Part B

1. Let A be a 2×2 matrix with real entries such that $|A| = \lambda_1\lambda_2 \neq 0$, $tr(A) = \lambda_1+\lambda_2$. Then $tr(A^2)$ is

a) $\lambda_1^2+\lambda_2^2$ b) $(\lambda_1+\lambda_2)^2 + 2\lambda_1\lambda_2$ c) $(\lambda_1+\lambda_2)(\lambda_1+\lambda_2)$

d) None.

2) Let A be a 5×5 matrix with real entries such that A has 3-distinct eigenvalues (non-zero eigenvalues) Then

a) $Rank(A) \geq 2$ b) $Rank(A) < 3$

c) $\dim(Ker(A)) \geq 1$. d) A is Never diagonalizable over $\mathbb{R}$.

3) Which of the following real quadratic forms on $\mathbb{R}^2$ is +ve definite

a) $Q(x,y) = xy$ b) $Q(x,y) = x^2 - xy + y^2$

c) $Q(x,y) = x^2 + 2xy + y^2$ d) $Q(x,y) = x^2 + xy + y^2$

4) Let A be a 5×5 matrix $\ni A^2 = A$. Then

a) $tr(A) \neq 0$ b) $tr(A) \neq rank(A)$

c) $tr(A) \neq rank(A^5)$ d) $tr(A^8) = rank(A^{100})$

2) Let $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a L.T $\ni$

$T\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}-1\\1\\1\end{pmatrix}$, $T^2\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}$, $T^2\begin{pmatrix}1\\1\\2\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}$

Then rank (T^2) is

a) 1 b) 2 c) 3 d) None.

b) Let A be a 3×4 matrix and B be a 4×3 matrix with real entries such that AB is non-singular. Then

a) $\ker(A) = 0$, b) $|BA| \neq 0$ c) $\ker(B) = \{0\}$

d) $\operatorname{rank}(AB) = \operatorname{rank}(BA)$.

7) Suppose that M is a 5×5 matrix with real entries and $p(\lambda) \triangleq \det(\lambda I - M)$

Then

a) $p(0) = |M|$ b) every eigen values of M is real if $p(1)+p(2)=0$
$p(2)+p(3)=0$

c) M^{-1} is necessarily a poly in M of degree 4 if M is invertiable

d) M is not inverttiable if $M^2 - 2M = 0$.

8) $T : (C[0,1], \|\cdot\|_\infty) \to (C[0,1], \|\cdot\|_\infty)$ be the L.O defined by $T(f(x)) = \int_0^x e^{-y} f(y)\,dy$

Then

a) $\|T\| = 1$ b) $I - T$ is inverttiable

c) T is surjective d) $\|I+T\| = 1 + \|T\|$.

9) Let $\dim(V) = n$ ~~and let V V.S.~~ let V be a vector space over $\mathbb{C}$. and $T : V \to V$ is a L.T $\ni$ Range (T) = Nullspace(T)

Then which of the followings are not true?

a) $\dim(V) = 2k$, for some $k \in \mathbb{N}$

b) 0 is the only eigen value of T

c) Both 0 and 1 are eigenvalues of T

d) $T^2 = 0$.

10) Let $P \in M_{m\times n}(\mathbb{R})$. consider the following statements

I. If $XPY = 0 \ \forall \ X \in M_{1\times m}(\mathbb{R})$ and $Y \in M_{n\times 1}(\mathbb{R})$ then $P = 0$

II. If $m = n$, P is symmetric and $P^2 = 0$ then $P = 0$

Then

a) I, II are true b) I - true but II is false

c) I is false but II - true

d) Both I and II are false.

11) Let $\dim(V) = n$, $\mathbb{F} = \mathbb{R}$, If W_1, W_2, W_3 are subspace of V. Then

a) If $W_1 + W_2 + W_3 = V$ then $\text{span}\{W_1 \cup W_2\} \cup \text{span}\{W_2 \cup W_3\} \cup \text{span}\{W_3 \cup W_1\} = V$

b) If $W_1 \cap W_2 = \{0\}$ and $W_1 \cap W_3 = \{0\}$ then $W_1 \cap \{W_2 + W_3\} = \{0\}$

c) If $W_1 + W_2 = W_1 + W_3$ then $W_2 = W_3$

d) If $W_1 \neq V$. then $\text{span}\{V \setminus W_1\} = V$.

12) Let A be an $n\times n$ matrix with rank$(A)=k$. Then

a) If A has real entries then AA^T necessarily has rank$(AA^T)=k$.

b) If A has complex entries then AA^T necessarily has rank$(AAT)=k$.

c) AA^T need not +ve definite matrix if $|A|\neq 0$

d) If $k=n$ then $\ker(A)\neq \ker(A^T)$.

13) Let $S=\{\lambda\in\mathbb{R} \mid tr(A)=\lambda$ for some $A\in M_{6\times 6}(\mathbb{R}) \ni A^2=A\}$

Then which of the following describes S?

a) $S=\{0,2,3,6\}$ b) $\{0,\frac{1}{2},\frac{1}{5},\frac{1}{3},6\}$

c) $S=$ set of all primes which are less than ≤ 7

d) $S=\{0,1,2,3,5,6\}$.

