=2}^{\infty} (-1)^n a_n$ is convergent.