Real Analysis

1) Let $f: \mathbb{R} \to \mathbb{R}$ be a continuously diffble fn $\ni |f(x)-f(y)| \geq |x-y|$ $\forall x, y \in \mathbb{R}$

Then the eqn $f'(x) = \frac{1}{2}$

a) has exactly one soln

b) has No soln

c) has a Countably infinite number of soln

d) has uncountably many soln.

2) How many zeros does the fn $f(x) = e^x - 3x^2$ have in $\mathbb{R}$?

a) 0 b) 1 c) 2 d) 3

3) An even fn $f(x)$ has left derivative 5 at $x=0$ then

a) the right derivative of $f(x)$ at $x=0$ need not exist.

b) The right derivative of $f(x)=0$ exist and is equal to 5

c) The right derivative of $f(x)$ at $x=0$ exist and is equal to -5

d) None of the above is necessarily true.

4) Let $p(x)$ be a poly of degree 3 with real co-effts. which of the following is possible ??

a) $p(x)$ has NO real roots

b) $p(x)$ has exactly 2 real root

c) $p(1)=-1$, $p(2)=1$, $p(3)=11$ and $p(4)=35$

d) $i-1$ and $i+1$ are root of $p(x)$.

5) Let $f:[0,1] \to \mathbb{R}$ be a fixed continuous fn such that f is diffble on $(0,1)$ and $f(0)=f(1)=0$ Then the eqn $f(x)=f'(x)$ admits

a) NO soln $x \in (0,1)$ b) More than one soln $x \in (0,1)$

c) Exactly one soln $x \in (0,1)$

d) Atleast one soln $x \in (0,1)$

6) Let $f:\mathbb{R} \to \mathbb{R}$ be diffble fn such that $\lim_{x \to \infty} f'(x) = 1$. Then

a) f is bdd b) f is increasing

c) f is unbdd d) f' is bdd.

7) Let $g: \mathbb{R} \to \mathbb{R}$ be defined by

$$g(x) = \begin{cases} \frac{\sin(x)}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

Then

a) g is not cont^s

b) g is cont^s but not diffble

c) g is diffble

d) g is not bdd

8) Let $f: \mathbb{R} \to \mathbb{R}$ be a twice diffble fn with $f(0) = f(1) = f'(0) = 0$. Then

a) $f''(x) \equiv 0$, b) $f''(0) = 0$. c) $f''(x) = 0$ for some $x \in (0,1)$

d) $f''(x) \neq 0, \forall x \in \mathbb{R}$.

9) Let the ff, $f: \mathbb{R} \to \mathbb{R}$ defined as

$$f(x) = \begin{cases} x^2 & x \in (0,2) \cap \mathbb{Q} \\ 2x-1 & x \in (0,2) \cap \mathbb{Q}^c \end{cases}$$

Then which of the following is not true

a) f is cts at $x=1$ b) f is diffble at $x=1$

c) f is not diffble at $x=1$

d) f is diffble only at $x=1$.

10) Let $f: \mathbb{R} \to \mathbb{R}$ be a diffble ff and consider the following

(i) $|f(x)-f(y)| \leq 1$, $\forall x, y \in \mathbb{R}$ with $|x-y| \leq 1$

(ii) $|f'(x)| \leq 1$, $\forall x \in \mathbb{R}$.

Then we have

a) (i) $\Rightarrow$ (ii) but (ii) $\not\Rightarrow$ (i) b) (ii) $\Rightarrow$ (i) but (i) $\not\Rightarrow$ (ii)

c) (i) $\Rightarrow$ (ii) and (ii) $\Rightarrow$ (i) d) (i) $\not\Rightarrow$ (ii) and (ii) $\not\Rightarrow$ (i)

11) which of the following are uniformly cont^{s} on (0,1)

a) $\sin(1/x)$ b) $x\sin(\frac{1}{\sqrt{x}})$ c) e^{-1/x^2}

d) $x\sin(1/x^{10})$

12) which of the following are uniformly cont^{s}

a) $\cos(x)\cos(\frac{\pi}{x})$ on (0,1)

b) $\sin(x)\cos(\frac{\pi}{x})$ on (0,1)

c) $f(x) = \sin^2(x)$ $x \in [0,\infty)$.

d) $f(x) = \sin(x^2)$ on $(0,\infty)$

13) Which of the followings are true??

a) $|\cos^2(x) - \cos^2(y)| \leq |x-y| \; ; \forall x,y \in \mathbb{R}$

b) If $f: \mathbb{R} \to \mathbb{R}$ ∋ $|f(x)-f(y)| \leq |x-y|^{\sqrt{2}}, \forall x,y \in \mathbb{R}$
then f must be constant fn.

c) Let $f: \mathbb{R} \to \mathbb{R}$ be continuously diffble
and such that $|f'(x)| \leq \frac{4}{5}, \forall x \in \mathbb{R}$
Then $\exists!_{b}\, x \in \mathbb{R}$ ∋ $f(x) = x$.

d) If $f: [a,b] \to (a,b)$ be onto
Then f has fixed point