① Let $X = (C[0,1], \|\cdot\|_\infty)$, $Y$ = Vector subspace of all diff'ble f's on $[0,1]$ w.r.t $\|\cdot\|_\infty$

suppose $D: X \longrightarrow Y \ni D(f) = f'$

Then

a) D is conts on X ~~b) D is diff'ble map.~~

b) D is Uniformly conts on X c) D is 1-1 fn.

d) D is onto.

② Let $f, g : \mathbb{R} \longrightarrow \mathbb{R}$ be conts fns whose graphs donot intersect Then for which fn below the graph lies entirely on one side of the x-axis

a) $f$ b) $g+f$ c) $g-f$ d) $gf$

③ The real root of $x^3+x+1=0$ lies between

a) $-2$ and $-1$ b) $-1$ and $0$

c) $1$ and $2$ d) $2$ and $3$

4) Suppose $f : [0, \infty) \to \mathbb{R}$ is conts

(a) Then $f$ is uniformly conts on $[k, \infty)$, $k > 0$

b) Then $|f|$ is U.C on $[0, \infty)$

c) If $f$ is uniformly conts on $[k, \infty)$ for some $k > 0$. Then $f$ is uniformly conts on $[0, \infty)$

d) If $f$ is decreasing then $f$ is uniformly conts on $[0, \infty)$

5) Let $f : [-1, 1] \to \mathbb{R}$ be a conts fn

$\ni \int_{-1}^{1} f(x)\,dx = 0$ Then

a) $f \equiv 0$ b) $f$ is an odd fn

c) $\int_{-1/2}^{1/2} f(x)\,dx = 0$ d) None ~~of the~~

6) Let $X \neq \phi$, $f: X \to X$ be a fn and let $A, B$ be subset of $X$. Then $f(A \cap B) = f(A) \cap f(B)$ is true

a) always  b) if $f$ is 1-1

c) If $f$ is onto  d) If $A \cup B = X$.

7) which of the followings are true?

Let $f$ be a fn from $[-1, 1]$ to $\mathbb{R}$

a) If $f$ is diffble at 0 with $f'(0) = 0$ then $f(0) = 0$

b) If $f(0) = 0$ then $f$ is diffble at 0

c) If $f(0) = 0$ then the x-axis is tangent to the graph of $f$ at 0.

8) Let $f(x) = 1 - x^{2/3}$ for $x \in [-1, 1]$ Then

a) $f'(c) = 0$ for some $c \in (-1, 0)$

b) $f'(c) = 0$ for some $c \in (0, 1)$

c) $f'(x)$ is never zero in $(-1, 0]$

d) $f'(x)$ is zero in $(0, 1)$ at two points.

9) Consider the following three conditions on a set $A \subset \mathbb{N}$

condition (1) := $A = \{ma + nb \mid m, n \in \mathbb{N}\}$ for some $a, b \in \mathbb{N}$ with $\gcd(a, b) = 1$.

condition (2) := $\mathbb{N} \setminus A$ is finite

condition (3) := $\exists\, n_0 \in \mathbb{N} \ni A = \{n \in \mathbb{N} \mid n \geq n_0\}$

Then

(a) (2) $\Rightarrow$ (3)  b) (3) $\Rightarrow$ (1) $\Rightarrow$ (2)

c) (1) $\Rightarrow$ (2) $\Rightarrow$ (3)  d) (1) $\Rightarrow$ (2)

10) Let $x_n = \frac{1}{n^2+1}$ and $y_n = \frac{1}{n \log(n)}$ Then

a) $\sum x_n$ is convergent, $\sum y_n$ is divergent

b) $\sum x_n$ is convergent, $\sum y_n$ is convergent

c) $\sum x_n$ is divergent, $\sum y_n$ is convergent

d) $\sum x_n$ is divergent, $\sum y_n$ is divergent.

11) Let $p(x)$ be a poly $\ni \alpha \in \mathbb{R}$, $p(\alpha)=0$ iff $\alpha = 2$ or $4$. Then

a) $\deg(p(x)) = 2$  b) $p(3) < 0$  d) $p'(\alpha) = 0$ for some $\alpha<4$

d) $p(x)$ is of the form $c(x-2)^n(x-4)^m$, where $c$ is a constant.

(12) Let $f: \mathbb{R} \to \mathbb{R}$ be conts fn with $f(0)=0$ and let $\{x_n\}$ be a seq in $\mathbb{R}$ with $\lim_{n\to\infty} f(x_n) = 0$. Then

a) $\lim_{n\to\infty} x_n = 0$  b) $\lim_{n\to\infty} x_{n_k} = 0$ for some subseq $\{x_{n_k}\}$

c) $\{x_n\}$ is bdd.

d) None.

