Test

Part C only

① Let $A \in M_n(\mathbb{R})$. pickout the true statement.

a) $\exists\ B \in M_n(\mathbb{R}) \ni B^2 = A$.

b) $\exists\ B \in M_n(\mathbb{R}) \ni B = B^T$ then $B = AA^*$

c) $\exists\ B \in M_n(\mathbb{R}) \ni B^4 = B^2 = A$.

d) suppose $A \in M_n(\mathbb{R})$ and $A = A^T$,
$\exists\ B \in M_n(\mathbb{R}) \ni B^3 = A$

② Let $A, B \in M_n(\mathbb{R})$. Pick the correct statements below.

a) If A, B are invertiable Then $AB - BA = I$

b) $\exists\ A, B \in M_n(\mathbb{R}) \ni AB - BA = 0$

c) $\exists\ A, B \in M_n(\mathbb{R}) \ni A^2B^2 - B^2A^2 = I$.

d) $\exists\ A, B \in M_n(\mathbb{R}) \ni AB - BA = 2I$.

3) Let $V = M_n(\mathbb{R})$, $\langle A, B\rangle = tr(B^T A)$

pick the true statements

a) $\forall A \in M_n(\mathbb{R})$, $\exists B \in M_n(\mathbb{R}) \ni \langle A, B\rangle = 0$.

b) $\forall A \in M_n(\mathbb{R})$, $\exists B \in M_n(\mathbb{R}) \ni \langle A, A\rangle = 1$

c) $\forall A \in M_n(\mathbb{R})$, $\exists B \in M_n(\mathbb{R}) \ni \langle A, B\rangle = 1$

d) $\forall A \in M_n(\mathbb{R}) \ni \langle A, A^2\rangle = 0$

Then $A \equiv 0$

4) Let $A \in M_n(\mathbb{R}) \ni A^2 + I = 0$ then

A can't be

(a) orthogonal

(b) skew-symmetric

(c) symmetric

(d) A is invertiable

5) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a 2×2 real matrix with $|A| = 1$. If A has No real eigenvalues then

a) $(a+d)^2 < 4$ b) $(a+d)^2 = 4$

c) $(a+d)^2 > 4$ d) $(a+d)^2 = 16$.

6. Let P be an $n\times n$ matrix with integer entries and $Q = P + \frac{1}{2} I$

Then Q is

a) $Q^2 = Q$ b) $|Q| \neq 0$ c) $Q^n = 0$

d) $Q - I$ is nilpotent.

7. Let $M = \begin{bmatrix} 1 & a & b \\ 0 & 2 & c \\ 0 & 0 & 1 \end{bmatrix}$, $a, b, c \in \mathbb{R}$.

Then M is diagonalizable $\Leftrightarrow$

a) $a = bc$ b) $b = ac$

c) $c = ab$ d) $a = b = c$.

8) Let A and B be $n \times n$ matrices with the same minimal poly. Then

a) $A \sim B$ b) A is diagonalizable if B is diagonalizable

c) $A-B$ is singular

d) A and B Commute.

9) Let $A \in M_{7\times 9}(\mathbb{R})$ and $B = A^T A$, Then

a) $y^T B y < 0 \ \forall y \neq 0$

b) $y^T B y \leq 0. \ \forall y \neq 0$

c) $y^T B y > 0 \ \forall y \neq 0$

d) $y^T B y \geq 0 \ \forall y \neq 0$!

