Real Analysis test

(1) which of the followings are true ??

If $\{a_n\}$, $\{b_n\}$ are real seq such that $\{a_n\} \longrightarrow l$ but $\{b_n\}$ diverges Then

a) $\{a_n \pm b_n\}$ is convergent

b) $\{a_n b_n\}$ Can be divergent

c) $\{a_n b_n\}$ Can be convergent

d) $\{a_n^2 + b_n\}$ may be divergent.

(2) Which of the followings are not true?

a) If $\{a_n^2\}$ convergent then $\{a_n\}$ is convergent

b) If $\{a_n\}$ is real seq and $\{a_n^2+1\}$ is convergent then $\{a_n\}$ is necessarily convergent.

c) If $\{a_n\}$ is real seq and $\{b_n\}$ is also real seq and if $\{a_n+b_n\}$ is convergent then $\{a_n\}$ or $\{b_n\}$ is convergent.

d) If $\{a_n\}$ is oscilates and $\{b_n\}$ convergent Then $\{a_n \pm b_n\}$ is oscilates.

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(3) Which of the followings are true?

a) $X = \mathbb{R}$, $d(x,y) = \begin{cases} \frac{1}{2} & x \neq y \\ 0 & x = y \end{cases}$

(X, d)-metric space?

b) $X = \mathbb{R}$, $d_1(x,y) = \dfrac{|x-y|}{1+|x-y|}$

Let $\{a_n\}$ be a seq in $\mathbb{R}$ $\ni$ $a_n \to l$ w.r.t usual metric then $\{a_n\}$ is convergent w.r.t d_1-metric.

c) $X = \mathbb{R}$, d-discrete metric

Let $\{a_n\}$ be a seq in such that $a_n \to l$ w.r.t usual metric then $\{a_n\}$ is convergent w.r.t discrete metric.

d) $X = \mathbb{R}$, d-discrete metric

Let $\{a_n\}$ be a seq in $\mathbb{R}$ $\ni$ $\{a_n\}$ is convergent w.r.t. discrete metric then $\{a_n\}$ is convergent w.r.t usual metric.

Choose only yes options.

a) Does $\exists$ a sequence $\{a_n\}$ in $\mathbb{R}$ $\ni$ $\{a_n^2\}$ diverges to ∞.

b) Does $\exists$ a seq $\{a_n\}$ in $\mathbb{R}$ $\ni$ $\{a_n^2\}$ is convergent but $\{a_n\}$ is divergent

c) Does $\exists$ a seq $\{a_n\}$ in $\mathbb{R}$ $\ni$ $\{a_n+b_n\}$ is convergent for every real seq $\{b_n\}$ is $\mathbb{R}$.

d) Does $\exists$ a seq $\{a_n\}$, ~~in~~ $\{b_n\}$ in $\mathbb{R}$ $\ni$ $\{a_n+b_n\} \to 10$ and $\{a_n\} \to 5$, $\{b_n\} \to 2$

5) Choose only yes options?

a) Does $\exists$ a metric in $\mathbb{R}$ $\ni$ every seq in $\mathbb{R}$ is convergent.

b) Does $\exists$ a seq $\{a_n\}$ converges to l and $\{-a_n\}$ converges l, for some $l \neq 0$.

c) Does $\exists$ a seq $\{a_n\}$ converges and $\{a_n^2\}$ diverges in $\mathbb{R}$?

d) Does $\exists$ a metric in $\mathbb{R}$ $\ni$ $\{a_n\} = \{n\}$ is converges

e) None of the above.

b) Which of the followings are true?

a) If A, B are subset of $\mathbb{R}$ and

A is countable ~~and~~ Then $A \cup B$ is Countable

b) ~~let~~ If $A \subseteq \mathbb{R}$ and A is uncountable

then A^c is Countable

c) If $A \subseteq X$ and A, A^c are Countable

in X then X is Countable.

d) Suppose $A \subseteq \mathbb{R}$ and $A \cup B$ is countable for every subset B of $\mathbb{R}$ Then

A is Countable.