(13) Evaluate $\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{k^2}{9n^2}$ is

(a) $\frac{1}{27}$  b) $\frac{1}{23}$  c) $\frac{1}{19}$  d) $\frac{22}{11}$

14) Which of the followings are uniformly conts on $(0,1)$?

a) $\frac{1}{x}$  b) $\frac{\sin(x^2)}{\sin^2(x)}$  c) $e^{\frac{1}{x}}$  d) $\sin(\frac{1}{x})$

15) $f(x) = \begin{cases} 1, & x \in \mathbb{Q} \cap [a,b] \\ 0 & x \in \mathbb{Q}^c \cap [a,b] \end{cases}$

Then ~~f is~~ $g(x) = (x^2+1) f(x)$ is

a) $g$ is R.I on $[a,b]$

b) $g^2$ is R.I on $[a,b]$

c) $f+g$ is R.I on $[a,b]$

d) $\not\exists\, g_1 : [a,b] \to \mathbb{R} \ni$

$\ni f + g_1$ is ~~No~~ R.I on $[a,b]$

16) Consider $f : \mathbb{R} \to \mathbb{R}$ defined as

$f(x) = \begin{cases} x^2 - 1 & \text{if } x \in \mathbb{Q} \\ 0 & x \in \mathbb{Q}^c \end{cases}$ Then

a) $f$ is discont<sup></sup>s only at $x = 1$

b) $f$ is cont<sup></sup>s only at $x = 1$

c) $f$ is disconts at $x = -1$ and $x = 1$

d) $f$ is disconts at $x = -1$

17) consider $f : [-2,1] \to \mathbb{R}$ defined as

$f(x) = |x| \;\forall x \in [-2,1]$. The total variation

of $f$ over $[-2,1]$ is

a) 0 b) 2 c) 1 d) 3.

17. If $f(x) = \begin{cases} 1-2x & \text{if } x \le 1 \\ 3x-4 & \text{if } x > 1 \end{cases}$ Then

a) f is conts at all points except at $x=1$

b) f is conts on $\mathbb{R}$ but not diffble at any point of $\mathbb{R}$

c) f is conts on $\mathbb{R}$ and diffble at all points except $x=1$

d) f is disconts at $x=1$.

18. $f(x) = \begin{cases} x^2+1 & \text{if } 0 \le x \le 1 \\ 2e^{x-1} & \text{if } x > 1 \end{cases}$

a) is conts and monotone increasing on $\mathbb{R}$

b) is disconts at $x=1$

c) is conts but not monotone increasing on $\mathbb{R}$

d) is monotone increasing but not conts on $\mathbb{R}$.

19. If $X = [0,1]$ and $Y = [0,1] \cup [2,3]$ and $f: X \to Y$ is a map then

a) f can't be conts   b) f can't be onto

c) f can't be 1-1   d) f can't be monotone increasing.

20) Let $\chi_E(x) = \begin{cases} 1 & x \in E \\ 0 & x \in E^c \end{cases}$

If $f(x) = 2\chi_{[0,\frac{1}{2}]} - 3\chi_{[\frac{1}{2},1]} + \chi_{[1,2]}$

Then $\int_0^2 f(x)\,dx$ is

a) $2/3$ b) $3/2$ c) $0$ d) $5/3$

21) If $f(x) = 5x^5 - 4x^4 + 3x^2 - 6x + 1$. Then

a) $f$ has a zero in $(0,1)$ b) $f(-1) < 0$

c) $f$ has 25 zeroes on $\mathbb{R}$ d) $f$ is diffble on $\mathbb{R}$

22) If $f : [a,b] \longrightarrow \mathbb{R}$ is a bdd R.I fn and $\int_a^b f(x)\,dx = 0$ Then

a) $f \equiv 0$ on $[a,b]$ b) $f \equiv 0$ on $[a,b]$ if $f$ is conts

c) $f \equiv 0$ on $[a,b]$ if $f(x) \geq 0$ $\forall x \in [a,b]$

d) $f$ can be non-zero over an interval of +ve length.