10) The quadratic form $2x_1x_2 + 4x_2x_3 - x_1^2 - 5x_2^2 - x_3^2$ is

a) Indefinite b) Negative definite

c) Positive semi-definite

d) Negative semi-definite

11) $f(x,y) = x^2 - 4xy + 5y^2$ on $\mathbb{R}^2$ is

a) symmetric and +ve definite

b) not symmetric, but positive definite

c) symmetric, but not positive definite

d) Neither symmetric nor positive defined

12) If $a+d = 1 = ad-bc$ and if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Then $A^3 =$

a) I b) $A=0$ c) A d) $=I$

13) A is a 3×3 matrix with all the eigenvalues are $1, -1, 0$. Then

$|I + A^{100}|$ is

a) 6 b) 4 c) 9 d) 100

14) If A and B are elments in $M_n(\mathbb{R})$

$\ni$ $AB=A$ and $BA=B$ Then

a) $A^2=A$, $B^2=A$

b) only $A^2=A$ c) only $B^2=B$

d) Both A and B are not idempotent.

15) A and B are n-square positive definite matrices. Then which of the following are true definite

a) $A+B$ b) ABA c) AB d) A^2+I.

16. Let A be a $n\times m$ matrix with strictly positive entires with $m<n$ and let $C=AA^T$ then we can conclude that

a) $C=C^T$ b) C is strictly +ve definite matrix.

c) C is non-singular

d) $\exists\ x\in\mathbb{R}^n \ni x^TCx=0$

17) Let $Q(x,y) = ax^2 + 2hxy + by^2$; $a,b,h \in \mathbb{R}$

Then $Q(x,y) = 1$ represents a hyperbola $\Leftrightarrow$

a) rank$(Q) = 2$ and signature $(Q) = 0$

c) rank$(Q) = 2$ and signature $(Q) = 2$

c) rank$(Q) = 1$ and signature $(Q) = 1$

d) rank$(Q) = 1$ and signature$(Q) = -1$

18) Let f be a quadratic form on $\mathbb{R}^n$; $x \in \mathbb{R}^n$, $\alpha \in \mathbb{R}$.

Then $f(\alpha x)$ is equal to

a) $\alpha^2 f(x)$ b) $\alpha f(x)$ c) $-\alpha f(x)$

d) $|\alpha| f(x)$.

19) which of the followings matrices have Jordan Canonical form equal to $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$?

a) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ b) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ c) $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

d) $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

Q20) Let $u, v \in \mathbb{R}^3$, $v \neq 0$.
Which of the following is false?

a) If $u^T w = v \cdot w$ $\forall w \in \mathbb{R}^3$ then $u = v$.

b) $u \cdot v = \frac{1}{2}\left(\|u+v\|^2 - \|u-v\|^2\right)$

c) $\|u+v\|^2 + \|u-v\|^2 = 2\left(\|u\|^2 + \|v\|^2\right)$

d) $\left|u \cdot \frac{v}{\|v\|}\right|$ is the length of the projection of u along v.

Q21) Let Q_1 and Q_2 be two positive definite quadratic forms. Then the following must also be a +ve definite quadratic form

a) $Q_1 Q_2$

b) $Q_1 + Q_2$

c) $Q_1 - Q_2$

d) $Q_1 \backslash Q_2$

22) Let A be a real symmetric $n\times n$ matrix. Then a necessary and sufficient condition for the map $\langle x,y\rangle = x^T A y$ to be an inner product on $\mathbb{R}^n$ is

a) $|A| \neq 0$ b) A is +ve definite

c) A is +ve semi definite

d) A is identity

23) The eigenvalues of the matrix of the quadratic form associated with the quadratic eqn

$$9x^2 - 4xy + 6y^2 - 10x - 20y = 5$$

are

a) 1, 2 b) 2, 5 c) 5, 10 d) -5, 10

24) Which of the followings are true?

a) A is +ve definite $\Leftrightarrow$ $A = B^T B$ for some square matrix

b) If A, B are +ve definite $\ni AB = BA$ Then AB is +ve definite

c) $\langle Ax, x \rangle > 0 \ \forall x \in \mathbb{R}^n$ Then $A \neq 0$

d) $\langle Ax, x \rangle = 0 \ \forall x \in \mathbb{R}^2$ then $A = 0$

25) Which of the followings are true?