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(a) $V = \{A \in M_2(\mathbb{R}) \mid tr(A) = 0\}$

(b) $V = \{A \in M_2(\mathbb{R}) \mid P_A(\lambda) = 0 \;\forall \lambda \in \mathbb{R}\}$

$P_A(x)$ - Char poly of A

(c) $V = \{A \in M_n(\mathbb{R}) \mid P_A(2) = 0,\ P_A(1) = 0\}$.

where $P_A(x)$ - Char poly of A.

(d) $V = \{A \in M_2(\mathbb{R}) \mid A - I = 0\}$

8. Which of the followings are uncountable ?

a) $V = \{ f \in C([0,1]) \mid f(0) = 0 \}$

b) $V = \{ f \in C([0,1]) \mid f(\alpha) = 0, \ \forall \alpha \in (0,1) \}$

c) $V = \{ f \in C([0,1]) \mid f(\alpha) = 0, \ \forall \alpha \in (0,1) \cap \mathbb{Q} \}$.

d) $V = \{ f \in C([0,1]) \mid f'(\alpha) = 0, \ \forall \alpha \in [0,1] \}$

9. Which of the following are not Countable ?

a) $V = \{ \{a_n\} \text{ in } \mathbb{R} \mid \{a_n\} \longrightarrow 5 \}$

b) $V = \{ \{a_n\} \text{ in } \mathbb{R} \mid \{a_n^2\} \longrightarrow 5 \}$

c) $V = \{ \{a_n\} \text{ in } \mathbb{R} \mid a_n \longrightarrow \infty \}$

d) $V = \{ \{a_n\} \text{ in } \mathbb{R} \mid \cancel{a_n \longrightarrow 0}, a_n^2 \longrightarrow 0 \}$

10) Which of the followings are true?

a) Let $\{a_n\}$ is bdd above and $\lim\limits_{n\to\infty} \sup\{a_n\} = 5$

Then $\{a_n\} \to 5$.

b) Let $\{a_n\}$ is bdd above and $\lim\limits_{n\to\infty} \sup\{a_n\} = 5$

then $\lim\limits_{n\to\infty} \sup\{a_{n_k}\} = 5$ for some subseq $\{a_{n_k}\}$

c) Let $\{a_n\}$ be a seq in $\mathbb{R}$ and $\lim\limits_{n\to\infty} \sup\{a_{n_k}\} = l$

for all subseq of $\{a_{n_k}\}$ in $\{a_n\}$ Then

$\{a_n\}$ is convergent.

d) Let $\{a_n\}$ be bdd seq and suppose

$\lim\limits_{n\to\infty} \sup\{a_{2n}\} = \lim\limits_{n\to\infty} \inf\{a_{2n-1}\} = l$ in $\mathbb{R}$.

Then $a_n \to l$.

(11) which of the followings are true?

a) Let $\{a_n\}$ be bdd seq in $\mathbb{R}$ and $\lim_{n\to\infty} \inf\{a_n\} = 2$

$\lim_{n\to\infty} \sup\{a_n\} = -2$ Then $\{a_n\}$ is convergent.

b) Let $\{a_n\}$ be a seq in $\mathbb{R}$ and if $\lim_{n\to\infty} \sup\{a_n\} = 5$.

then $\exists$ a subseq $\{a_{n_k}\}$ in $\{a_n\}$ $\ni$ $a_{n_k} \longrightarrow 5$.

c) suppose $\lim_{n\to\infty} \inf\{a_n\} = 2$, then $\exists$ a subseq $\{a_{n_k}\}$

in $\{a_n\}$ $\ni$ $a_{n_k} \longrightarrow 2$.

d) suppose $\lim_{n\to\infty} \cancel{\sup}\{a_n\} = l_1$ and $\lim_{n\to\infty} \sup\{b_n\} = l_2$

then $\lim_{n\to\infty} \sup\{a_n + b_n\} = l_1 + l_2$.