23) If $(X,d)$ is a metric space, $A \subseteq X$, and $x \in \bar{A}$ Then

a) $x \notin A^c$ b) $x \in A$

c) $\forall \varepsilon > 0$, $\exists y \in A \ni d(x,y) < \varepsilon$

d) $\exists y \in A \ni d(x,y) > 1$

24) Let $f: \mathbb{R} \to \mathbb{R}$ be conts with $f(0) = f(1) = 0$ which of the following is not possible??

a) $f([0,1]) = \{0\}$ b) $f([0,1]) = [0,1)$

c) $f([0,1]) = [0,1]$ d) $f([0,1]) = [-\frac{1}{2}, \frac{1}{2}]$

25) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{i\sqrt{n}\,x}}{n^2}$ converges

(a) only at $x=0$ b) only for $|x| \leq 1$

(c) Converges pointwise for all $x \in \mathbb{R}$ but not uniformly

d) Converges uniformly on $\mathbb{R}$.

26) Let $f: \mathbb{R} \to \mathbb{R}$ be a $f^n$. If $|f'|$ is bdd, Then

a) $f$ is bdd b) $\lim_{x\to\infty} f(x)$ exists.

c) $f$ is uniformly conts

d) The set $\{x \mid f(x) \neq 0\}$ is closed.

27) Let $f: [0,\infty) \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} \dfrac{x}{1-e^{-x}} & \text{if } x > 0 \\ 0 & x = 0 \end{cases}$$

Then the $f^n$ $f$ is

a) Conts at $x=0$ b) bdd

c) Increasing d) zero for atleast one $x > 0$.

28) Let $f: \mathbb{R} \to \mathbb{R}$ be a twice contly diff'ble fn with $f(0) = f(1) = f'(0) = 0$ Then

a) $f''$ has no zeros in $[0,1]$

b) $f''(x) = 0$ for some $x \in (0,1)$

c) $f''(0)$ is always 0

d) $f''(0)$ is always $\neq$

29. Which of the followings is/are true for a series of real numbers $\Sigma a_n$?

a) If $\Sigma a_n$ converges then $\Sigma a_n^2$ converges

b) If $\Sigma a_n^2$ converges then $\Sigma a_n$ converges

c) If $\Sigma a_n^2$ converges then $\Sigma \frac{1}{n} \cdot a_n$ convgs

d) If $\Sigma |a_n|$ converges then $\Sigma \frac{1}{n} a_n$ converges.

30) Which of the following fns are uniformly conts on $\mathbb{R}$?

a) $f(x) = x$ b) $f(x) = x^2$ c) $f(x) = \sin^2(x)$

d) $f(x) = e^{-|x|}$

31) Which of the followings are true?

a) $\exists f : \mathbb{R} \to \mathbb{R} \ni f(-1) = -1$, $f(1) = 1$ and ~~$|f(x) - f(y)| \le$~~

$|f(x) - f(y)| \le |x-y|^{3/2} \ \forall x, y \in \mathbb{R}$

b) Let $f : [0,1] \to \mathbb{R}$ be conts fn $\ni f(x) \geq x^3 \ \forall x \in [0,1]$

with $\int_0^1 f(x)\,dx = \frac{1}{4}$ Then $f(x) = x^3$, $\forall x \in \mathbb{R}$.

c) Suppose f is ~~conts~~ continuously diffble fn on $\mathbb{R}$

$\ni f(x) \to 1$ and $f'(x) \to b$ as $x \to \infty$ Then $b = 1$.

32) pick out the true statements

a) $|\cos^2(x) - \cos^2(y)| \leq |x-y|, \forall x,y \in \mathbb{R}$

b) If $f: \mathbb{R} \to \mathbb{R}$ satisfies $|f(x) - f(y)| \leq |x-y|^{\sqrt{2}}$ $\forall x,y \in \mathbb{R}$ then $f$ must be a constant fn.

c) let $f: \mathbb{R} \to \mathbb{R}$ be continuously diff'ble and such that $|f'(x)| \leq 4/5$ $\forall x \in \mathbb{R}$. then $\exists x \in \mathbb{R} \ni f(x) = x$.

d) If $f: \mathbb{R} \to \mathbb{R} \ni |f(x) - f(y)| \geq |x-y|$ $\forall x,y \in \mathbb{R}$ then $f$ has atleast one $x \in \mathbb{R} \ni f(x) \neq x$.

33. let $\{x_n\}$ be a seq of real numbers. pick out the cases which imply that the seq is cauchy

a) $|x_n - x_{n+1}| \leq \frac{1}{n}$ $\forall n \in \mathbb{N}$

b) $|x_n - x_{n+1}| \leq 1/n^2$ $\forall n \in \mathbb{N}$

c) $|x_n - x_{n+1}| \leq 1/2^n$ $\forall n \in \mathbb{N}$

d) $|x_n - x_{n+1}| \leq \sin(1/n)$, $\forall n \in \mathbb{N}$.