a) $\cancel{\exists A \in M_3(\mathbb{R})}$

a) $\forall A \in M_3(\mathbb{R})$, $\exists u \in \mathbb{R}^3 \ni Au = \lambda u$, for some $\lambda \in \mathbb{R}$.

b) $\forall A \in M_2(\mathbb{R})$, $\exists u \in \mathbb{R}^2 \ni Au = 1 \cdot u$

c) $\langle Ax, x \rangle < 0, \ \forall x \in \mathbb{R}^3$ Then A is negative definite

d) $\cancel{\langle Ax, x \rangle > 0 \text{ is I.P} \Leftrightarrow |A| \neq 0}$.

d) $\langle Ax, x \rangle > 0 \ \forall x \in \mathbb{R}^3 \Leftrightarrow |A| \neq 0$.

(21) Let, $A \in M_2(\mathbb{R})$ be a matrix which is not a diagonal matrix. Which of the following statements are true?

a) If $tr(A) = -1$ and $det(A) = 1$ then $A^3 = I$

b) If $A^3 = I$ then $tr(A) = -1$ and $det(A) = 1$

c) If $A^3 = I$ then A is diagonalizable over $\mathbb{R}$

d) If $A^4 = I$ then A^2 is not diagonalizable over $\mathbb{R}$

(22) Let, $x \in \mathbb{R}^n$ be a non zero column vector. Define $A = xx^T \in M_n(\mathbb{R})$. Which of the followings are not incorrect?

a) rank (A) may be k, $1 < k < n$

b) rank(A) is exactly 1

c) If $x^T x = 1$ Then $I - 2A$ is ~~diagonal~~ orthogonal matrix.

d) If $I - 2A$ is ~~diagonal~~ orthogonal matrix Then $x^T x = 1$

23) Let, V be a finite dimensional real vector space and let, $A : V \to V$ be a linear map $\ni : A^2 = A$. Assume that $A \neq 0$ & $A \neq I$. Which of the followings statements are not true?

a) $\ker(A) \neq \{0\}$ b) $V = \ker(A) \oplus \text{Range}(A)$

c) The map $I + A$ is invertible

d) $\text{rank}(I - A) = \text{tr}(I - A)$

24) Let, V be a finite dimensional real vector space and let, f and g be non zero linear functionals on V. Assume that $\ker(f) \subset \ker(g)$. Which of the following statements are true?

a) $\ker(f) = \ker(g)$

b) $f = \lambda g$ for some real number $\lambda \neq 0$.

c) The linear map $A : V \to \mathbb{R}^2$ defined by $Ax = (f(x), g(x))$, $\forall x \in V$ is onto.

d) $\rho(f) \neq \rho(g)$ where, $\rho(A)$ is denotes rank of A.

(35) Let, W be the subspace of $\mathbb{R}^4$ spanned by the vector $(1,0,-1,1)$ and $(2,3,-1,2)$. write down a basis for $W^{\perp}$, the orthogonal complement of W in $\mathbb{R}^4$ with respect to the usual inner product on $\mathbb{R}^4$

(36) which of the following statements are not true?

a) Let, $A \in M_3(\mathbb{R})$ be such that $A^4 = I$, $A \neq \pm I$ Then $A^2 + I = 0$

b) Let, $A \in M_2(\mathbb{R})$ be such that $A^3 = I, A \neq I$ Then $A^2 + A + I = 0$

c) Let, $A \in M_3(\mathbb{R})$ be such that $A^3 = I$, $A \neq I$ Then $A^2 + A + I = 0$

d) Let, $A \in M_2(\mathbb{R})$ & λ be an eigen value of A Then λ^2 is an eigen value of A^2

32) Let, $A \in M_n(\mathbb{R})$, $n \geq 2$. Which of the following statements are true?

a) If $A^{2n} = 0$ then $A^n = 0$

b) If $A^2 = I$ then $A = \pm I$

c) If $A^{2n} = I$ then $A^n = \pm I$

d) If $A^2 = A$ then $A = 0$, (or) $A = I$

33) Which of the following statements are not true?