14) let $\dim(V)=n$, let $T: V\to V$ be a L.T $\ni$ Rank$(T)\leq$ Rank(T^4)

Then

a) $\ker(T)=$ Range(T) b) $\ker(T)\cap$ Range$(T)=\{0\}$

c) $\exists$ a non-zero subspace W of $V \ni \ker(T)\cap$ Range(T) is W

d) $\ker(T)\subseteq$ Range(T). e) none

15) Let $C^{\infty}(\mathbb{R}) = V$ be v.s. over $\mathbb{R}$. Let $T: V \to V$ be a L.T defined by $T(f) = f + f'$ is,

a) T is 1-1 but not onto c) Neither 1-1 nor onto

b) T is onto but not 1-1 d) Both T is 1-1 and onto

16) The matrices $A = \begin{pmatrix} 0 & i & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 & 0 \\ -i & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

Notation $\sim$ - similar

Then a) $A \sim B$ b) $A \nsim B$ c) $A^T \nsim B$

d) $A^2 \sim A$.

17) Suppose A, B, C are 3×3 real matrices with $\text{Rank}(A) = 2$, $\text{Rank}(B) = 1$, $\text{Rank}(C) = 2$

Then a) $\text{Rank}(ABC) = 1$, b) $\text{Rank}(ABC) = 2$

c) $\text{Rank}(ABC) = 3$ d) $\text{Rank}(AB) = 2$.

18) Let A be a 2×2 complex matrix $\ni$ A is invertiable and diagonalizable And $\cancel{A^2, A}$ A^2, A has same char poly then a) $A = I$, b) $A \neq I$ c) No A exist

d) $A^2 = I$.

19) If $A \in M_{10}(\mathbb{R}) \ni A^2 + A + I = 0$ Then

a) $\text{Rank}(A^2) = \text{Rank}(A)$ b) A is diagonalizable over $\mathbb{R}$

c) $\text{Rank}(A) \leq \text{tr}(A)$ d) A^2 is diagonalizable over $\mathbb{R}$.

20) $S = \{A \in M_n(\mathbb{F}) \mid A$ has only eigenvalues $\}$

Then

a) $(A-5I)^n = 0 \ \forall A \in S$.

b) $A^n = 5^{n-1} I \ \forall A \in S$.

c) S has skew-matrix

d) $|A-5I| \neq 0$.

21 $A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

a) Char poly A = Minimal poly of A.

b) Minimal poly is linear factor

c) A^2 is diagonalizable over $\mathbb{R}$.

d) Minimal poly is $(x-2)^2(x-5)$.

22) Let A be a non-zero 5×5 complex matrix ∋ $A^2 = 0$. Then largest possible rank(A) is

a) 2 b) 3 c) 4 d) ~~Rank (A)~~ 1

23) Let A be a 2×2 orthogonal symmetric matrix such that $|A| = -1$ Then

a) $tr(A) \neq 0$, b) $tr(A) = rank(A)$

c) $\exists \ v \in V \ni Av = v$, $v \neq 0$

d) ~~dim (E)~~ d) $\dim(E_1) = 2$.

24) which of the following is non diagonalizable matrix?

a) $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$ c) $\begin{pmatrix} 2 & 3 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{pmatrix}$ d) $\begin{pmatrix} 2 & 0 & 0 \\ 2 & 3 & 0 \\ 3 & 2 & 1 \end{pmatrix}$

25) Which of the following is a linear functional on $\mathbb{R}^3$ which annihilates the subspace $\{(x, y, z) \mid z = 0\}$

a) $f(x, y, z) = x - y + z$ b) $f(x, y, z) = x - z$

c) $f(x, y, z) = y - z$ d) $f(x, y, z) = z$

(26) Let A be a 5×7 matrix and B be a 7×5 matrix over $\mathbb{R}$. Then

a) $|AB| \neq 0$

b) $|AB| = 0$

c) $|BA| \neq 0$

d) $|BA| = 0$

(27) Let A and B be $n \times n$ matrices. Which of the following equals $tr(A^3B^2)$

a) $tr(A(AB)^2)$ b) $tr(B^2A^2)$

c) $tr(BABABA)$ d) None.

(28) Let $T: P_3(\mathbb{R}) \longrightarrow P_3(\mathbb{R})$ be a L.O on $P_3(\mathbb{R})$. Suppose $T(1)=0$, $T^2(x)=0$, $T^3(x^2)=0$, $T^4(x^3)=0$

Then which of the following are false?

a) $tr(T) \neq 0$ b) $|T| \neq 0$

c) T diagonalizable d) T is never diagonalizable over $\mathbb{R}$

24) Let A be an 10×15 matrix of rank 9 with real entries then

a) $Ax = b$ has soln for every b.

b) $Ax = b$ has No soln for some $b \in Im(A)$.

c) $Ax = b$ has unique soln for some b.

d) $Ax = b$ has soln. Then it ~~has~~ has infinitely many soln

35) Let $S = \{ A \in M_3(\mathbb{R}) \,/\, AA^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \}$

Then which of the followings are false?

a) S - contains rank 1 matrix

b) S - contains rank 2 matrix

c) S - contains a nilpotent matrix

d) S - contains rank 3 matrix.