① Which of the followings are true??

a) Any non-singular $K \times K$ matrix with real entries can be made singular by changing exactly one entry

b) Let $A, B, C \in M_3(\mathbb{R})$ be such that A commutes with B, B-commutes with C and B is not Scalar matrix Then A is commutes with C

c) If $A = -A^T$ then $|A| = 0$

d) If $A = A^T$ and $A = -A^T$ then A is diagonal

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2. Which of the followings are true?

a) Let $A, B \in M_n(\mathbb{R}) \ni \text{rank}(A) = \text{rank}(B) = n$

Then $\text{rank}(A+B) \leq n$

b) Let $A, B \in M_n(\mathbb{R}) \ni \text{rank}(A) = \text{rank}(B) = n$

Then $\text{rank}(A^3 B^2 A) \neq n$.

c) If $\text{rank}(A_{3\times3}) = 2$ and $\text{rank}(B_{3\times3}) = 1$

Then $\text{rank}(AB) \geq 1$

d) If $A \in M_n(\mathbb{R})$ Then $\det(\text{adj}(A)) = |A|$.

3. Let $A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ Then A^{-1} is

a) diagonalizable over $\mathbb{R}$

b) A^{-1} has eigen value $\ni |\lambda| > 1$

c) minimal poly of A^{-1} is linear factor over $\mathbb{R}$

d) $\exists\ v_1, v_2 \in \mathbb{R}^6 \ni \langle v_1, v_2 \rangle = 0$, where $v_1 \neq v_2$

4. Let $A \in M_{m\times n}(\mathbb{R})$ and $b_0 \in \mathbb{R}^m$ suppose the system of eqns $Ax = b_0$ has unique soln Then

a) $Ax = b$ has soln for every $b \in \mathbb{R}^m$

b) If $Ax = b$ has a soln Then it is unique

c) $A^T x = 0$ has a unique soln

d) A has rank $m-1$

5) Let $M_2(\mathbb{R})$ be the space of all 2×2 real matrices Then pick correct statements

a) $\exists$ two different matrices $A, B \in M_2(\mathbb{R})$

$\ni AB + BA = I$

b) $\exists$ two different matrices $A, B \in M_2(\mathbb{R})$

$\ni AB - BA = I$

c) $\exists$ a singular matrix $A \in M_2(\mathbb{R}) \ni$

$A^2 + I = 0$

d) $\exists A \in M_2(\mathbb{R}) \ni A^{2+1} \neq 0$ but $A^4 = 0$

b) Let $M_{4\times 3}$ be a real matrix and let $\{e_1, e_2, e_3\}$ be standard basis of $\mathbb{R}^3$. Then

a) If Rank$(M) = 1$ then $\{Me_1, Me_2\}$ is L.I

b) If Rank$(M) = 2$ then $\{Me_1, Me_2\}$ is L.I

c) If Rank$(M) = 2$ then $\{Me_1, Me_3\}$ is L.I

d) If Rank$(M) = 3$ then $\{Me_1, Me_3\}$ is L.I

7) Let $f : [0,1] \to \mathbb{R}$ be a conts fn such that $f\left(\frac{1}{2}\right) = -\frac{1}{2}$ and

$|\cancel{f(x) - f(y)}|$

$$|f(x) - f(y) - (x-y)| \leq \sin\left(|x-y|^2\right) \quad \forall x, y \in [0,1]$$

Then $\int_0^1 f(x)\,dx$ is

a) $-1/2$ b) $-1/4$ c) $1/4$ d) $\frac{1}{2}$

8) Let $f(x,y) = \begin{cases} x^2\sin(1/x) + y^2\sin(1/y) & xy \neq 0 \\ x^2\sin(1/x) & \text{if } x \neq 0, y = 0 \\ y^2\sin(1/y) & \text{if } y \neq 0, x = 0 \\ 0 & x = y = 0 \end{cases}$

Then

a) f is conts at $(0,0)$

b) $\frac{\partial f}{\partial x}$ is conts but $\frac{\partial f}{\partial y}$ is not conts

c) f is not differentiable

d) f is differentiable but both $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are not conts.