34. Which of the following fns defined on $\mathbb{R}$ are diffble?

a) $f(x) = x|x|$ b) $f(x) = [x] \sin^2(\pi x)$

c) $f(x) = \sqrt{|x|}$ d) $x[x]$

35) Which of the followings are true?

a) Let $f$ be contly diffble in $[a,b]$ and twice differentiable on $(a,b)$. If $f(a) = f(b)$ and if $f'(a) = 0$ Then $\exists$ $x_0 \in (a,b) \ni$ $f''(x_0) = 0$

b) Let $f$ be ~~consto~~ continuously diffble in $[a,b]$. If $f(a) = f(b)$ and if $f'(a) = f'(b)$ Then $x_1, x_2$ in $(a,b) \ni x_1 \neq x_2$ and such that $f'(x_1) = f'(x_2)$

c) Let $f$ be ~~constly~~ continuously diffble on $[0,2]$ and twice differentiable on $(0,2)$. If $f(0) = 0$, $f(1) = 1$ and $f(2) = 2$ Then $\exists$ $x_0 \in (0,2) \ni f''(x_0) = 0$

d) If $f: \mathbb{R} \to \mathbb{R}$ be diffble fn $\ni$ $f(x) \leq x^3 + x + 1$, $\forall x \in \mathbb{R}$ Then $f$ is increasing ~~on R~~ or decreasing on $\mathbb{R}$.

36. Which of the followings relations are true?

a) $(-1)^{\frac{n(n-1)}{2}} = (-1)^{\frac{n(n+1)}{2}}$

b) $(-1)^{\frac{n(n-1)}{2}} = (-1)^{\left[\frac{n}{2}\right]}$

c) $(-1)^{\frac{n(n-1)}{2}} = (-1)^{n^2}$

d) $(-1)^{\frac{n(n+1)}{2}} = \frac{1}{(-1)^{\frac{n(n+1)}{2}}}$

37. Let $f(x) = e^{\frac{1}{x}}$. In which of the following domains is it uniformly conts?

a) $(0,1)$ b) $(1,\infty)$ c) $(1\,2)$ d) $(0,\infty)$.

38. Let $f(x) = 2x^3 - 9x^2 + 7$ Then

a) $f$ is 1-1 on $[-1,1]$

b) $f$ is 1-1 on $[2\,14]$

c) $f$ is not 1-1 on $[-4,0]$

d) $f$ is not 1-1 on $[0\,14]$.

39) Let $\{a_n\}$ be a seq of +ve real numbers. Suppose that $l = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$. Which of the followings are true?

a) If $l = 1$ then $\lim_{n\to\infty} a_n = 1$

b) If $l = 1$ then $\lim_{n\to\infty} a_n = 0$

c) If $l < 1$ then $\lim_{n\to\infty} a_n = 0$

d) If $l < 1$ ~~$\lim_{n\to\infty}$~~ then $\lim_{n\to\infty} a_n = 1$

40) Let $a, b \in \mathbb{R}$ and $a<b$ Which of the following statements is/are true?

a) $\exists$ a conts fn $f:[a,b] \to (a,b) \ni f$ is 1-1

b) $\exists$ a conts fn $f:[a,b] \to (a,b) \ni f$ is onto

c) $\exists$ a contr fn $f:(a,b) \to [a,b] \ni f$ is 1-1

d) $\exists$ a contr fn $f:(a,b) \to [a,b] \ni f$ is onto.

41) Let $f:[0,1] \to \mathbb{R}$ be a fixed conts fn $\ni f$ is diffble on $(0,1)$ and $f(0)=f(1)=0$

Then the eqn $f(x)=f'(x)$

a) No soln $x \in (0,1)$

b) More than one soln $x \in (0,1)$

c) Exactly one soln $x \in (0,1)$

d) At least one soln $x \in (0,1)$

42) Which of the followings define a metric on $\mathbb{R}$

a) $d(x,y) = \dfrac{||x|-|y||}{1+|x||y|}$

b) $d(x,y) = \sqrt{|x-y|}$

c) $d(x,y) = |f(x)-f(y)|$, where $f:\mathbb{R}\to\mathbb{R}$ is strictly increasing fn

d) $d(x,y) = |e^x - e^y|$.