a) If $A \in M_n(\mathbb{R})$ $\ni$ $\langle Ax, x \rangle = 0$ $\forall x \in \mathbb{R}^n$ then $A = 0$

b) If $A \in M_n(\mathbb{C})$ $\ni$ $\langle Ax, x \rangle = 0$ $\forall x \in \mathbb{C}^n$ Then $A = 0$

c) If $A \in M_n(\mathbb{C})$ $\ni$ $\langle Ax, x \rangle \geq 0$ $\forall x \in \mathbb{C}^n$ Then $A = A^*$

d) If $A \in M_n(\mathbb{R})$ $\ni$ $\langle Ax, y \rangle = 0$, $\forall x, y \in \mathbb{R}^n$ Then $A = 0$

(34) Let $a_n \geq 0$ be a sequence of real numbers. Which of the following statements are true?

a) If $\sum_{n=1}^{\infty} a_n < \infty$ then $\sum_{n=1}^{\infty} a_n^K < \infty$, $K > 1$ & $K \in \mathbb{R}$

b) If $\sum_{n=1}^{\infty} a_n^3 < \infty$ then $\sum_{n=1}^{\infty} a_n < \infty$

c) If $\sum_{n=1}^{\infty} a_n^{3/2} < \infty$ then $\sum_{n=1}^{\infty} \frac{a_n}{n} < \infty$

d) If $\sum_{n=1}^{\infty} a_n < \infty$ then $\sum_{n=1}^{\infty} \cos n . a_n < \infty$

(35) Let A be an orthogonal 3x3 matrix with real entries. pick out the not true statements?

a) The determinant of A is rational number

b) $d(Ax, Ay) = d(x, y)$ $\forall x, y \in \mathbb{R}^3$, where $d(u, v)$ denotes the usual Eclidean distance b/w vectors u and v on $\mathbb{R}^3$

c) All entries of A are +ve

d) All the eigen value of A are real

Schedule 08:00 09:00 10:00 11:00 12:00 13:00 14:00 15:00 16:00 17:00

36) Which of the following matrices are non-singular?

a) $I+A$, where A be a non zero real $n\times n$ skew symmetric matrix, $n \geq 2$

b) Every skew-symmetric non zero real 5×5 matrix

c) Every skew symmetric non zero real 2×2 matrix

d) Every non zero symmetric matrix of order n, $n \geq 2$

37) Evaluate:

$$\lim_{n\to\infty} \left\{ \left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\cdots\left(1+\frac{n}{n}\right) \right\}^{1/n}$$

38) Let, A be an $n\times n$ upper triangular matrix with Complex entries. pick out the true statements?

a) If $A \neq 0$ & if $a_{ii} = 0 \ \forall \ 1 \leq i \leq n$ then $A^n = 0$

b) If $A \neq I$ and if $a_{ii} = 1 \ \forall \ 1 \leq i \leq n$ then A is not diagonalizable over $\mathbb{C}$

c) If $A \neq 0$ then A is invertible

d) If $A \neq 0$ then A is may be diagonalizable over $\mathbb{C}$

(29) Which of the following statements are true?

a) If a real symmetric $n \times n$ matrix is non nagative definite then all of its eigen values are non nagative

b) If a real symmetric $n \times n$ matrix has all its eigen values non-nagative then it is non-nagative definite

c) If $A \in M_n(\mathbb{R})$ then AA^T is non nagative definite

d) If $A \in M_n(\mathbb{R})$ then $A^T A$ is +ve definite if A is invertible

(45) Which of the following matrices is non negative definite?

a) $\begin{pmatrix} 5 & -3 \\ -3 & 5 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix}$

b) $\begin{pmatrix} 1 & -3 \\ -3 & 5 \end{pmatrix}$ d) $\begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}$