9) Let $f(x) = 2x^3 - 9x^2 + 7$. Then

a) f is 1-1 in $[-1, 1]$

b) f is 1-1 in $[2, 4]$

c) f is not 1-1 in $[-4, 0]$

d) f is not 1-1 in $[0, 4]$

10) Let $A = \lim_{n\to\infty}\left(\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \cdots + \frac{(n-1)}{n^2}\right)$

$B = \lim_{n\to\infty}\left(\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}\right)$

Then a) $A > B$ b) $A < B$ c) $AB = \ln(\sqrt{2})$

d) $\frac{A}{B} = \ln(\sqrt{2})$

11) Let f be a real-valued fn on a real variable, such that $|f^{n}(0)| \leq K$, $\forall n \in \mathbb{N}, K > 0$. Then

a) $\left|\frac{f^{n}(0)}{n!}\right|^{1/n} \longrightarrow 0$ as $n \to \infty$

b) $\left|\frac{f^{n}(0)}{n!}\right|^{1/n} \longrightarrow \infty$ as $n \to \infty$

c) $f^{n}(x)$ exist $\forall x \in \mathbb{R}$ and $\forall n \in \mathbb{N}$

d) $\sum_{n=1}^{\infty} \frac{f^{n}(0)}{(n-1)!}$ is absolutely convergent

(12) Let $f : \mathbb{R} \to (0, \infty)$ be an infinitely differentiable fn with $\int_{-\infty}^{\infty} f(t)\,dt = 1$. Then,

a) $f(t)$ is bdd b) $\exists\, t_0 \in \mathbb{R} \ni f(t_0) \geq f(t),\ \forall t \in \mathbb{R}$

c) $\lim_{|t| \to \infty} f'(t) = 0$ d) $f''(a) = 0$ for some $a \in \mathbb{R}$.

13) ~~No~~ pick true statements

a) No group of order 625 is simple

b) $GL_2(\mathbb{R})$ is simple

c) Let G be a simple group of order 60. Then G has exactly six subgps of order 5

d) Let G be a group of order 60 Then G has exactly seven subgps of order 3.

14) Let v a fixed unit vectors in $\mathbb{R}^3$.

Let $M := I - 2vv^T$. Assume that v is column vector

Then

a) 0 is an eigenvalue b) $M^2 = I$

c) 1 is an eigenvalue

d) $\dim(E_{-1}) = 2$..

15) $X = C[0,1]$, $d(f,g) = \sup\{|f(x) - g(x)| \mid x \in [0,1]\}$

Let $Y = \{f \in X \mid f([0,1]) \subset [0,1]\}$

$Z = \{f \in X \mid f([0,1]) \subset [0,\frac{1}{2}) \cup (\frac{1}{2},1]\}$

Then

a) Y is compact b) X and Y are connected

c) Z is not compact d) Z is path-connected

16) Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ be analytic on the unit disk D with $a_1 \neq 0$. Suppose that $\cancel{\sum_{n\geq 2}^{\infty} |a_n|}$ $\sum_{n=2}^{\infty} |n a_n| < |a_1|$. Then

a) There are only finitely many such f

b) $|f'(z)| > 0, \ \forall z \in D$

c) if $z, w \in D$ are such that $z \neq w$ and $f(z) = f(w)$ then $a_1 = -\sum_{n\geq 2} a_n (z^{n-1} + z^{n-2}w + \cdots + w^{n-1})$

d) f is 1-1 on D

17. Which of ~~fo~~ the following are class equations for a finite group?

a) $1+3+3+3+3+13=39$

b) $1+1+2+2+2+2+2+2=14$

c) $1+1+3+3+7+7=22$

d) $1+1+1+1+2+5+5=15$

18. Which of the followings subsets are F_σ-type in $\mathbb{R}$.

a) $\{\pi+n \mid n\in\mathbb{Z}\}$ b) $\{1/n \mid n\in\mathbb{N}\}$

c) $\{m+n\sqrt{2} \mid m,n\in\mathbb{Z}\}$ d) $\{\sin(n) \mid n\in\mathbb{Z}\}$

19. which of the followings are not Uniformly conts on A?

a) $A=(0,1)$, $f(x)=\log(x)$.

b) $\tan(x)$, on $(0, \pi/2)$

c) $\sin(1/x)$ on $(\frac{1}{10}, 1)$

d) $\sin(x^2)$ on $\mathbb{R}$.

20) which of the followings are Uniformly conts?

a) $\sqrt{x}$ on $(0, \infty)$

b) $\sin^2(x)$ on $[0, \infty)$

c) $\sin(x \sin x)$

d) $e^{\sin(x^2)}$

(21) Consider the subsets A and B of $\mathbb{R}^2$ defined by $A = \{(x, x\sin(1/x)) \mid x \in (0,1]\}$ and $B = A \cup \{(0,0)\}$. Then

a) A is compact b) A is connected

c) B is compact d) B is connected.

(22) Let E be a connected subset of $\mathbb{R}$ with atleast two elements. Then the number of elements in $\overset{\circ}{E}$ is

a) Exactly two b) More than two but finite

c) Countably infinite

d) Uncountable.

(23) Which of the following are correct??

a) The set of interior points of $\mathbb{N}$ is empty

b) $\mathbb{Q}^{\circ} = \phi$ c) $\mathbb{Z}^{\circ} = \phi$ d) $\mathbb{R}^{\circ} = \phi$.

