Topology Test

1. Which of the following subsets are connected in $\mathbb{R}^2$ w.r.t supnorm metric

$d((x,y),(a,b)) = \max\{|x-a|, |y-b|\}$.

a) $\{(x,y) \in \mathbb{R}^2 \mid x < y\}$

b) $\{(x,y) \in \mathbb{R}^2 \mid xy = 0\}$

c) $\{(x,y) \in \mathbb{R}^2 \mid y = 1/x, x > 0\}$

d) $\{(x,y) \in \mathbb{R}^2 \mid ax + by = 1\}$

2) Which of the followings are Compact w.r.t usual top on $\mathbb{R}^3$

a) $\{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 - z^2 = 1\}$

b) $\{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 = 1\}$

c) $\{(x,y,z) \in \mathbb{R}^3 \mid 2x + 3y + 4z = 0\}$

d) $\{(x,y,z) \in \mathbb{R}^3 \mid \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{7} = 1\}$

3) Which of the followings sets are closed in $C([0,1])$ w.r.t supnorm topology?

a) $\{ f \in C([0,1]) \mid f(0) = f(1) = f(1/2) = 0 \}$

b) $\{ f \in C([0,1]) \mid f'(x) = f(x) \}$

c) $\{ f \in C([0,1]) \mid f(x) = 0, \forall x \in [0, 1/2] \}$

d) $\{ f \in C([0,1]) \mid f(1/n) = 0, \forall n \in \mathbb{N} \}$

4) Which of the followings are open in $C([0,1])$ w.r.t 1-norm topology?

i.e, $d(f,g) = \int_0^1 |f-g| dt$

a. $\{ f \mid f(1) = 2 \}$

b. $\{ f \mid f(0) = f(1) = 4 \}$

c) $\{ f \mid f'(x) = f(x) \}$

d) $\{ f \mid f(\frac{1}{n}) = 1, \forall n \in \mathbb{N} \}$

5. Which of the followings are path-connected sub space of $\mathbb{R}^3$ w.r.t usual top?

a) $\{(x,y,z) \mid x=y=0\}$

b) $\{(x,y,z) \mid x^2+y^2+z^2=1\}$

c) $\{(x,y,z) \mid x^2+y^2+z^2 \neq 1\}$

d) $\{(x,y,z) \mid 2x+2y+z \neq 0\}$

6. Which of the followings are Hausdorff space?

a) $\{\phi, X, \{a\}, \{b\}, \{a,b\}\}$

When $X = 1,2,3,4,\ldots$

b) Let X be finite set, w.r.t Co-finite topology on X:

c) ~~Let~~ X be infinite ~~the and~~

~~$(X, d\text{-disc}) \cong$~~ $(X, \mathcal{J}_f)$ Then $(X, \mathcal{J}_f)$ Hausdorff sp.

d) $X = \mathbb{R}$, $\mathcal{J} = \{(a,\infty) \mid a \in \mathbb{R}\} \cup \{\phi\} \cup \mathbb{R}$

Then $(X, \mathcal{J})$ - is Hausdorff space?

7. Which of the following are have non-empty interior w.r.t usual top in $M_2(\mathbb{R})$?

a) $S = \{A \in M_2(\mathbb{R}) \mid tr(A) = 0\}$

b) $S = \{A \in M_2(\mathbb{R}) \mid |A| \neq 0\}$

c) $S = \{A \in M_2(\mathbb{R}) \mid |A| = \pm 1$

d) $S = \{A \in M_2(\mathbb{R}) \mid A \neq A^T\}$

8. Which of the followings are dense in $M_2(\mathbb{R})$ w.r.t usual topology ?

a) $\{A \in M_2(\mathbb{R}) \mid A = -A^T\}$

b) $\{A \in M_2(\mathbb{R}) \mid tr(A) = 0\}$

c) $\{A \in M_2(\mathbb{R}) \mid A + A^T = 0\}$

d) $\{A \in M_2(\mathbb{R}) \mid A^2 = I\}$

9. $Y = X = \mathbb{R}$, w.r.t. usual top.

$f : X \longrightarrow Y$.

Which of the followings are homeomorphism

a) $f(x) = ax + b$, $a \neq 0$.

b) $f(x) = x^3$

c) $f(x) = x^3 + 1$

d) $f(x) = x^5$

10. Which of the followings are T_1-space?

a) $(X = \mathbb{R}, \mathcal{J}_f)$

b) $(X = \mathbb{R}, \mathcal{J}$- discrete top$)$

c) $(X = \mathbb{R}, \mathcal{J}$ - V.I.P containing '0'$)$

d) $(X = \mathbb{R}, \mathcal{J}_{\text{co-countable top}})$