(24) The limit point of the seq $\{1 + 1/n\}$ is w.r.t discrete metric on $\mathbb{R}$?

a) ~~Countable in number~~ a) ϕ b) 2

b) ~~Uncountable in number~~

c) 0 d) 1.

(25) which of the followings are true?

a) $f(x) = \begin{cases} x^2-1 & x \in \mathbb{Q} \\ 0 & x \in \mathbb{Q}^c \end{cases}$ is conts exactly two points

b) $f(x) = \begin{cases} 0 & x \in \mathbb{Q} \\ \sin(x) & x \in \mathbb{Q}^c \end{cases}$ is conts infinitly many points of $\mathbb{R}$.

c) $f(x) = \begin{cases} 0 & x \in \mathbb{Q}^c \text{ or } x = 0 \\ \frac{1}{q} & \text{if } x = p/q,\ p \in \mathbb{Z}, q \in \mathbb{N}, \gcd(p,q)=1 \end{cases}$

is conts uncountable many points of $\mathbb{R}$.

d) every bijection $f: \mathbb{R} \to [0,\infty)$ has infinitly many points of discontinuity.

Q 26 Let $f:(0,\infty)\to\mathbb{R}$ be uniformly conts fn Then.

a) $\lim\limits_{x\to 0^+} f(x)$ and $\lim\limits_{x\to\infty} f(x)$ exists

b) $\lim\limits_{x\to 0^+} f(x)$ exists but $\lim\limits_{x\to\infty} f(x)$ need not exists

c) $\lim\limits_{x\to 0^+} f(x)$ need not exists but $\lim\limits_{x\to\infty} f(x)$ exists

d) Neither $\lim\limits_{x\to 0^+} f(x)$ nor $\lim\limits_{x\to\infty} f(x)$ need exist

Q. 27 Let $S = \{ f : \mathbb{R} \to \mathbb{R} \mid \exists\, \varepsilon > 0 \ni \forall\, \delta > 0,\ |x-y| < \delta \Rightarrow |f(x) - f(y)| < \varepsilon \}$

Then

a) $S = \{ f : \mathbb{R} \to \mathbb{R} \mid f \text{ is conts} \}$

b) $S = \{ f : \mathbb{R} \to \mathbb{R} \mid f \text{ is uniformly conts} \}$

c) $S = \{ f : \mathbb{R} \to \mathbb{R} \mid f \text{ is bdd} \}$

d) $S = \{ f : \mathbb{R} \to \mathbb{R} \mid f \text{ is constant} \}$.

28) Let $S = \{x \in [-1, 4] \mid \sin(x) > 0\}$

Then

a) $\inf\{S\} < 0$ b) $\sup\{S\}$ does not exist

c) $\sup\{S\} = \pi$ d) $\inf\{S\} = \pi/2$

(29) Let (X, d) be a metric space and Let $f: X \to X$ be a fn such that $d(f(x), f(y)) \leq d(x, y), \forall x, y \in X$

Then

a) f is conts b) f is injective

c) f is surjective d) f is neither injective nor surjective

30) Consider the seq $\{a_n\}$, where

$$a_n = 3 + 5(-1/2)^n + (-1)^n \left(\frac{1}{4} + (-1)^n 2/n\right)$$

Then the interval $\left(\lim_{n\to\infty} \inf\{a_n\}, \lim_{n\to\infty} \sup\{a_n\}\right)$ is

a) $(-2, 8)$ b) $\left(\frac{11}{4}, \frac{13}{4}\right)$ c) $(3, 5)$ d) $\left(\frac{1}{4}, 7/4\right)$

(31) If $f : [a,b] \longrightarrow \mathbb{R}$ be conts fn

and $|f(x) - f(y)| \leq ||x| - |y||$, $\forall x, y \in [a,b]$

Then

a) f is uniformly conts on $[a,b]$

b) f is diffble on $[a,b]$

c) f is R.I on $[a,b]$

d) f is bdd variation on $[a,b]$.

(32) Let $A \subseteq \mathbb{R}$ and let $f : \mathbb{R} \rightarrow \mathbb{R}$ be conts

Then

a) If A is closed then $f(A)$ is closed

b) If A is closed and bdd Then $f(A)$ is closed and bdd.

c) If A is bdd Then $f(A)$ is bdd

d) If A is bdd then $f^{-1}\{A\}$ is bdd.

33) In which of the following cases does there exists a conts and onto fn $f: X \to Y$

a) $X = (0,1)$, $Y = [0,1]$ b) $X = [0,1]$, $Y = (0,1]$

c) $X = (0,1)$, $Y = \mathbb{R}$ d) $X = (0,2)$, $Y = \{0,1\}$